ESSENTIALS OF PHYSICS

FOR

COLLEGE STUDENTS

A TEXTBOOK FOR UNDERGRADUATES AND A LECTURE COURSE AND REFERENCE WORK FOR TEACHERS AND OTHER STUDENTS OF PHYSICS

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PREFACE TO SECOND EDITION.

In this revised edition, the author has corrected typographical errors and made changes which no doubt will enhance the value of the book. The original purpose has been to present the principles of physics in a reasonably complete form, so selected as to round out the information of a well-educated man who is not aiming to qualify as an engineer or technical expert, but who nevertheless desires to be well informed in this branch of science. Especially has the author endeavored to make the subject of human as well as of technical interest to the student. It is hoped that this may help to make the book attractive to some who may feel that their knowledge of the subject is becoming impaired, as well as to those who are first acquiring their knowledge of it.

In presenting a subject so broad and so varied as physics, no matter what material is selected there will always arise the question whether certain things that are omitted would not better have been included and others that are included might not have better have been omitted. No choice can be made that will meet with universal approval. Quite as important, it seems to the author, is it to present any topic in a way that will give the student a proper conception of it and of its relation to the rest of the science.

To accomplish this without a knowledge of higher mathematics on the part of the reader is a difficult task, and one that is definitely limited in its range for want of such knowledge. Where the author has felt obliged to stop short of a complete exposition, or to omit a rigid demonstration, he has usually opened the way for the teacher or student, by reference to more

extended treatises. These references are of the widest variety, from those wholly popular in character to the most narrowly technical. However excellent it may be in itself, it is only when thus supplemented, at least in some measure, that a general treatise of moderate size can attain its fullest usefulness. Instead of giving numerous separate tables of physical data, one comprehensive view of physical properties of many common substances is appended, in tabular form, and so arranged that each column is a table in itself. This will meet, in large measure, the occasional desire to refer to such tables, and will constitute a valuable addition to the book. The partially classified list of books on page x does not include all that are referred to in the text, yet this list, even if considerably abridged, would provide a student with an admirable library of physics, general and special, elementary and advanced.

D. W. HERING.

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PREFACE TO FIRST EDITION.

THIS work is the outgrowth of a course of lectures which the author has delivered for several years past to undergraduate students in the University College, in the Collegiate Division, and in the Summer School of New York University.

In the provision that has been made for physics in high schools, in universities, and in schools of technology, little regard has been had for that considerable class of students preparing to fill the position of educated men and women who are not specialists in science.

These should have an opportunity to become acquainted with the principles of physics in more than an elementary form, yet without the fulness of detail and the more difficult mathematical demonstration that would be required by the engineer.

Although the material in this volume was prepared in the first place as a course of lectures, it has been arranged in paragraphs and topics to make it suitable also for a textbook of recitations.

As here given the whole can be included in about sixty lectures of fifty minutes each; but with the text as a basis of recitations, the lectures as illustrations, the problems for practice and drill, and with occasional tests and a final review, the course may be much more extended. On the other hand, it has been found flexible enough to make from it a course of half the length by judicious selection.

Sufficient numerical examples are given to illustrate the principles; for more extended problem work, recourse may be had to any of the collections of problems published separately.

No higher mathematics is required than the elements of algebra, geometry, and plane trigonometry; and the course may be taken with profit by an earnest student who has not studied

the subject before, but a knowledge of elementary physics would be a desirable preparation. In some instances details have been purposely omitted, to leave more latitude both to the teacher and to the student.

The experiments suggested for demonstration purposes have been placed at the ends of the chapters, so as not to break the continuity of the text.

In most cases experiments of a simple character have been chosen, so as to serve the lecturer whose cabinet of apparatus is scantily equipped; but any capable teacher will be able to vary those here suggested, or increase their number by using others more to his liking. Elaborately arranged experiments are usually not the kind best suited to the illustration of general physics.

It is hoped that the numerous references throughout the book will be of service, especially to teachers.

The Author tenders his thanks to Dr. Charles Forbes for the cut of the Columbia Wave Machine, and to the *Scientific American*, the Weston Electrical Instrument Co., and others who have generously contributed material for the illustrations.

D. W. HERING.

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CONTENTS.

BOOKS OF REFERENCE.

Among the numerous good works on physics in English the following would be serviceable for the principles of physics (not laboratory manuals).

Advanced:

*Watson's *Textbook of Physics*.
*Hastings and Beach, *General Physics*.
Daniell's *Principles of Physics*.
Barker's *Advanced Physics*.
Crew's *General Physics*.
Ames' *Textbook of General Physics*.
Carhart's *University Physics*.
*Ganot's *Physics*.

Secondary:

Mumper's *Textbook in Physics*.
Hoadley's *Physics* (text and laboratory).
*Millikan and Gale, *First Course in Physics* (text and laboratory).
*Carhart and Chute, *First Principles of Physics*.

Special:

*Cajori, *History of Physics*.
*Maxwell's *Matter and Motion*.
Comstock and Troland, *The Nature of Matter and Electricity*.
*Lodge's *Elementary Mechanics*.
*Edser's *Heat for Advanced Students*.
*Starling's *Electricity and Magnetism*.
Wood's *Physical Optics*.
Schuster's *Theory of Optics*.
*Barton's *Textbook of Sound*.
Smithsonian Institution, *Physical Tables*.
*Kaye and Laby, *Physical and Chemical Constants*.

Questions and Examples:

Snyder and Palmer, *One Thousand Problems in Physics*.
*Shearer, *Notes and Questions in Physics*.
Jones' *Examples in Physics*.
Miller's *Progressive Problems in Physics*.

An excellent treatise, but more advanced than any of the above works on general physics, is a *Textbook of Physics*, in five separate volumes, by Poynting and Thomson.

Hahn's *Freihandversuche*, I, II, Berlin, is a mine of simple experiments illustrating the subject matter of Chapter I, following.

Of periodicals, apart from the standard journals of physics, both American and foreign, much valuable information is to be had from *School Science and Mathematics*, Chicago; and *Die Zeitschrift für den physikalischen und chemischen Unterricht*, Berlin.

* Those marked with an asterisk would constitute a small but good and comprehensive library of physics.

ESSENTIALS OF PHYSICS

CHAPTER I.

PROPERTIES OF MATTER; MECHANICS.

1. Physics: Ways of Viewing the Subject. — Physics is likely to be viewed in one of three ways:

(1) By the student who is anxious to master the subject for its own sake, it is to be studied in detail, nothing in it being too little to be examined, nothing too big to be encountered, no difficulties to be evaded and no simple points to be ignored.

To one who is not thus devoted to the subject this kind of treatment is a task not merely unattractive but actually repellent.

(2) Physics may be studied for the most part as a pure science, but with direct and frequent reference to the ways in which it enters into our daily experience. For this, the fundamental principles must be learned and their relations to one another. The science must be treated systematically, but carried only so far as we please; i.e., it must be correct but not necessarily complete. It is not to be regarded as a mere set of disjointed phenomena, but all conclusions must rest ultimately upon observed facts.

(3) Physics may be and sometimes is regarded simply in its applications, so called, meaning usually its most obvious applications, such as trolley cars, electric lights, an echo, the microscope, etc. This is a superficial and inadequate view, because at almost every point in our experience with the world around us we come in contact with physics, not always obvious, but always there. And, moreover, to attempt to gain a knowledge of physics which we can ourselves apply, a more thorough treatment is needed than is afforded by a casual study of one or two branches of the subject, because in physics, as in any other branch of applied science, it is useless to hope to apply what we do not command. One cannot apply mathematics if he has no mathematics to apply, nor can he apply physics if he has no physics.

Therefore, it is the second of these three heads under which we may hope to make the study of physics both interesting and profitable. The former is comparatively easy to do, and the latter will not be doubtful if questions or discussions here given serve to arouse other inquiries or discussions among the students.

2. Why Study Physics? — Partly because it is a subject well worth knowing for its own sake, though there are many to whom that idea does not appeal. Then, because it affords a kind of training which, it is true, if one has only that, will make him as one-sided as any other exclusive line of thought, but which is disciplinary in character, and which gives him an orderly, logical view of relative values, — a view which rests upon facts instead of mere speculation, and the conclusions from which may be verified or disproved by facts external to the observer. He can hardly become dogmatic. A thing is never so because he says so, but if it is so, no amount of disputing on his part can make it otherwise. Thus it is a good corrective to arrogance, an incentive to humility, and a support to confidence.

If this also is an insufficient reason for giving attention to this particular branch of science, there is still a third which is more constantly in evidence. Physics in some form or other touches human everyday life so continually, that one who is ignorant of this subject is in constant danger of blunders that make him ridiculous if they do not affect him more seriously. It is true that chemistry bears closely upon numerous phases of life and upon industrial processes, but chemical actions are molecular in character, and not so easy to get at as physical; biology deals with questions of life itself, but these questions are very recondite, and can hardly be approached intelligently without a knowledge of many of the principles of physics. The daily experience of everybody is a series of occurrences in physics, and to write or to read (especially to write) daily papers intelligently, or to comment upon the affairs of the day, requires a knowledge of physics.

It is not pleasant to see an otherwise intelligent person make himself ridiculous by ignorance or error in regard to some simple matter of science. It is not as if he blundered in a matter of high art, or of deep philosophy, or of some special professional ability; for all these are special, and one needs not venture into them if he does not choose to do so; but with physics it is not so. He cannot get away from that. So long as the atmosphere is for us "the breath of life;" so long as the rainbow shall delight the eye, or the harmonies of a symphony charm the ear; so long as we shall care to annihilate time and distance by electric signaling, or avoid weariness by improved means of locomotion; so long as civilization shall tend to greater convenience and comfort of living by applying the forces of nature, — so long shall we be under the dominion of physics, and the only alternative by which we can escape from it is to revert to barbarism or else to vanish from the earth.

3. Fundamental Ideas. — Our view of the universe is comprehended in three concepts, viz., *space*, *time*, *matter*. Each has been regarded as a "primary concept," and so not capable of definition in more simple terms.

4. **Space.** — Space might be regarded as the aggregate of all possible answers to the question "Where?" It would thus be the sum total of all places or regions, but as these are no simpler in conception separately than collectively, the idea of space is regarded as a primary concept. The concept itself depends on our consciousness of exploring, in our own persons, regions differently situated *relatively to ourselves*. "We may add to the small region which we can explore by stretching our limbs, the more distant regions which we can reach by walking or by being carried. To these we may add those of which we learn by the reports of others, and those inaccessible regions whose position we ascertain only by a process of calculation, till at last we recognize that every place has a definite position with respect to every other place, whether the one place is accessible from the other or not" (Maxwell, *Matter and Motion*, "On the Idea of Space").

5. **Time.** —Like space, time also is treated by the physicist as a primary concept, admitting of no definition. As to the idea of time, Maxwell suggests that in its most primitive form it is probably *the recognition of an order of sequence in our states of consciousness* (*Matter and Motion*). The following, also from the same source, is adapted from Newton's *Principia*, Scholium to Definition VIII (q.v.): "Absolute, true and mathematical time is conceived by Newton as flowing at a constant rate, unaffected by the speed or slowness of the motions of material things. It is also called *duration*. Relative, apparent and common time in duration is estimated *by the motion of bodies*, as by days, months and years." And again: "As there is nothing to distinguish one portion of time from another except the different events which occur in them, so there is nothing to distinguish one part of space from another except its relation to the place of material bodies. We cannot describe the time of an event except by reference to some other event, or the place of a body except by reference to some other body. All our knowledge, both of time and place, is essentially relative."

Thus we get an idea of intervals of time by associating the succession of events with periods of duration. Now if we can

find that certain kinds of events occur at equal intervals of time, we can get a means of measuring time. But we must first have a means of deciding the equality of intervals. Experience gives us such confidence in Nature's fidelity to herself — in her constancy — as to lead us to expect that upon the repetition of the conditions we shall get like results.

From the earliest period of history, the succession of day and night, and the earth's passage through a cycle of positions relatively to the sun and stars, have been observed. As the earth describes its orbit around the sun, the plane of the meridian through any given place on the earth passes through the sun three hundred and sixty-five times, before the earth has quite reached the position relatively to the sun and stars, from which its orbital motion was reckoned. That is, the earth makes 365 + rotations relatively to the sun in making one revolution about the latter. It makes one more rotation relatively to any of the stars, and it makes just as many with reference to one star as to another, and between any two instants of observation the same portion of a rotation is indicated, no matter to what star it is referred. These sidereal rotations are therefore found to be all equal in duration. Those with reference to the sun, however, are not made in equal periods of time, but the average of all the solar periods in one orbital revolution of the earth is called a *mean solar day*, and is taken as a standard for comparing intervals of time.

Suppose a clock to be constructed whose action is not dependent in any sense upon the position of the earth in its orbit, but only upon the earth itself or the materials of which the clock is made. Now if the revolution of the earth about the sun is repeated in equal periods, and the action of the clock by virtue of any qualities of its material is uniform, then the return of the earth to any initial position in its orbit will always be correctly told by the same number, 365 + days recorded by the clock. If the earth's return is irregular and the clock is regular, or *vice versa*, they will in general not agree. If both are irregular, they could only agree for all points in the orbit by having their irregu-

larities to vary in precisely the same way, — a state of things scarcely conceivable, and the less so when various types of clocks are used. In fact they agree very closely, and the agreement has been closer as the mechanism of clocks has been improved. *Their agreement* is a strong indication of the constancy in the period represented by a day, and in the agencies or in the properties of the materials by which the clocks act. It is justly pointed out by Maxwell, however (*Matter and Motion*), that our fundamental conception of time is not based upon the rotation of the earth, from the fact that we sometimes ask whether the length of the day has changed during the last two or three thousand years. (See Garnett's *Elementary Dynamics*, Chap. I, Arts. 1 and 42.)

Note. — Astronomers have endeavored to answer the question of invariability in the length of a day, with as yet not very accordant results. The conclusion reached by the English astronomer, Mr. Adams, and frequently quoted, is that the day is growing longer by about 22 seconds in a hundred years. This total gives an average of 0.22 second per year for one hundred years, but if the earth's motion is undergoing a regular rate of retardation, then in the period of a century the actual retardation would change from nothing at the beginning to 0.44 second per year at the end, and 0.22 second per year at the middle. Now, as there are $365\frac{1}{4} \times 86{,}400$ seconds in a year, in one second the earth performs $\frac{1}{86{,}400 \times 365\frac{1}{4}}$ part of its motion for the year. If it were retarded by one second per year, its rate of rotation would be diminished by the above fraction of itself, and 0.44 second per year would mean a diminution in rate of rotation by $\frac{44}{100}$ of $\frac{1}{86{,}400 \times 365\frac{1}{4}}$ or $\frac{1}{71.7 \times 10^6}$ as compared with its rate one hundred years ago. But a review of Mr. Adams' astronomical data by Mr. G. H. Darwin corrects his figures to 23.4 seconds in a century, while corresponding data deduced by the late Professor Newcomb give as little as 8.3 seconds, or only a little more than one-third as much as Mr. Adams' result. The astronomers themselves regard the figures as too uncertain to be relied upon with any confidence. (See Thomson and Tait, *Natural Philosophy*, Part II, Art. 830 and Appendix G.) Even taking the largest value given, the rate at which the length of the day would seem by that to be increasing is too small to be perceived in any ordinary series of observations.

6. Matter. — We have used the terms "body" and "matter" without defining them, but without danger, thus far, of any misconstruction. It is sufficient to say of a body that it is "a portion of matter which is limited in every direction" (Garnett, *Elementary Dynamics*).

Matter itself is not so easily defined. Attempts to give an adequate definition, by metaphysicians as well as by physicists, have taken a wide range, from declaring it by the former to be not substantial but only ideal, to resting content with the declaration by the latter that the meaning of matter is a primary concept, and therefore the term is not capable of definition. With the concept once realized as to matter in any form at all, we may and must extend our study of matter to all forms of it, whether they appeal to any of our six or seven special senses, to all of them, or to none.

Note. — For the propriety of considering a temperature sense apart from the sense of touch, see Sir William Thomson's lecture on "The Six Gateways of Knowledge," *Popular Lectures and Addresses*, Vol. I. Also, a so-called muscular sense is sometimes urged as the means of our becoming cognizant of resistance; but this might perhaps be shown to be ultimately the same as the sense of touch, by which we become aware of the presence of matter in many of its forms.

Thus, space involves the idea and determination of the *where* of an event or a thing (and so, of its size); time, of the *when;* and matter, of the *what* and how, and why.

7. Entities of Nature. — External nature makes us aware of only two entities, or things having an objective existence, viz., matter and energy; and the latter we are able to recognize only in association with the former. We are alike ignorant of the ultimate nature of both. Energy may, however, be defined provisionally as "a capability of matter" in virtue of which any definite portion of it may be made to effect changes in other definite portions. (See Barker's *Advanced Physics*, p. 4.)

8. Significance of Dynamics in Physics. — There is a tendency to rest all physical phenomena upon mechanical principles. The late Professor Dolbear writes (*Matter, Ether, Motion*): "As all physical phenomena are reducible to the principles of mechanics, atoms and molecules are subject to them as much as masses of visible magnitude; and it has become apparent that however different one phenomenon is from another, the factors of both are the same, . . . matter, ether and motion." It is best, perhaps, not to be too sweeping in our generalizations. A later pronouncement upon this subject (*Elements of Mechanics*, Franklin and Macnutt), in a section on "The Science of Physics," has this to say: "But what is physics? That is the question. One definition at least we must repudiate: it is not 'the

science of masses, molecules, and the ether.' Bodies have mass and railroads have length, and to speak of physics as 'the science of masses' is as silly as to define railroading as the 'practice of lengths,' and nothing as reasonable as this can be said in favor of the conception of physics as the science of molecules and the ether; it is the sickliest possible notion of physics even if a student really gets it, whereas the healthiest notion, even if a student does not wholly grasp it, is that physics is the science of the ways of taking hold of things and pushing them!" But if this is a fact, we must not forget that the mere statement of a truth about anything is not the same as giving a definition of it.

In the first chapter of the excellent manual, Glazebrook and Shaw's *Practical Physics*, the authors say: "We cannot then do better than urge those who intend making physical experiments to *begin* by obtaining a sound knowledge of those principles of dynamics which are included in an elementary account of the science of matter and motion. For us it will be sufficient to refer to Maxwell's work on *Matter and Motion* as the model of what an introduction to the study of physics should be."

So fully is all this now recognized as to make it the basis of the distinction between physics and chemistry, so far as a distinction is required. Without making such a distinction we have Maxwell's statement (*Matter and Motion*, Chap. VI): "The discussion of the various forms of energy,—gravitational, electromagnetic, molecular, thermal, etc.,—with the conditions of transference of energy from one form to another, and the constant dissipation of the energy available for producing work, constitutes the whole of physical science in so far as it has been developed in the dynamical form, under the various designations of Astronomy, Electricity, Magnetism, Optics, Theory of the Physical States of Bodies, Thermodynamics and Chemistry." Daniell's *Physics* opens with the statement, "Natural philosophy, or physics, may be briefly defined as the science of matter and energy." The author, however, recognizing that this necessarily includes chemistry and biology, proceeds to the closer definition of physics as "the systematic exposition of the phenomena and properties of matter and energy in so far as these phenomena can be stated in terms of definite measurement and explained by

reference to mechanical principles or laws," thus placing physics more upon the basis of mechanics.

The distinction between physics and chemistry is further elaborated by both Professor Daniell and Professor Barker through a further extension of this idea. That is, chemistry is regarded as the science of matter as to its forms and transformations, while physics is the science of energy as to its forms and transformations. The former has for its fundamental principle the conservation of matter, the latter the conservation of energy. The two overlap in this wise: *If changes in matter* are studied with especial reference to the phenomena of energy involved, it is chemistry, but it is specified as physical chemistry; *if changes of energy* are studied with especial reference to the kinds of matter concerned, it is physics, but is specified as chemical physics.

The intimate association of matter and energy is further seen in the fact that no change in matter can be effected without a simultaneous energy-change of some form, so that every chemical change necessarily involves physical changes; but the converse of this is not true.

9. **Definition of Physics.** — Evidently, therefore, physics regards matter solely as the vehicle of energy. And hence, from this point of view, physics may be defined as that department of science whose province it is to investigate all those phenomena of nature which depend either upon the transference of energy from one portion of matter to another, or upon its transformation into any of the forms which it is capable of assuming. In a word, physics may be regarded as the *science of energy*, precisely as chemistry may be regarded as the *science of matter*. (Barker, pp. 5 and 6.) Physics is not, however, to be regarded as simply a number of detached phenomena.

10. **Science and Measurement.** — It has been aptly said that "science is measurement," for although intelligent ideas as to the kinds of relations subsisting between supposed causes and effects are often important and sometimes necessary to further investigation, in advance of the exact statement of natural laws, yet knowledge cannot be called truly scientific until those

relations can be expressed in numbers. To this end it is necessary to be able to measure each thing, whether cause or effect, so as to compare the measurements. Measurement always involves the idea of difference; either a difference between two things, or between two states of the same thing. Physics as an exact experimental science depends upon the detection and measurement of change, either as to the amount of change or rate of change.

"When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of *science*, whatever the matter may be" (Sir William Thomson, *Popular Lectures and Addresses*, Vol. I, p. 80).

11. **Units.** — For measurement are required units, since a unit is that physical magnitude that is applied to another magnitude of the same sort to determine its size. All the interactions that occur between bodies can be ultimately expressed in terms of three fundamental magnitudes as factors, involved to various degrees. When measurements are expressed in such terms they are called *absolute measurements*, and a system of units of various kinds, each of which is defined in terms of the three fundamental quantities, is called an absolute system of units. The absolute system in common use in science is known as the centimeter-gram-second (c.g.s.) system, and is based upon the centimeter as the unit of length, the gram as the unit of mass, and the second as the unit of time. (See Glazebrook and Shaw, *Practical Physics*, pp. 17 to 24.)

12. **Physical Law.** — Our confidence in the constancy of Nature, which constancy is the basis of the laws of Nature, is sometimes expressed by the statement that "the same causes will always produce the same effects." This expresses a physical impossibility, since no event ever occurs more than once, and Maxwell explains the meaning of the statement to be that "if the causes differ only as regards absolute time or the absolute place at which the event occurs, so likewise will the effects," and submits the following as a more explicit statement of the prin-

ciple: "The difference between one event and another does not depend on the mere difference of the time or the places at which they occur, but only on differences of the nature, configuration, or motion of the bodies concerned" (*Matter and Motion*, p. 31). The relation which any event bears to the "nature, configuration, or motion of the bodies concerned," or the relation between the above-named "differences," is what is meant by a *physical law*.

13. **Nature and Properties of Matter.** — A definition which should embody a statement of the essential character of matter has often been attempted by metaphysicians as well as by physicists. Tait, *Properties of Matter*, pp. 12 and 13, and Appendix I, gives numerous such definitions and hypotheses (q.v.), but says for himself that "an exact or adequate conception of matter itself, could we obtain it, would almost certainly be something extremely unlike any conception of it which our senses and our reason will ever enable us to form."

It is probable that there are some, and possible that there are many, forms of matter of which we are not cognizant because we have no especial sense organ to appreciate them, or have not yet recognized the conditions favorable to making them appreciable by the organs which we have.

It is not, however, always necessary to have a special sense organ to perceive a special form of matter. We may be assured of the existence of the matter, by knowing that it performs functions which are exclusively those of matter, and we may learn something of its attributes by the manner in which it performs those functions. (See Maxwell, *Matter and Motion*, "Test of a Material Substance," pp. 165, 166.) Vortex motion in a hypothetical perfect fluid gives to the portions in motion distinct properties which may serve so to distinguish these portions from other portions not in such motion as to constitute what we call matter, that is, to make a distinction between a material substance and a nonmaterial substance. (See Holman's *Matter, Energy, Force and Work*, Chap. IV. Read from Watson's *Physics*, last half of Art. 124, "The Constitution of Matter.")

Experiment No. 1, p. 85. — Vortex Rings. To form the rings, place side by side in the box a vessel containing a little hydrochloric acid, preferably warmed (or else a little common salt on which is poured sulphuric acid), and one containing strong ammonia. Remember that the ammonium salt thus formed *has nothing to do with the rings except to make them visible.*

14. **Forms of Matter.** — Although the ultimate nature of matter is unknown, its structure and the forms under which we encounter it are to a considerable extent familiar. Possibly there is but one primitive and primary form, of which all others are modifications; but matter as we know it presents itself under five so-called forms or states (sometimes called states of aggregation), viz., solid, liquid, gaseous, ultragaseous, and ethereal, and in each, except perhaps the last, there are various kinds of matter differing in certain distinctive characteristics or "properties." But each kind of matter is *continuous* at least throughout the range of the first-named four states of aggregation.

Paying no heed for the present to any question of molecules and their relative motions, or freedom to move, we may recognize purely physical distinction of solids, liquids and gases.

The first division of matter as to form is into solid and fluid; the latter, again, into liquid and gaseous forms. "A body that will sustain a longitudinal pressure, no matter how slight, without lateral support, is a solid; one that will not thus sustain pressure is a fluid." Also, "A liquid is a fluid with which a vessel may be *partly* filled; a gas will wholly fill a vessel though but a small quantity be introduced" (Maxwell). These conceptions are sufficient for all dynamical considerations of matter. Under them fluids may be harder than solids.

Illustrations. — A rod of tallow will sustain a small longitudinal pressure without giving way or needing lateral support. It is thus a soft solid. Pitch, though hard, will yield under the slightest pressure, in time, unless supported laterally. It is a hard fluid. Its mobility is increased (or viscosity diminished) by addition of turpentine.

Most gases are invisible, but are usually perceptible by the sense of smell. They can contain and transmit energy, as shown by steam or air under pressure.

Ultragaseous matter is shown by Crookes' tubes. A comparison of the electric discharge through Crookes' tubes and Geissler's tubes shows difference not in the matter itself but in the states of aggregation. The same is shown by the long Crookes' tube containing caustic potash. This when warmed reduces the vacuum and the matter changes from ultragaseous to gaseous. But ultragaseous form is vehicle of energy, as shown by radiometer, mill-wheel tube, etc. (Lecturer exhibit.)

Ether is not perceptible to the senses, but is a vehicle for the transmission of energy, especially heat and light, or radiation. Radiant energy would pass across a space that is a vacuum, or that contains quiet material like air or water at rest, or that has matter like air or water sweeping through it in either direction; so that ether is the vehicle of energy apart from the other matter traversed. If that matter obstructs or checks the radiation, then it acquires the energy of which the ether separately was possessed. Example: The sun's energy radiated to the earth.

15. **Stress.** — Actions between different portions of matter, so far as we know, always produce motion or a change of motion in bodies as a whole, though the body be but a molecule or an atom; or else they produce distortion, which is a movement of some parts of a body relatively to the other parts. In the case of molecular motions, they may be interpreted to our consciousness as sound, heat, light, or other physical phenomena; but motion or deformation of extended bodies we appreciate physiologically in the first instance by a sense of pulling or pushing, — a sense of exertion sometimes called our muscular sense. No body has ever been known to move itself or change its motion in any way, but only to undergo such effect when under the influence of some other portion of matter and in turn exerting a corresponding influence upon it. And, further, no body has ever been known to fail in moving under the influence of any portion of matter if free from all other external influences of a restraining nature.

The mutual action between two portions of matter is called *stress*, and may exist between different bodies or between different portions of the same body.

The tendency of a body to fall to the earth, or of the earth to fall to the sun, is ascribed to an action of the earth and the body, or of the sun and the earth, upon each other called the attraction of gravitation; and although the nature of this action is not understood, it is believed to take place between them in some way by stress through the medium of the universal ether.

The stress subsisting between the molecules of any given substance is called an attraction of cohesion, which may possibly be of the same nature as gravitation. (See Thomson's *Popular Lectures and Addresses*, Vol. I, lecture on "Capillary Attraction," Appendix B.)

The mutual actions of magnetized bodies or of electrified bodies upon one another are called attractions or repulsions as the bodies are influenced to approach or to separate from one another, and are also supposed to indicate a stress in the ether as the medium connecting the bodies.

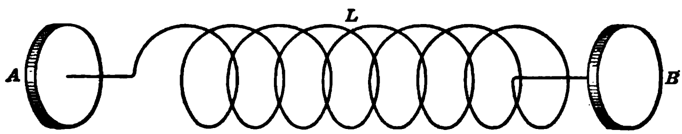


Fig. 1. Stress in Coiled Spring.

16. Force. — Stress viewed as affecting one of the bodies or one of the parts between which the stress exists is *force*. In Fig. 1, A and B are bodies between which is the spring L in a state of tension or compression. A and B may be the hands of the experimenter. They are each subjected to a force as well as L. The force on A or that on B is one aspect of the stress existing in the spring connecting the two bodies. It must be remembered that a stress of any kind is always dual in character. Two bodies or two parts of one body are always concerned in it, and if one pushes or pulls upon the other, so the other pushes or pulls equally upon the one. The stress, viewed in its relation to one body, is frequently called the *action upon* that body, while the reciprocal relation is called the *reaction by* the same body or

upon the other body. Action and reaction always exist together, being two aspects of one and the same stress, and are evidently equal. Action and reaction occur between bodies.

Instances are abundant and varied. A book supported on the hand exerts pressure, but the hand reacts upon the book to hold it up, or if it is descending, to hold it by just so much as its slow fall is attended by pressure on the hand. One cannot push open a door without pushing back upon the floor; he cannot press in the piston of an air chamber without a corresponding pressure from the piston upon the hand. The recoil of a gun when discharged is due to the reaction corresponding to the action of the powder on the bullet. In the case of bodies not contiguous, as magnets or electrified bodies which exert attraction or repulsion, the action is mutual; and so with astronomical bodies. If the earth attracts an apple, equally the apple attracts the earth. (Perhaps neither "attracts," but whatever the action between them, it is considered as mutual.) In all cases the action and reaction between the bodies are equal and opposite; no matter what the nature of the bodies acting upon one another, or what the medium connecting them and through which the action is transmitted, the stress acting upon either body is *force*, and so far as we can tell is of only one sort: *we do not know of different kinds of force.*

Stresses themselves, and therefore force as one aspect of a stress, are consequences rather than causes; but as the effect upon either body concerned, so far as regards motion or distortion, is always commensurate with the stress that is developed, instead of saying that the action of other bodies upon the one under consideration has caused the ensuing motion or distortion, it is customary to say the motion or distortion is *in obedience to the force* or is caused by it. In this way the force is regarded as the agent and its magnitude is determined by the effect upon the body. Forces are then said *to act upon bodies* with various intensities, at different points and in any direction; and thus conventionally all phenomena of mechanics have been referred to the fictitious agency of forces.

The convention is convenient enough to justify its retention, even when we recognize its fictitious character; so the misuse of terms serves to avoid tiresome circumlocution, and is general, but only excusable on the ground of convenience. (See further on this subject, W. K. Clifford in *Nature* for June 10, 1880; also Soddy, *Matter and Energy*, pp. 105–108.)

Where it is practicable we shall generally prefer to regard forces as exerted upon or applied to bodies instead of acting upon them.

Since the only consequence of the application of force is a change in a body with respect to its motion if it is free to move, or with respect to its size or shape or both if it is not free, the change of the body in either of these respects might be made a means of comparing or measuring forces. We should reach the same value for the same force by whatever method we employ to measure it, and therefore either of these two will answer, provided they always agree with each other. Generally the one depending upon the motion of bodies is preferred. We need only in the outset so far assume the permanence of the properties of matter as to treat equal moderate distortions of a spring as equal manifestations of force; also to assume that, other conditions being the same, the mass of different portions of homogeneous material is proportional to their volume.

17. First Law of Motion. — *Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled by impressed force to change that state.*

This law is also known as the law of inertia, since it states that no body alters its state of rest or motion without the intervention of some outside influence; and this fact we express in scientific language by saying tersely that "matter has inertia."

Inertia, then, is a general property of matter with the exception, possibly, of ether, which is itself a limiting form of matter. Inertia is implied in the first law of motion. As we shall see, its character is that of a *time function* of matter rather than a quantitative property of matter. But because some bodies require a greater force than others to bring them to a given motion very quickly, they are said to possess greater (or more) inertia, and

some writers (as Thomson and Tait, construing Newton) go to the length of saying that "matter possesses an inherent power of resisting" the agent that is brought to act upon it.

Note. — This is much like the so-called electrical *resistance* which a substance is supposed to possess because it does not perfectly conduct; and matter, in like manner, can be said to resist motion only because it does not facilitate it.

In this view inertia is sometimes treated quantitatively, and masses are compared by their inertias, the inertia itself being determined by the force necessary to overcome it *in any stated measure*. But the only measure possible is the velocity acquired, and this in its turn is dependent upon the length of time during which the body is acted upon. With all this understood, it is possible to build up a consistent system of masses, forces, and motions on inertia as a basis; but where mass is of chief consequence in ultimate results, it is also possible and more convenient to regard inertia *simply as a fact* concerning matter rather than as a measurable property of matter, and then measure masses by a direct comparison as to forces and resulting accelerations.

This seems the more reasonable because matter *does not resist*, it yields. It is not as if a body refused absolutely to yield or to move until the force applied to it reached a certain magnitude, and then it suddenly started off at a definite speed. The slightest force is able to move (overcome the inertia of) the greatest mass, and we prefer to say, then, that the rate at which the motion is changed is dependent not upon the inertia but upon the quantity of matter. In our view masses are the things considered, masses having already been defined, and may be measured without regarding inertia quantitatively if the latter is only viewed as a fact common alike to all matter.

To say that a large amount of matter has more inertia than a small amount is like saying that in a silent house a large room has more silence than a small one, or that in a dark hall one portion has more darkness than another portion, or that a dead elephant has more death than a dead mouse. Even if it does require more light to illuminate a large room than a small one to the same degree, when the least amount of light is admitted into the

largest of rooms some illumination is effected and it is no longer dark; and so, too, when the slightest force is applied to the greatest mass it moves it.

The decisive point, and that which brings both modes of treatment to the same result, is that a definite length of time is necessary to bring about a definite change of motion. The body which in one statement has the slightest finite amount of inertia, or in the other the least finite mass, would require an infinite force to give it a finite velocity *instantaneously*. This gradual bringing up of a body to a given speed is expressed by Professor Lodge as "reluctance of matter to change its state," but this is to intimate that matter will not rather than cannot so change. Perhaps the former is true; the latter is in accord with ordinary views. The fact is simply that matter *does* not change its state of itself.

The fact of inertia, then, is expressed in the statement that a finite force gives to a finite mass a finite velocity *only in a finite time;* if time is infinitesimal, mass must be infinitesimal or force must be infinite. If time is infinite, mass may be infinite or force infinitesimal.

Examples of inertia: Pulverizing small masses by sharp blows in mid-air, destruction from collision unless buffers, air cushions or springs are interposed, etc.

Experiment No. 2, p. 85.—The breaking of a cord above or below a weight suspended from it, to show, first, the fact of inertia, and next, the part time plays in it.

Experiment No. 3, p. 86.—Ball or coin on a card. When card is slowly moved it carries the object along; suddenly flipped, it leaves the object undisturbed.

Experiment No. 4, p. 86.—Rigidity of chain on pulley to show both inertia and the fact that motion confers upon the chain properties it did not possess when at rest.

18. How We Know the Earth Turns Round.—Inertia has been employed to demonstrate the rotation of the earth upon its axis. If the earth rotates, the top of a tower or mast moves faster than the bottom. The dome of the Pantheon in Paris is 272 feet above the floor; the latitude is 48° +, and for the earth to rotate in 24 hours the top *T*, Fig. 2, moves to the east faster than the bottom *P* moves, 0.18″ per second. A body requires 4.1 seconds to fall from the top of the dome to the floor. Theoreti-

cally it should strike 0.75″ to east of point under plummet. By repeated experiments a ball thus dropped always fell to the east from ½″ to 1″.

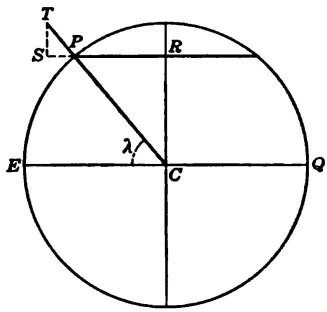


Fig. 2. Proof of the Earth's Rotation.

This experiment may be interpreted either way, for if we regard the rotation of the earth sufficiently well proved, we may regard this as confirming the principle of inertia. (For fuller statements on this particular subject see Hall in *Physical Review* for September, 1903; and also Cajori, *History of Physics*.)

Foucault's Pendulum Experiment.—The most celebrated demonstration of the rotation of the earth is by a method devised by Léon Foucault. A pendulum set oscillating in a given plane will, owing to the principle of inertia, continue to oscillate in the same plane no matter how its point of suspension may be turned.

Let A and B (Fig. 3) be two positions of a dial fixed in a horizontal plane, i.e., tangent to a meridian. Suppose a pendulum to be set oscillating in the meridian plane AP and to trace a mark across the dial beneath it. As the absolute direction of oscillation will not be altered, when the dial moves to B the trace first made will now be in the line BP, but if the pendulum now makes a trace it will be in the line BS, parallel to its original direction of swinging in space, or at an angle with the trace BP equal to the angle APB or α.

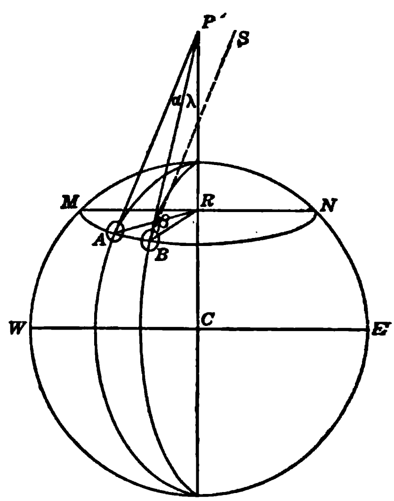


Fig. 3. Demonstration of the Principle of Foucault's Pendulum.

Draw AR and BR in the plane perpendicular to the axis CP; call $AR = BR = r$; and $AP = BP = p$; angle $ARB = \beta$. $\alpha = \frac{\text{arc } AB}{p}$; $\beta = \frac{\text{arc } AB}{r}$; dividing, $\frac{\alpha}{\beta} = \frac{r}{p}$. Angle BPR = latitude, λ, and $\frac{r}{p} = \sin \lambda$; therefore, $\alpha = \beta \sin \lambda$. For one hour of rotation β should equal 15°; therefore, the hourly rotation of the dial, or the angle α, should equal $15^\circ \times \sin \lambda$. Results of carefully conducted experiments agree closely with this. At New York, latitude $40^\circ\ 50'$, the hourly angle should be $9^\circ\ 48'$.

Experiment No. 5, p. 86. Foucault's Experiment.

EXAMPLES. —

1. At New York University, latitude $40^\circ\ 52'$, the angle described by a Foucault pendulum in fifty minutes was $8^\circ\ 20'$; what does this give as the period of the earth's rotation? *Ans.* 23 hrs. 33 min.

2. If the earth rotates in 24 hours, what is the hourly angle described by the pendulum at the poles? What at the equator?

19. Is There Such a Thing as "Force of Inertia?" — No, if we mean a definite force appertaining to a definite body, a resistance which must be overcome before the body's state of motion can be altered; for *any* force will change the motion (overcome the resistance) of *any* body, and, on the other hand, we know of no instance in which a body's motion was changed by its inertia. The force employed in overcoming the inertia of a given body depends upon the time allowed for producing a definite change in the rate of motion. If inertia were a resistance, then with a given inertia no change of motion would occur until the applied force reached a definite amount, as in the case of friction, where the force of friction increases with the increase in the applied force until the latter is sufficient to overcome the maximum value of the former.

20. Second Law of Motion. — While the first law cites inertia in a negative manner, the second law presents the other side and shows the responsiveness of matter to the application of force. It says:

Change of motion is proportional to the impressed force, and takes place in the direction in which the force acts.

This law also is an expression of the behavior of bodies due to inertia, but the quantitative relation of force and motion is determined by the quantity of matter in the body and the time rate of change in its motion. Observe, however, that this law declares not a reluctance but a willingness of matter to yield to the application of force; not a refusal but a compliance; not an opposition but an obedience. No instance is known of the refusal of a free body to obey the application of a force in changing its state of rest or motion.

This law gives us to understand that if several forces are applied to a body simultaneously the body follows the law with regard to each force, finally reaching a place or a condition to which it might have been brought by one single force. Such single force is called *the resultant* of the given forces. The forces may be represented by vectors, and the resultant obtained by vectorial addition, or by geometrical construction thus:

Let two forces, applied at a point, be represented in magnitude and direction by two straight lines drawn to scale; their resultant, i.e., a single force that would accomplish just what these two do, would be represented by the diagonal of the parallelogram of which the lines representing the given forces are adjacent sides. This process of uniting two forces into one is called the *composition of forces*, and is not restricted to two, but may be extended to any number of forces. A body to which such forces are applied will be held in equilibrium by a single force that is applied at the same point, and which is equal to their resultant in magnitude and opposite to it in direction.

From this it is easy to see that any force may be replaced by two others that are applied at the same point as the given force, and are represented by two sides of a parallelogram if the given force is represented by the diagonal. This process of separating one force into two others, styled components, is called the *resolution of forces*, and may be extended until the given force has been resolved into any number of components. The commonest method is to resolve the force into two compo-

nents at right angles to each other. These are rectangular components, and the parallelogram then becomes a rectangle.

By velocity is meant the rate at which space is traversed; and the average velocity v is the ratio of the distance s to the time t occupied in traversing the distance; or, $v = \frac{s}{t}$. The velocity of a moving particle at any instant is measured by the distance that would be traversed by the particle in a unit of time (say one second), if the particle moved on for the one second without going faster or slower.

An exact idea of velocity includes direction of motion as well as distance and time, the term "speed" being used for the rate of travel when direction is disregarded; but the distinction is not adhered to rigidly. A body may move at a given rate or with a given speed, but the phrase "rate of speed" is meaningless and should be avoided.

If the velocity of a moving body is changed its motion is said to be accelerated, and the measure of the acceleration is the rate at which the velocity is altered. If the *increase or decrease* of velocity in the time t is v, the acceleration is $\frac{v}{t}$. In general, since acceleration is the velocity gained or lost per second, if a body starts from rest the velocity at the end of one second will equal the acceleration, at the end of two seconds twice the acceleration, and after t secs. t times the acceleration; i.e., if acceleration is f and velocity acquired is v, then $v = ft$. If the velocity acquired in t secs. from nothing amounts to v, then the average velocity for these t secs. is $\frac{v}{2}$, and the distance traveled is the average velocity $\times$ time, or, under uniform acceleration, distance $s = \frac{1}{2}ft \times t = \frac{1}{2}ft^2$. If the acceleration is that of gravity g, and space is height h, $h = \frac{1}{2}gt^2$.

EXAMPLES. —

1. An aëronaut flies from Paris to Rheims, a distance of 160 km., in $2\frac{2}{3}$ hours. What is his average speed? *Ans.* 60 km. per hr.

2. A body travels 900 meters in $1\frac{1}{4}$ hours. Show that its average velocity is 20 cm. per second.

3. A body has a velocity of 50 cm. per second; 6 seconds later its velocity is 290 cm. per second. By how much was the velocity changed?

Ans. 240 cm. per second.

4. Using the term "acceleration" to indicate the rate at which velocity is changed, what was the acceleration in Ex. 3?

Ans. 40 cm. per sec. per second.

5. A body has a velocity of 50 cm. per second forward; 8 seconds later it has a backward velocity of 190 cm. per second. What is its acceleration?

Ans. -30 cm./sec.^{2}.

Motion as mentioned in the second law is to be understood as Newton explained it, viz., the combined measure of mass and velocity, i.e., their product, which we call momentum; and it is the change of this in a given time that is proportional to the force that is acting during that time; or, the force itself is proportional to the *rate of change in momentum.*

Momentum is the *effect on the body*, due to the force; the action of the latter is measured by the force taken in conjunction with the time it is in effect; i.e., by their product, which is called the *impulse.*

Impulse expresses the *action that produces the effect.* The second law of motion declares the equality of impulse and the momentum it produces. If velocity is changed from v_0 to v_1 and the mass is M, momentum is changed by $M(v_1 - v_0)$, and if this was effected by a force F in the time t, we have the equation $Ft = M(v_1 - v_0)$. If the body starts from rest and is brought to the velocity v, or starts with the velocity v and is brought to rest, the change of momentum is Mv; and

$$Ft = Mv.$$

Hence
$$\text{force} = \frac{\text{change of momentum}}{\text{time}} \tag{A}$$
$$= \text{time rate of change of momentum.}$$

Again, the equation might be written,

$$F = M\frac{v}{t};$$

but $\frac{v}{t}$ is rate of change in velocity and is called acceleration, hence

$$\text{force} = \text{mass} \times \text{acceleration.} \tag{B}$$

21. Measure of Force. — (A) and (B), Art. 20, show at once how to measure force and what is a unit force. From (A), force may be determined at any instant by the rate at which momentum is then being altered; or if the change of momentum, either gaining or losing, is going on uniformly for a length of time, the average force is equal to the entire change of momentum divided by the time during which the change is effected. (B) gives the same result by multiplying the mass by the rate at which velocity is being altered; or in prolonged uniform change of velocity, by dividing the entire change of velocity by the entire time, and multiplying this quotient by the mass moved. The result thus determined will be the number of units of force only under the definition of unit force as the force which will give to unit mass unit velocity in unit time. It presupposes the definition of unit mass, unit distance, and unit time. In common usage there is but one unit time, — the second; there are two standard units of mass in use, — the English, called the pound, and the French, the gram; and there are two units of length, — the English foot and the French centimeter. In the English units, unit force is such force as will give one pound a velocity of one foot per second in one second; it is called a poundal. In the French units, unit force is such a force as will give to one gram of matter a velocity of one centimeter per second in one second; it is called a dyne. These are so-called "absolute units" of mass, length, time, velocity and force. Another set, known as gravitation units, is presented in a table on p. 32, Art. 24. Here we may simply say that the *weight* of a pound is about 32.2 poundals, and the *weight* of a gram about 980 dynes.

Note. — If a moving body is stopped by impact with another body, the question what is the force of the blow delivered is too indefinite to admit of an answer. It depends upon the time required to stop the body, i.e., to change its momentum from something to nothing; or else upon the distance passed over in bringing the body to rest.

Some interesting examples may be drawn from common experience.

(*a*) What additional pressure is exerted upon the surface of the earth (or a roof), by falling rain?

On the afternoon of July 5, 1901, New York was visited by a phenomenal downpour of rain. During the storm there was one period of five minutes in which the rainfall amounted to 0.38 inch. To compute the effect on one square foot of surface, this depth of 0.38 inch would have a mass of 2 pounds, or $m = 2$. If we assume that the average velocity with which the drops strike the earth is 100 feet per second, or $v = 100$, the momentum mv, destroyed in five minutes (or 300 seconds) by one square foot of surface, is $2 \times 100 = 200$ units. The average force, then, $F = \frac{200}{5 \times 60} = 0.67$ poundal per square foot. On a building 50 feet by 100 feet this would make 3350 poundals, or a weight of a little over 100 pounds. On an acre the force would be 29,040 poundals, or 902 pounds — nearly half a ton in weight.

When we come to consider the pressure of a gas according to the Kinetic Theory, we shall have recourse to precisely the same form of computation.

(*b*) The principle that force equals the rate of change of momentum serves to show that the force with which the wind presses against a surface varies as the square of the velocity of the wind; for doubling the velocity doubles the mass of air that impinges in a unit of time, and as the mass has double velocity the momentum destroyed per unit of time is $2\,m \times 2\,v = 4\,mv$, or four times the original momentum. Similarly, if velocity is trebled, momentum equals $9\,mv$; quadrupled, $16\,mv$; and so on.

(*c*) If a car have a given velocity and at the beginning of each of 60 successive seconds a man weighing 150 pounds steps aboard, how much greater force must the motor supply that the speed may not be diminished by this steadily increasing load?

If v = velocity (say 10 ft. per sec.), and $m = 150$, the momentum to be added each second is $mv = 1500$; and since $Ft = mv$, $F \times 1 = 1500$, whence $F = 1500$ poundals, or 50 pounds, nearly. This does not mean that the additional force is to be applied every time a person gets on, but that if such force were added

without the accession of passengers there would be an increase of speed; and against this the calculated rate of increase in momentum would just equalize the increased force and keep the speed constant.

(*d*) From Watson's *Physics*, p. 881: "A small sphere of mass one milligram travels backwards and forwards between two parallel planes with a constant speed of 1000 cm./sec. If the distance between the planes is 5 cm., find the force which must be applied to the planes to keep them from moving under the influence of the impacts. Obtain this force (1) supposing the diameter of the sphere to be negligible, and (2) when the diameter of the sphere is 2 mm."

m = 0.001 gram; distance traveled from one impact to the next against the same plane = 10 cm.; time to go this distance = $\frac{10}{1000}$ = 0.01 sec.; *change* of velocity on impact = 2000 cm./sec.; change of momentum = mv = 0.001 × 2000 = 2 units; as this occurs in 0.01 sec. the change of momentum per second is $\frac{2}{0.01}$ = 200; this is the measure of the force against the plane, which is therefore 200 dynes.

If the diameter of the sphere is 0.2 cm., the distance traveled between impacts is 2 × 4.8, or 9.6 cm.; the time for this is $\frac{9.6}{1000}$, or 0.0096 sec.; the change of mv per second is $\frac{2}{0.0096}$, or 208.3, and the force is 208.3 dynes.

22. Comparison of Forces and of Masses. — From the second law we see that if a force be applied to various bodies, by a fixed distortion of a given spring, their masses will be proportional to the accelerations they receive, and equal masses will be those which receive equal acceleration under the same or equal forces. If two masses whose equality has thus been established be subjected to various forces, these forces will be proportional to the accelerations they produce; and equal forces will be those which give to the same mass or to equal masses equal accelerations.

23. Third Law of Motion. — *To every action there is always an equal and contrary reaction; or the mutual actions of any two bodies are always equal and oppositely directed.*

This has already been considered in the treatment of stress, and might as well have been stated first as third.

Note. — In the fuller consideration of what Newton meant by the terms "action" and "reaction," it appears that he included much of what is now treated under the equivalence of work and energy.

For illustration of second and third laws of motion, etc.:

Experiment No. 6, p. 88, shows that a body obeys each force impressed upon it as if others were not also impressed on it, by the fact that two balls dropped from a given point reach the floor at the same time, though one has a *horizontal* impulse and the other has not.

Experiment No. 7, p. 88, using *spring balance* for forces, shows resultant of two forces, or three forces in equilibrium.

EXAMPLES. — (For Examples 1 to 7 refer to Arts. 20 and 21.)

1. A force of 980 dynes acts upon a mass of one gram for one second. What velocity does it generate? *Ans.* 980 cm. per sec.

2. A force of 1,000,000 dynes acts upon a body for 20 seconds and gives it a velocity of a meter per second. What is the mass of the body? *Ans.* 200,000 grams.

3. How long must a constant force of 150 dynes act upon a kilogram to generate in it a velocity of 30 cm. per second? *Ans.* 3 min. 20 sec.

4. What force acting upon a mass of 120 grams for one minute will produce a velocity of 35 cm. per second? *Ans.* 70 dynes.

5. A jet of water impinges perpendicularly against a wall with a velocity of 20 meters per second. If 50 kg. of water strike the wall each second, what pressure will be exerted against the wall (*a*) if the water does not rebound; (*b*) if it rebounds with a velocity of 5 meters per second? *Ans.* (*a*) 10^8 dynes; (*b*) 1.25×10^8.

6. A mass of 50 grams is moving with a velocity of 90 cm. per second. After a certain force has acted upon it for 5 seconds, its velocity is 250 cm. per second. How great is the force? *Ans.* 1600 dynes.

7. A mass of 49 grams moving at the rate of 20 meters per second is opposed by a force of 980 dynes. In what length of time will this force bring the body to rest? *Ans.* 1 min. 40 sec.

8. A ball weighing 500 grams falls upon a steel slab with a velocity of 500 cm. per second and rebounds with a velocity of 400 cm. per second. What is the average force between the ball and the slab if the impact occupies $\frac{1}{100}$ sec.? *Ans.* 45 million dynes.

9. A spring stretched to a certain elongation pulls upon a mass of 1000 grams, free to move, and in 2 seconds gives it a velocity of 50 cm. per second; the same force gives to another body in 5 seconds a velocity of 200 cm. per second. What is the mass of the second body? (Art. 22.)

Ans. 625 grams.

10. Two forces F_1 and F_2, of 3000 dynes and 4000 dynes respectively, at right angles, both in the plane of the paper, are applied to a particle. Construct a diagram to scale, and determine from the diagram, and also by calculation, the magnitude and direction of a third force that will hold the particle in equilibrium. (Experiment No. 7.)

Ans. 5000 dynes, making an angle with F_1 of $126° 52'$.

11. In Fig. 33*b*, what is the pull on the horizontal chain, and the thrust on the oblique strut, to support a suspended weight of 800 grams, if the horizontal chain (and balance) is half as long as the oblique strut? Solve by drawing and numerically.

Ans. Horizontal, 462 grams; oblique, 924 grams.

24. Work and Energy. — (For this entire subject refer to Barker's *Physics*, Chap. III; or to Maxwell, *Matter and Motion*, Chap. V, pp. 101–106, middle paragraph of p. 107, and on to p. 110, then pp. 133 and 134; but for this course give simply the following.)

"Work is the act of producing a change of configuration in a system (or body), in opposition to a force which resists that change" (or when force is required to produce such change). Something, therefore, is moved when work is done. The work is measured by the product of the force by the distance through which it moves the body on which work is done.

"Energy is the capacity for doing work."

A material system is a conservative system; i.e., when work is expended on the system to change its configuration, it possesses then additional energy by which it is capable of doing an exactly equal amount of work, in regaining its former condition. Potential energy is its energy due to any advantage of position it may have as a whole or in the distribution of its parts (configuration). Kinetic energy is energy due to motion, and energy in either of these divisions may belong to a body *en masse*, or to its molecules individually.

"All kinds of energy are so related to one another that energy of any kind can be changed into energy of any other kind." This is the *correlation of energy.*

"When one form of energy disappears, an exact equivalent, of other form or forms, always takes its place, so that the sum total of energy is unchanged." This is the *conservation of energy.* As thus stated, the changes of energy must not be confounded with the effect produced if the system of bodies under consideration be affected by extraneous forces as additional. The total amount of energy in any body or given system of bodies is *constant,* unless the system is acted upon by forces from without. In such case the change of energy is at the expense of some other system (producing the change) by an exactly equal amount, and so it results in saying that the total energy in the universe is an unchanging and unchangeable quantity. The form of energy and the way in which it is manifested may alter, but not the grand total, stated by Maxwell thus: "The total energy of any material system is a quantity which can neither be increased nor diminished by any action between the parts of the system, though it may be transformed into any of the forms of which energy is susceptible." Now, considered as deduced from observation and experiment, that, of course, only asserts that no system has yet been discovered in which the principle is not true, but "as a science-producing doctrine it is always acquiring additional credibility from the constantly increasing number of deductions which have been drawn from it, and which are found in all cases to be verified by experiment. If, by the action of some agent external to the system, the configuration of the system is changed, while the forces of the system resist this change of configuration, the external agent is said to do work on the system. In this case the energy of the system is increased by the amount of work done on it by the external agent. If, on the contrary, the forces of the system produce a change of configuration which is resisted by the external agent, the system is said to do work on the external agent, and the energy of the system is diminished by the amount of work which it does.

"Work, then, is *a transference of energy from one system to another;* the system which gives out energy is said to do work on the system which receives it, and the amount of energy given out by the first system is always exactly equal to that received by the second. If, therefore, we include both systems in one larger system, the energy of the total system is neither increased nor diminished by the action of one partial system on the other." The doctrine of the conservation of energy has been declared "*the one* generalized statement which is found to be consistent with fact, not in one physical science, but in all.

"When once apprehended, it furnishes to the physical inquirer a principle on which he may hang every known law relating to physical actions, and by which he may be put in the way to discover the relations of such actions in new branches of science" (Maxwell, *Matter and Motion*).

The facts that any form of energy may have an exact equivalent in any other form, and that there is no absolute destruction of energy, would lead us to suppose that perpetual motion, for instance, would be not only possible but inevitable; and it would be possible if we could always control the exact character of energy that would result in the case of any transformation, but that we cannot do. The different modes in which energy is manifested, as muscular effort, chemical action, heat, etc., form, as it were, a succession of steps by which energy descends from a higher to a lower plane. Such descent is called degradation. Energy can be fully changed from a higher to a lower form, but in attempting to restore it we succeed in restoring only a part, the remainder reappearing in a still lower form, and thus not available for the accomplishment of work requiring the higher form of energy. It follows, therefore, that a degradation of energy is thus continually going on, and we are obliged to confront the startling prospect that eventually energy, on the earth, will exist only in one form. That form in which it is continually reappearing as a residuum is heat.

To illustrate external and internal work, as also conservation

of energy: If the system, Fig. 4, is in equilibrium, S is under stress and has potential energy of strain. If W_2 is drawn down while W_1 is *held*, energy is increased in S. The work thus done may be seen by releasing W_1 while W_2 is kept in its lowered position; W_1 rises a measurable height h corresponding to work W_1h and increasing the energy of W_1 *relatively to the earth* by that amount. The total energy of the system, including the earth, is now the same as originally, since the energy of position of W_2 is less than at first by just so much as that of W_1 is greater; and the energy of S is the same as at the beginning, the whole system being again in equilibrium. There is no change in the energy of a system by action between the parts themselves.

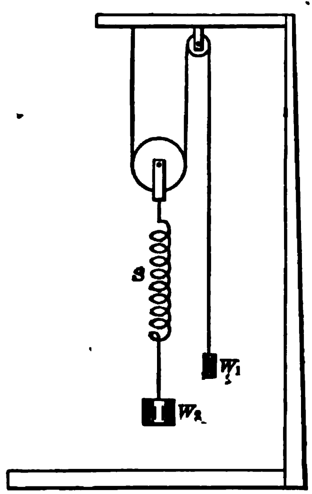


Fig. 4. Transference of Energy.

A change in the potential energy of a system is the equivalent of the work done in changing the configuration of the system.

A change in kinetic energy is the equivalent of the work done in changing velocity.

If a force F is steadily applied to a body of mass m, the latter, starting from rest, acquires in the time t a velocity v, and passes over a distance s equal to that which it would have traversed in the same time with the average velocity $\frac{v}{2}$; i.e.,

$$s = \frac{v}{2}t,$$

and since $F = m\frac{v}{t}$ (Art. 20), multiplying these two equations member by member,

$$Fs = \tfrac{1}{2}mv^2.$$

Fs is the measure of the work done by the force F, and $\frac{1}{2}mv^2$ is the measure of the kinetic energy lost or gained by the body upon

which the work is done. Therefore, $F = \frac{\text{work}}{s}$, or $\frac{\text{k.e.}}{s}$. Hence, force may be regarded as the *space rate* of doing work or of transferring energy.

The various forms in which energy is manifested may be arranged in two groups as follows:

"I. *Potential Energy:*

1. Strain, whether extension, compression or distortion.
2. Gravitative separation.
3. Chemical separation.
4. Electrical separation.
5. Magnetic separation.

II. *Kinetic Energy:*

1. Translatory or rotatory motion.
2. Vibration, including sound.
3. Radiation, including light, etc.
4. Heat, both latent and sensible.
5. Electricity in the form of current."—*Barker.*

Energy of *strain* is shown in a bent or wound-up spring. The energy is displayed as the spring is relaxed, doing work, and "strain" disappears. Energy of *gravitative separation* is in a raised weight or a waterfall; the energy is displayed when the body descends and there is no longer "separation." Energy of *chemical separation* is in carbon and oxygen when apart. The energy is evidenced when they combine, burning, and the "separation" gives place to combination. There is energy of *electrical separation* when the earth and a cloud are oppositely charged, and the electricities are separate. The work of effecting this separation was gradual and quiet but cumulative; energy is displayed when the lightning stroke occurs, and "separation" gives place to combination. Energy of *magnetic separation* is the equivalent of the work required to separate two magnetic poles, and then they can do work by reason of being separate.

Similarly, illustrations may be given of energy due to motion. These latter forms are all evidenced in the work or physical

change they are able to effect; e.g., 1, in the work required to deprive a body of its motion, or the resistance the body can overcome in moving through a distance; 2, the internal or molecular work by which vibrations are steadily damped, or the production of sound; 3, heat produced by the interception of radiation; 4, mechanical work accomplished by heating a gas; 5, magnetic and heating effects of a current.

Under I, Nos. 1, 2, 4 and 5 are distinctly mechanical; and likewise under II, Nos. 1 and 2 are reducible to the mechanical form in most cases, and always expressible as the product of two factors. (See Lodge's *Elementary Mechanics*, Art. 93, p. 118; also article on "Energetics" in *New International Encyclopedia*.)

Note. — Mechanics we may understand to be that portion of physical science which deals with the action of matter upon matter, so far as such action affects the motion, size or shape of bodies, no matter how large or how small they may be.

Mechanical actions are such actions as affect the motion, size or shape of bodies, no matter how large or how small they may be.

We are now prepared for a systematic view of the three principal dynamic units.

SCHEME OF UNITS. (Discuss.)

	Absolute.		Gravitation. (See Art. 21.)	
	Metric c.g.s.	British f.p.s.	Metric c.g.s.	British f.p.s.
Mass.......	Gram.	Pound.	980.2 grams.	32.2 pounds.
Force.......	Dyne.	Poundal.	Gram weight.	Pound weight.
Work.......	Erg.	Foot-poundal.	Gram-centimeter.	Foot-pound.

A force of 980.2 dynes equals the weight of one gram at New York.

Note.—It must be remembered that force and mass cannot both be expressed as pounds in the same problem. If mass is pounds, then force is poundals, and 32.2 poundals equals the weight of one pound. On the other hand, if it is desired to express the force in pounds, the mass will be the $\frac{\text{lbs. weight}}{32.2}$.

In like manner, in c.g.s. units, if mass is grams, force is dynes, and 980.2 dynes equals the weight of one gram, but if it is desired to express force in grams, then the mass will be $\frac{\text{gms. weight}}{980.2}$.

Energy of position is often converted into that of motion, as in a descending pendulum, or the weights of a clock; in water falling to drive machinery; also strain is so converted when springs impel a mechanism.

Energy of motion is converted into energy of position in ascending pendulum or other body, and owing to inertia it may be seen in *mobile* substances as liquids; e.g., when water is taken into the tank of a locomotive by the motion of the latter (Fig. 5). The

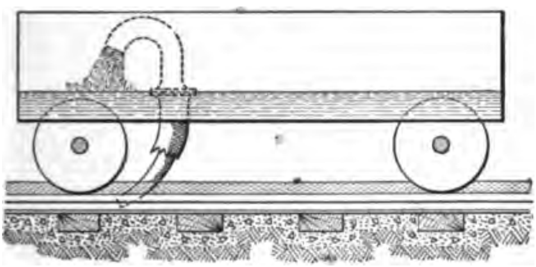


Fig. 5. Tender of Locomotive taking Water while in Motion.

energy of motion of the train is expended to increase the energy of position of water, relatively to the earth, by *lifting* it from the track to the tank of the engine, if the speed is sufficient for the height. To lift any mass m requires a force mg; to raise it a height h, work equal to mgh must be done. Enough energy must be supplied by the velocity v that $\frac{1}{2} mv^2 = mgh$; i.e., if the train stood still and the water flowed into the feed pipe, it would require a speed v, giving it energy $\frac{1}{2} mv^2$ equal to mgh; v represents the relative speed of train and water. If the latter is stationary, then v is speed required by the train. The equation gives $v = \sqrt{2gh}$, and taking g to be 32 ft./sec.^{2}, if h is, say, 8 ft., then $v = 22.5$ ft. per second, or 15.3 miles per hour, making no allowance for friction of the water in the pipe.

(*Example from College Entrance Examinations.* — A weight w descending a height h drives a pile into soil a distance d. Find the average resistance of the pile.

If the weight w is the force, say pounds, and h the height, say feet, w expends energy equal to wh foot-pounds. If d is the distance, also in feet,

which the pile is driven while offering a resistance of r pounds, the work is rd foot-pounds. Since $wh = rd$, $r = \frac{wh}{d}$.)

EXAMPLES. —

1. A bag containing 50 kg. of sand lies on a scaffold 25 meters above the ground. (*a*) What is its potential energy relatively to the ground? (*b*) It falls; what is its kinetic energy when it strikes the earth? (*c*) What is its velocity? (*d*) What is its momentum?

Ans. (*a*) 1225×10^8 ergs.
(*b*) The same.
(*c*) 2213.6 cm./sec.
(*d*) $11{,}068 \times 10^4$ gm.-cm./sec.

2. A bullet of mass 120 grams is discharged with a velocity of 400 meters per second from a rifle, the barrel of which is one meter in length. What is the energy of the bullet when it leaves the muzzle, and the average pressure exerted upon it within the barrel by the exploding powder?

Ans. Energy, 9.6×10^{10} ergs. Force, 9.6×10^8 dynes.

3. How much work must be done on a mass of one ton to give it a velocity of 20 feet per second? ($g = 32$.) *Ans.* 12,500 foot-pounds.

4. What is the energy of a train of 40 tons, moving at the rate of 30 miles an hour? What force will bring it to a stop in a distance of 500 feet?

Ans. 2,420,000 foot-pounds; 4840 pounds.

Two equal masses are at the top of two inclined planes of the same height but unequal lengths. Which mass has the greater potential energy, relative to the base of the plane? If both slide down without friction, which will have the greater velocity on reaching the base?

6. A baseball whose mass is 150 grams and velocity is 2000 cm./sec. is stopped by the hand of the catcher in moving back 30 cm. after impact. What is the average pressure of the hand against the ball? (Suggestion, $Fs = \frac{1}{2} mv^2$.)

Ans. 10,000,000 dynes.

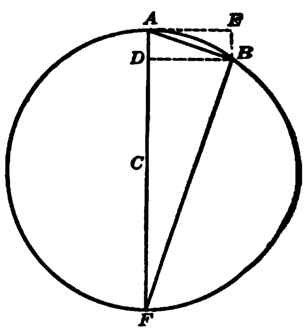


Fig. 6. Central Acceleration.

25. Central Acceleration; Centripetal Force. — A body moving uniformly in a circular path of radius r with speed v has a central acceleration $\frac{v^2}{r}$ as shown thus:

In Fig. 6, AB = small arc described in time t; then $AB = vt$. In time of going from A to B the body has descended towards

center the distance AD; this is the extent of departure from the direction of the original motion while describing the actual path AB. If velocity in circumference is uniform, so also is this change in direction toward center, or central acceleration. Call this acceleration f. Then in time t, $AD = \frac{1}{2}ft^2$ (see p. 21); and $AF = 2r$. If AB represent a *very short* distance, it is virtually equal to the chord. Also in triangle ABF, $AB^2 = AD \times AF$; or, substituting values,

$$v^2t^2 = 2r \times \frac{1}{2}ft.^2, \text{ whence, } f = \frac{v^2}{r}$$

By the first law of motion, the body would only be deflected from the tangential path by the application of some force, the effective component of which is at right angles to the tangent or towards the center of the circle. The measure of this force is the product of the mass by the acceleration, or $\frac{mv^2}{r}$; and the reaction *which the body exerts* against such force is known as centrifugal force. It is necessarily equal and opposite to the centripetal force.

Experiment No. 8, page 89. Centrifugal Force.

EXAMPLES. —

1. A mass of 200 grams is whirled round in a circle at the end of a string 120 cm. long, fastened at the center of the circle, at a rate of 2 revolutions per second. What is the pull upon the string?

Ans. 379×10^4 dynes, or 3866 grams.

2. A car of 20 tons is moving round a circular curve of 1000 feet radius at a speed of 40 miles per hour. What is its outward pressure against the track? *Ans.* 137,671 poundals, or 4275 pounds.

26. Simple Harmonic Motion. — This may be defined provisionally as the projection of uniform circular motion upon a diameter. A body moving with S.H.M. has an acceleration that is proportional to its displacement.

Let P, Fig. 7, be a point moving with uniform velocity v in circle of radius r. A point as M, traversing the diameter AB, exactly keeping pace with P, will describe S.H.M. through the center O. The distance of M from O is called the displacement.

If ω is the angular velocity of P, $v = r\omega$. For any position of P call the angle $POA = \theta$. If T is the period of one complete revolution of P or of one complete to-and-fro movement of M, $T = \frac{2\pi}{\omega}$. The acceleration of M along AO is equal to the parallel component of the acceleration of P along PO. The acceleration along PO, by preceding article, is $\frac{v^2}{r}$, which equals $\frac{r^2\omega^2}{r}$ or $r\omega^2$. This, multiplied by the cosine of POA, gives the acceleration of M along AO.

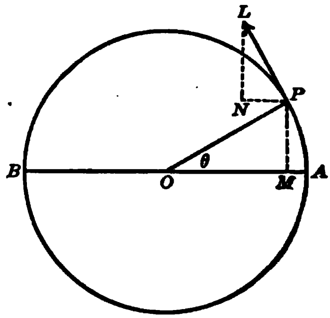


Fig. 7. Simple Harmonic Motion.

$$\cos POA = \frac{OM}{OP} = \frac{\text{displacement}}{r}.$$

$\therefore$ Acceleration along $AO = \omega^2 \times$ displacement.

Q.E.D., since ω is constant; and this law of acceleration may be used *to define* Simple Harmonic Motion.

Now the period of the S.H.M. is $T = \frac{2\pi}{\omega}$;

whence $$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}.$$

Since a body moving with S.H.M. has acceleration proportional to displacement, it is always under a force which varies as the displacement (for force varies as acceleration). Under dynamics the converse of this is shown to be true, viz., *a body subject to a force varying as its displacement has S.H.M.*

But this law of variation of force is a law of elasticity (as determined by experiment), therefore elastic bodies or bodies whose movement is controlled by an elastic medium describe S.H.M., or have vibratory motion. The actual motions of vibrating elastic bodies or parts of bodies, then, are periodic and consist of

one or a combination of more than one S.H.M. The diagram of S.H.M. is the sinusoid.

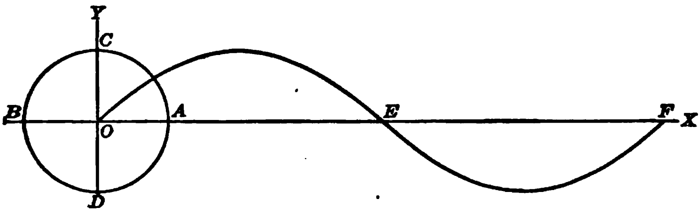


Fig. 8. Wave Form representing Simple Harmonic Motion.

In Fig. 8, if abscissæ (or horizontal distances from OY) represent time, and ordinates (or vertical distances from OX) displacements, then for a particle moving in DC with S.H.M. beginning at O, the curve representing one complete vibration has the wave form OEF. The motion of A to which the S.H.M. along DC corresponds may be circular, or elliptical, with longest axis vertical, reaching the path CD as its limit, in which case the vibration is said to be *transverse* (i.e., to the direction OF in which waves due to the motion of O proceed); or the longest axis may be horizontal, reaching the path AB as its limit, in which case the vibration is *longitudinal*.

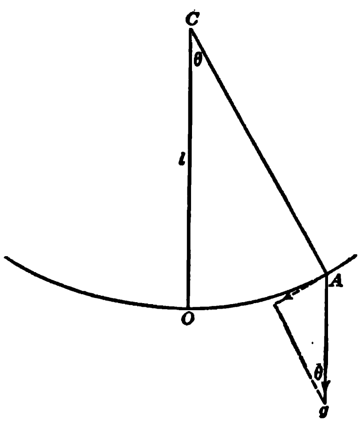


Fig. 9. The Simple Pendulum.

The pendulum is an approximate example of S.H.M. If the bob is at A, Fig. 9, with mass m, the force is mg acting vertically. The component of g acting in the path AO is $g \sin\theta$. For small values of θ, $\sin\theta \propto \theta$. The displacement is $AC \times \theta = l\theta = OA$.

The period of complete swing is

$$T = 2\pi\sqrt{\frac{l\theta}{g\sin\theta}}, \quad \text{in which} \quad \frac{\theta}{\sin\theta}$$

approaches unity as its limit as θ decreases, so that for an infinitesimal arc $t = 2\pi\sqrt{\frac{l}{g}}$. Thus t is dependent only on l and g. In strict S.H.M., where the ratio of displacement $\div$ acceleration is constant, T is the same for any amplitude, or vibrations large or small are isochronous.

EXAMPLES. —

1. At New York the length of a simple pendulum beating seconds is 99.3 cm. If it is shortened one millimeter, how much will it gain in one day? *Ans.* 43.2 seconds.

2. How many swings per minute will be made by a simple pendulum 20 feet long? (Take $g = 32$ ft./sec.^{2}.)

3. Assuming the length of the seconds pendulum to be 98 cm., find the length of a pendulum in the same locality that will oscillate 120 times in 30 seconds. *Ans.* One-sixteenth of the length of the seconds pendulum.

4. By how much must a pendulum be shortened to make it oscillate twice as rapidly?

27. Constants of Nature. — We often speak of the constancy of nature, and we sometimes speak of the necessity of stating natural processes in some definite measure, some exact quantitative terms whereby we distinguish physics as an exact science. The possibility of doing this leads us to the knowledge and use of so-called "constants of nature." It must be understood, however, that the constancy of these "constants" requires that well-defined conditions shall be met. Examples are the freezing or the boiling point of a liquid; the coefficient of expansion of a substance, or its elasticity, or its density; the dielectric constant of a medium, or its refractive or dispersive power, etc. Volumes of these constants have been compiled for reference.

One of the most important of these quantities is the acceleration due to gravity; meaning here by "gravity" the attraction of the earth. This is usually symbolized by g. It means the rate at which the velocity of a freely falling body increases. It is not a force, therefore, but it is numerically equal to the number of units of force which gravity exerts upon a unit mass.

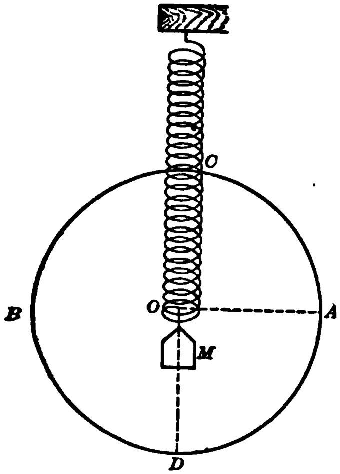


Fig. 10. Determination of g by Helical Spring.

A helical spring, Fig. 10, if extended, will develop a force that is proportional to the extension. If a weight whose mass is m be suspended from such a spring, it will produce a definite elongation, say e, coming to rest at O. With this elongation, the force pulling on the weight by the spring is just equal to the weight, or mg. If the weight be drawn further, as to D, and then released, it will oscillate through the mid-position O under a force that varies as its distance from O; i.e., it will have S.H.M. If A, having mass m, were describing the circle of reference $ACBD$ in the period of oscillation T, its velocity would be $\frac{2\pi r}{T}$, and, as its acceleration towards the center is $\frac{v^2}{r}$, this becomes $\frac{4\pi^2 r}{T^2}$, and the force directing it towards the center is m times this, or $\frac{4\pi^2 mr}{T^2}$, or $\frac{\text{force}}{r} = \frac{4\pi^2 m}{T^2}$, r being the extreme distance from the mid-position in the S.H.M. of M. But in S.H.M. for *any* displacement x, of the oscillating body, the ratio of the force F to the displacement x, or $\frac{F}{x}$ = const., sometimes called the "spring constant." Thus in this oscillating weight, when the displacement is e the force is mg, and

$$\frac{mg}{e} = \frac{4\pi^2 m}{T^2}, \quad \text{whence} \quad g = \frac{4\pi^2 e}{T^2}.$$

Experiment No. 9, page 90. — By measuring the elongation produced in such a spring by any weight, and timing the oscillations of that weight, the above equation determines g.

It must be remembered, however, that in the above discussion the effect of the mass of the spring itself upon the period of oscillation is ignored, so that a determination of this kind is only approximate, and not very close unless the period of the spring itself is negligible.

28. Gravitation; Why a Heavy Body Falls No Faster than a Light One. — The falling of a body to the earth is attributed to the earth's attraction for it, and the weight of the body is a force that is the measure of that attraction.

It is observed that in a vacuum all bodies fall at the same rate, i.e., with equal accelerations. (This is not saying they are all equally attracted by the earth.) They are all subjected to the attraction of the same body, the earth, and at the same distance from it (i.e., from its center). We have seen that in accordance with the second law of motion the acceleration of a body is proportional to the force and inversely proportional to the mass, or $a = k\frac{F}{M}$. Now with various masses M, the acceleration, a, could not be the same unless the force F varied in just the same proportion that M varied, that is, unless $\frac{F}{M}$ is constant; so that if the accelerations are alike, as shown by experiment, then the ratio of F to M for each and all is the same, and hence the attraction of the earth for bodies, i.e., their weight, is proportional to the quantity of matter in them, i.e., their mass. This force, therefore, or the *weight*, is a proper basis for the comparison of masses.

Newton established the fact that the earth attracts bodies with a force that is proportional to their mass, by suspending hollow spheres of the same size from strings of equal length, *filling them with various substances*, and observing that *they all swung in the same period*, i.e., with equal accelerations under the earth's attraction (*Principia*, Bk. III, Prop. VI, Theorem VI). Such an experiment helps to confirm the second law of motion; but if we consider the law as sufficiently well established and also that the attraction of the earth for any body at a given place is

proportional to the mass of the body, then these principles would demonstrate that necessarily a heavy body and a light one would fall at the same rate. For by heavy (*gravis*) we mean having gravitation force, and so the force F due to gravity varies just as the mass M of the body; or for all bodies the ratio of the force of gravity to the mass, or $\frac{F}{M}$, is constant. But this ratio, by the second law, is the acceleration due to gravity.

Experiment No. 10, page 90, with water hammer, shows that the liquid falls in bulk as a solid, or when it breaks into drops at the throat the drops fall with a click, like a metal. Also bodies falling in a vacuum tube.

29. Universal Law of Gravitation. — Newton's experiments with the pendulum showed that the attraction of the constant body, the earth, at a constant distance from another body varied as the mass of the other body, but this was only a partial expression of the general law of gravitation. This law says that bodies behave as if "every particle of matter" attracted "every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them." Treating bodies as particles, they may be considered as each at a point. The formula to express this law is $F = k\frac{mm'}{d^2}$. That the law applies to bodies throughout space was established by Newton's demonstration that the motion of the moon conformed to it; then it was extended to the entire solar system. Newton's great work, then, in connection with gravitation, was not the determining of the principle, but of its universality (Lodge, *Pioneers of Science*). That it applies to small terrestrial bodies under the influence of the earth has just been shown; that it applies to such small bodies in their action upon each other at short distances was first established experimentally by the famous "Cavendish experiment." This experiment, so named from Henry Cavendish who first (1797–98) successfully performed it, was designed by Rev. John Mitchell, but he died before he could try it.

30. Gravitation Constant. — The fundamental formula expressing the law of gravitative attraction is

$$F = K\frac{mm'}{d^2}. \qquad \text{(A)}$$

In this, we can so choose our units of mass, distance and force that K shall be unity, and then in such units the force of gravitative attraction between masses would be

$$F = \frac{mm'}{d^2}.$$

For instance, if we define as our *unit of force* the force which a unit mass exerts upon another unit mass at a unit distance, then in formula (A) F, m, m' and d are all unity at the same time, and, with that understanding, the equation makes $K = 1$. But such a unit force would be different from any in common use.

Having in mechanics already defined our unit of mass, of distance and of time (and therefore of acceleration), the unit of force, as derived from its effect in producing acceleration, is of necessity the force which produces unit acceleration in unit mass. In the c.g.s. system we have already determined this to be the dyne, and we do not know this to be the gravitation unit above described. In fact, since the acceleration of a gram of matter, under the attraction of the *whole earth* (many grams), at a distance of 4000 miles is 980 cm./sec.^{2}, and therefore the force equals 980 dynes, it is probable that the attraction due to one gram is much less than a dyne even at as small a distance as one centimeter; so our problem is to see what K in the above equation would be, to give the number of *dynes* in F, if m and m' are each one gram and d is one centimeter. With that determined, if we use that value of K, then the equation will always correctly give the force in dynes, due to the attraction of masses expressed in grams, at distances expressed in centimeters; and K, thus determined, is called the "gravitation constant." (It might also be determined for the f.p.s. system.)

31. The Cavendish Experiment. — The apparatus, Fig. 11, consists of a delicate torsion suspension fiber w, a light arm a,

at the ends of which are two equal small balls of known mass m. The angle a through which the fiber is twisted by a force of one dyne applied on each side of the suspension fiber in opposite directions at half a unit distance from the suspension, i.e., with unit distance between the forces, is measured. Then two equal large balls of mass m' are placed at a distance d (center from center) from the balls m, and the angle of twist θ, when the balls are at this distance, is observed. The observation is repeated with the balls m' changed to the opposite side of m. Whatever the two equal rotative forces applied at right angles to a, the twist is proportional to one force multiplied by the distance between the two; and various pairs of forces applied at m and m' will be proportional to the angles of twist which they produce. In the first case above,

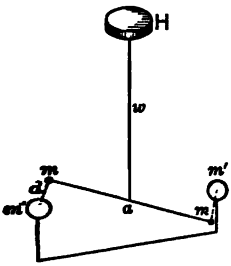


Fig. 11.
Torsion Balance.

$$a = c\ (1) \cdot (1) \tag{B}$$

where c is a constant depending on the elastic quality of the suspension fiber. In the second case,

$$\theta = cfa. \tag{C}$$

But

$$f = \frac{Kmm'}{d^2}. \tag{D}$$

$$\therefore \quad \theta = c\frac{Kmm'}{d^2}a. \tag{E}$$

Then

$$\frac{\theta}{a} = \frac{Kamm'}{d^2},$$

or

$$K = \frac{d^2}{amm'} \cdot \frac{\theta}{a}. \tag{F}$$

Note. — Any two parallel equal forces in opposite directions are called a couple. The effect of a couple is solely to produce rotation, and the rotative effect, called the moment of the couple, is measured by the product of either force by the perpendicular distance between them. When this product is unity the couple is called a unit couple. In such an illustration as the present, the unit couple would be obtained in practice by using a couple consisting of any known equal forces (not necessarily unity), at any known distance apart, having a calculable moment

and producing an observed twist. Then the twist for unit moment would be this observed twist divided by the moment of the couple producing it. The known force, then, need not be unity and might be applied at m.

a and θ are determined by turning a milled head H until the twist produces the required force at m, i.e., holds m and m' apart the required distance d. Thus all the quantities in the second member of Eq. (F) are known, and K is determined. Since the time of Cavendish the experiment has been often repeated with great care, the best results up to this time giving for K in c.g.s. units the value 6.6579×10^{-8}. It is thus seen that, as compared with magnetic or electric attractions or repulsions, gravitation is an extremely feeble force, although when acting between the enormous masses of the heavenly bodies it is sufficient to control their movements.

32. **Weighing the Earth.** — If by weight we mean the force with which the earth attracts a body, then the weight of the earth itself is an indefinite expression; but usually the process of weighing is a comparison of the mass of a body with that of some standard body, and in this sense weighing the earth would be determining how many units of mass it contains. That being determined, if the volume also is known, then the ratio of the mass to the volume is its mean density.

Since at the surface of the earth the attraction of the earth for one gram is g dynes, we have, calling mass of the earth M,

$$g = \frac{K(1)M}{R^2}.$$

Substituting for R its value (3962 miles) in cms., for g its value in cm./sec.^{2} (981), and for K its value as above determined, we find

$$\begin{aligned} M &= 6272 \times 10^{24} \text{ grams} \\ &= 6272 \times 10^{18} \text{ tonnes.} \end{aligned}$$

(10^{18} is a billion of billions.)

In this sense Cavendish is said to have been the first to "weigh the earth."

This mass in grams divided by the volume of the earth in cubic centimeters gives the number of grams per cubic centimeter

in the earth. This is called the mean density of the earth and is 5.5268 g./c.c.

33. **Center of Gravity.** — Gravity acting on every particle of a body constitutes a system of parallel forces, the weight of the body being the resultant of these forces, which, in the case of parallel forces, is their sum. For any given body the resultant of these forces will always pass through one point, no matter what may be the position of the body. (Shown from principles of mechanics.) This point is called the center of gravity of the body, and may be outside of the material composing the body, as, for example, in a ring or hoop. Regarding the entire body in its mechanical relations to other bodies, it may be treated as if it were all at its center of gravity. Considered in its parts, each part may be regarded as concentrated at its own center of gravity.

For stability of position, if a vertical line through the center of gravity passes outside of the line circumscribing the base on

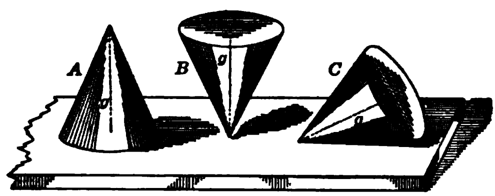


Fig. 12. Stable, Unstable and Indifferent Equilibrium.

which the body stands, i.c., the circumscribing line with no re-entrant angle, the body will topple over or be unstable; but if the vertical passes within the base the position is stable. In the former case it is easily seen that a toppling of the body necessarily means a lowering of the center of gravity, and in the latter case a raising of it. The body is stable, therefore, when a movement of overturning raises the c.g., and unstable when such movement lowers the c.g. There is a third possibility, in which a turning neither raises nor lowers the c.g.; this position is called neutral or indifferent. The three conditions are illustrated in Fig. 12 by a cone on its base, its apex and its side, respectively.

34. Potential Energy Tends to a Minimum. — When the parts of a body or of a system of bodies are in any degree free to adjust themselves under forces that exist within the system, they will always so rearrange themselves as to make the potential energy of the system as small as possible. From the last article we see that, treating a given body and the earth as a system, the potential energy is the greatest when the c.g. of the body is farthest from the earth; and, in accordance with this energy principle, a body will always so move under the action of gravity as to bring its c.g. to the lowest possible position. In doing this a body may apparently roll uphill, at least for a short distance.

Experiment No. 11, page 91. Mechanical Paradox. —

Loaded cylinder of wood; the c.g. being eccentric, the cylinder rolls uphill a part of a revolution; also in the "mechanical paradox" the double cone apparently rolls up an inclined plane; also various toys.

35. Mechanical Powers. — In machines the relation of the applied force P to the resistance W that is overcome is determined by the principle that the energy acquired by the body moved equals the work done in moving it. If S_p denote the distance moved by the point of application of the force, and S_w the distance through which a weight W is raised or a resistance W is overcome in the direction of the resistance,

$$F \times S_p = W \times S_w. \qquad (1)$$

The first member of this equation represents the work done by the applied force, and the second member is an expression of the work against the resistance, and for equilibrium these two are equal. This part of the problem is mechanics. The equation can be applied, provided it is possible to determine what space will be traversed by the resistance when the point of application of the force moves over a given distance in the direction of the force. This depends on the arrangement of the mechanism, and this part of the problem is not mechanics but geometry. Therefore, examine the arrangement of the mechanism, and from the geometry of it determine the distance through which the re-

sistance is moved when the point to which the moving force is applied has moved a known distance. If free from friction, these forces will be to each other inversely as the distances.

The principle may be illustrated with lever, straight or bent, with pulley, inclined plane, wheel and axle.

If F is the force, parallel to an inclined plane of height h and length L, to just support a weight W, since

$$F \times L = W \times h, \quad \frac{h}{L} = \frac{F}{W} = \frac{ma}{mg},$$

where a is the acceleration with which W would freely descend the plane. This gives Galileo's mode of determining the value of g.

Experiment No. 12, page 91, Determination of g. —

With pulley on wire, if

$$L = 950 \text{ cm.}, \quad h = 95.6 \text{ cm.}, \quad t = 4.4 \text{ sec.}$$

Since $L = \frac{1}{2}\, at^2$, $a = \frac{2\,L}{t^2} = 98.0$, and from above equation $g = a\,\frac{L}{h} = 980$, nearly.

The above relation also leads to the principle of virtual velocities, a purely geometrical conception (but the basis of Lagrange's *Analytical Mechanics*).

Illustrate with rolling-pin model; also differential windlass.

Experiment No. 13, page 91, Rolling-pin Model. —

If the earth and a weight on an inclined plane be regarded as a system, and the weight be drawn up the plane, work is done *on the system* (by external force) and the energy of gravitative separation is thus increased. The force F necessary for this does work by being exerted through a distance s to raise a weight W against gravity a height h, so that $Fs = Wh$. Observe that the potential energy of the system is now increased by the amount of work that has been done upon it, and the weight so raised will return to its former position if free to do so, or *the potential energy tends to a minimum.* (In such decrease of potential energy, what becomes of the energy?)

Equation (1) above is applicable to every contrivance for raising a weight.

EXAMPLES. —

1. The arms of a lever make a right angle at the fulcrum. What horizontal force applied to the vertical arm at 33 cm. from the fulcrum will support a weight of 10 kg. suspended from the horizontal arm at 10 cm. from the fulcrum? *Ans.* 3030 gm.

2. If a boy weighing 90 pounds has a lever 5 ft. 5 in. long, how should he use the lever and his weight to raise a stone weighing 300 pounds? Supposing he can lift 100 lbs., show another way for him to place the bar so as to raise the stone. (Neglect the weight of the lever.)

3. In a hoisting apparatus the hand, pulling with a force of 20 pounds upon a rope, draws it 50 ft. in raising a weight a height of 4 ft. What is the weight? *Ans.* 250 pounds.

4. The crank arm of a windlass is 50 cm. long, and the shaft, around which is wound a rope to draw up a weight, is 18 cm. in diameter. What force must be applied to the end of the crank to raise a weight of 110 kg.? *Ans.* 19.8 kg.

5. If the roller *R*, Fig. 38, p. 87, weighs 120 g. and the diameters are 2 cm. and 7 cm., what must be the weight of *W* to just sustain *R*? What weight would be required if the two diameters were equal? *Ans.* First, 48 g.; second, infinity.

36. Rotation; Moment of a Force. — If all parts of a body move in equal parallel paths it is said to have a motion of translation; the velocity or change of velocity is the same for every part of the body; but without the body as a whole changing its locality, it may turn about some point or line as an axis. This is a motion of rotation. In it all parts of the body describe a definite portion of a complete circumference about the axis in the same time, although the parts far from the axis move in their respective paths faster than those near the axis. The parts have the same angular velocities, but their velocities in their circular paths, i.e., their linear velocities, are proportional to their distances from the axis. The angular velocity of the whole body is expressed by the same number as the linear velocity of a particle at one unit distance from the axis. If the axis is fixed, the body may have rotation, although it cannot have translation.

To set a body in rotation or to change its rate of rotation requires force just as in linear motion, but the effectiveness of a force in producing rotation depends not only on the magnitude of the force, but also upon the distance from the axis of its line of direction. Every one knows that a given effort exerted at the rim of a wheel is more effective in turning the wheel than if applied near the axle. So the turning effect of a force is measured by the product of the force by the perpendicular distance from the axis to the line of the force. This distance is called the lever arm of the force, and this product is called the *moment* of the force. A force of 20 pounds acting at a distance of 5 inches from the axis, would have as great a turning effort as one of 100 pounds at one inch from the axis; and if these two forces were so directed as to tend to rotate the body in opposite directions, they would exactly counter-act each other, and the body would be in equilibrium so far as rotation is concerned.

If a body is to be in complete equilibrium it must have no tendency to change of motion, either in translation or rotation. The latter is secured by having *the moments* of the forces tending to produce rotation in one direction equal to *the moments* of those tending to produce rotation in the opposite direction, no matter what point is chosen as an axis.

Also if one force is truly a "resultant" of two or more other forces, it must be able to accomplish just what they could do, and therefore its moment must be equal the sum of their moments.

This principle is readily applied to many of the problems already considered. Apply the principle of moments to Examples 1, 2, and 4, in Article 35, neglecting the weight of the parts of the apparatus.

37. Moment of Inertia; Energy of Rotation. — Let P be a particle in a body of any shape, Fig. 13, which is rotating about C under the action of forces applied at one or more points of the body. If the parts of the body are rigidly connected to one another, a force applied at any point is communicated to all parts of the body by internal action from particle

to particle. Among these particles action and reaction are mutual, and if rotation is accelerated, the acceleration will be in obedience to just so much force on each particle as equals the mass of the particle multiplied by its acceleration in the line of its motion, i.e., the tangential direction. Let m_1 = the mass of a particle, r_1 its distance from the axis of rotation, α the angular acceleration, which is alike for all particles. Its linear acceleration is $r_1\alpha$, and the force upon the particle in the direction tangential to the circular path in which it is moving is $m_1r_1\alpha$. The moment of this force is $m_1r_1^2\alpha$; similarly for any other particle of mass m_2 at distance r_2, the moment of the effective force on it is $m_2r_2^2\alpha$, and for all the particles the total moment of the tangential forces is $\Sigma mr^2\alpha$, where Σ simply stands for the sum of all the like quantities; or since α is a common factor multiplying each mr^2, it may be written $\alpha\Sigma mr^2$. The sum Σmr^2 is called the moment of inertia of the body with reference to the axis C.

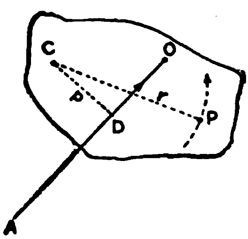


Fig. 13.

Suppose the acceleration of the body is due to a single force F, whose direction is along a line AO, passing at a distance $CD = p$ from the axis C. The force may be regarded as applied at any point in its line of action as, say, at O, and its effect upon the speed of rotation is measured by its moment Fp. Then we should have $Fp = \alpha\Sigma mr^2$. If, instead of a single force, there are various forces F, each at its own distance p from the axis, the total moment of rotating forces is ΣFp, and this will equal the total moment of effective forces on the particles, or $\Sigma Fp = \alpha\Sigma mr^2$. That is, *the sum of the moments of the applied forces is equal to the angular acceleration they produce, multiplied by the moment of inertia of the body.* This is one of the most important principles in the dynamics of rotation.

Determining how much the moment of inertia of a body amounts to is a mathematical process with no regard to physics. The mass of every element of the body is multiplied by the square of its distance from the axis, and these products are added together. There can be a definite distance from the particle to the axis only if the particle is infinitely small, so the process of exact computation is essentially one of infinitesimal calculus.

The idea of moment of inertia is applied to geometric figures, whether plane or solid, by using the area or volume instead of the mass.

For a body of definite mass and figure it is evident that the moment of inertia will depend upon the position of the axis with reference to which the moment is computed,—in a rotating body, the axis of rotation. In a plane figure the axis may be a point or a line in the surface; in a solid the axis of rotation is a straight line, and in advanced dynamics it is shown that the moment of inertia is less for an axis through the center of gravity than for any other axis in the same position, i.e., parallel to it.

If ω = the angular velocity of the body, $r_1\omega$ = linear velocity of m_1, and as the kinetic energy of a body is the product of one-half the mass by the square of its linear velocity (p. 30), $\frac{1}{2}m_1r_1^2\omega^2$ = the kinetic energy of m_1, and since ω is alike for all the particles, the total energy of rotation, the sum of energy of all the particles, is $\frac{1}{2}\omega^2\Sigma mr^2$; or one-half of the product of the square of the angular velocity by the moment of inertia of the body. If a mass of as many units as the Σmr^2 were all concentrated at a distance of one unit from the axis its moment of inertia would be the same number as its mass and if it rotated with the same number of turns per second as the body it would have the same energy of rotation. That is the physical significance of the moment of inertia of a rotating body.

A rotating body of given mass will have greater moment of inertia and consequently greater energy of rotation for a given speed, the farther its material is from the axis. A flywheel of small weight, if the material is mostly in the rim, would have more energy than a heavier one of the same diameter if concentrated in or near the hub. So a flywheel of an engine, whose purpose is to steady the action of the mechanism, is constructed on that principle, and if the driving effort of the steam is variable, or applied by means of a crank, so that at points it is contributing nothing to turn the wheel, the flywheel gives up some of its energy to carry the rotation past the "dead centers" with little change of speed, and if the motive power of the engine were cut off entirely the flywheel would keep up the motion until as much additional work was done as equalled its energy of rotation.

38. **Friction.**—The relation of force and resistance in elementary machines, as presented in the preceding article, is on the assumption that the entire work is expended in overcoming the final single resistance. In fact, some of it is expended in overcoming intermediate prejudicial resistance causing a waste of work. The chief of such resistances is friction. Its investigation must be left to closer study of mechanics, but while we

usually think of friction as something undesirable and therefore to be got rid of, we must not overlook its advantages, which are simply inestimable.

Example from Watson's Physics, Ex. 9 of Chap. X. — "A kilogram weight sliding down an inclined plane 9 cm. high reaches the bottom with a velocity of 5 cm. per second. How much energy has been rubbed out of it during the descent ($g = 980$)?"

Mass $M = 1000$; $g = 980$; $h = 9$ cm. Potential energy of M at top of plane $= Mgh = 8{,}820{,}000$ ergs. At foot of plane, $v = 5$ cm./sec.; kinetic energy $= \frac{1}{2} Mv^2 = 12{,}500$ ergs. Loss by friction is the difference, or $8{,}820{,}000 - 12{,}500 = 8{,}807{,}500$ ergs.

39. Properties of Matter. — Besides those properties which attach to bodies as a whole, sometimes called molar properties, and which are commonly the subject of mechanics, physics is concerned with others called molecular properties, which represent conditions due to the structure of matter. These are chiefly such as to indicate internal forces in the body.

That matter is discrete is indicated by its porosity. This is exemplified by its compressibility or expansibility. Either of these qualities alone would not demonstrate porosity, for we might conceive of its being due to a shrinking or an enlargement of the particles themselves; but when an intimate mixture of two substances is attended by a diminution of volume, the inference is greatly strengthened that such a rearrangement of the molecules has taken place as to bring them closer to one another.

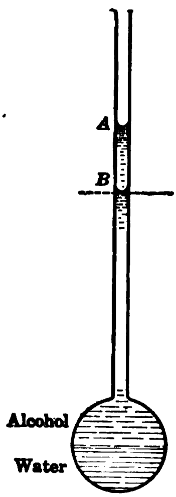


Fig. 14. Contraction of Liquids on Mixing.

This may be shown by carefully pouring upon water an equal volume of alcohol, as in Fig. 14, and then mixing the two liquids. If the two liquids fill the vessel to the mark A, after mixing they will only rise to a point B, showing a contraction in volume of about three per cent. Similar contraction occurs with other liquids. Porosity is also indicated by the amalgamation of gold or other metals with mercury.

One of the chief effects of the interaction between molecules is elasticity, and this accounts for a good many phenomena in physics. The law of elasticity, known as Hooke's law (*ut tensio sic vis*), is that the force of distortion is proportional to the distortion; or, briefly, stress is proportional to strain.

Since, then, when a particle of an elastic body is displaced the force tending to restore it to its first position is directly proportional to the displacement of the particle, a body or particle moving under elastic reaction will have S.H.M. All vibrations, then, that are due to elasticity are S.H.M. They include nearly all the phenomena of sound and sounding bodies. (Refer to Maxwell, *Matter and Motion*, pp. 122–125.)

40. Fluids. — Some phenomena characterize fluids generally, others are peculiar to liquids or to gases separately.

Mechanics of liquids in general is *hydrodynamics;* of liquids at rest, *hydrostatics;* of liquids in motion, *hydrokinetics* or *hydraulics.*

41. Pascal's Principle. — Fluids transmit (not necessarily exert) pressure equally in all directions. This might be inferred from the mobility of a fluid. The principle is useful for multiplication of force, as in a hydrostatic press.

Pressure of a fluid at rest is normal to the surface pressed; otherwise, if resultant pressure were oblique, by reason of the perfect mobility of a fluid the particles pressed obliquely would move along the surface and the fluid would not be at rest.

Since the ocean surface is not that of a sphere, normals to the "level" ocean surface do not pass through the center of the earth. As a consequence of the freedom of movement of the particles of a fluid, it cannot have shearing elasticity, but only volume elasticity. Of fluids, only liquids have a "free surface." (See definition of liquid and gas, Art. 14.)

42. Pressure of a Liquid Due to Weight. — Under gravity, the pressure of a liquid is proportional to the depth below the free surface. Also, *the pressure upon any submerged surface equals the weight of a column of the liquid whose base is the area pressed, and whose height is the distance from the center of gravity of the area to the free surface of the liquid.* Thus, in a cubical vessel filled

with liquid, the pressure on each side is one-half the weight of the contents of the vessel, and the total pressure on the sides and bottom equals three times the weight of the liquid. A tall but very narrow column of liquid will suffice to burst a strong cask, though only a small quantity of the liquid is needed to fill a tube containing such a column to a considerable height.

43. Center of Pressure. — This is a point of a surface under pressure at which the total pressure on the surface might be regarded as concentrated. It is then a point at which an equal opposite pressure would maintain the surface in equilibrium. If *AB*, Fig. 15, represents a strip (or line) as a vertical element in the side of a vessel under pressure from the liquid of depth *AB*, and *D* is the center of pressure, then a contrary pressure *R* at *D* equal to the total outward pressure on *AB* will exactly counterbalance that outward pressure.

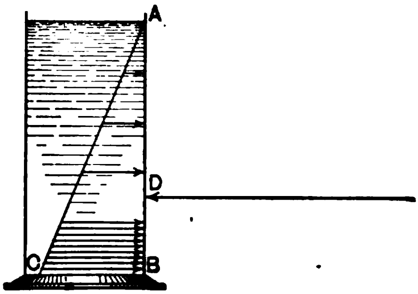


Fig. 15. Center of Pressure.

The center of pressure is always below the center of gravity of the surface pressed unless the latter is horizontal, in which case all its points are at the same depth below the surface of the liquid. For a strip of uniform width, as *AB*, since the pressure is proportional to the depth, if p is the pressure at the depth unity, then at any depth d the pressure is pd, and may be represented by the arrowheaded lines which increase in length uniformly from *A* to *B*. The bounding line *AC* is a straight line of uniform slope, and the total pressure against *AB* would be represented by the sum of all the pressure lines from *A* to *B*, i.e., by the area of the triangle *ABC*. If the triangle were laid upon *AB* as a base and its weight were equal to the total pressure of the liquid, then the base *AB* of the triangle would have the pressure distributed upon it just as the pressure of the liquid is distributed upon the vertical face *AB*. But the weight of this triangle as a whole may

be considered as acting at the center of gravity of the triangle, which is two-thirds of the distance from A to B, or the center of pressure on AB is at two-thirds of the depth of the liquid. The c.g. of the filament AB is at one-half the depth AB.

Note. — The center of pressure, in statics, corresponds to the center of percussion in dynamics. The determination of this point belongs to higher mechanics, but it may be located in this wise.

Regarding the portion of the surface under pressure as a detached plane surface suspended from the line of its intersection with the upper surface of the liquid, and

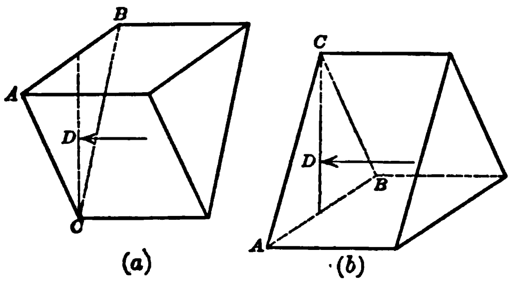


Fig. 16. Center of Pressure and of Percussion.

oscillating about that line as an axis, the vertical distance from the axis to the center of pressure is equal to the length of the simple pendulum oscillating in the same period with it. If the surface were the triangle ABC, in Fig. 16, (*a*) and (*b*), in the first the center of pressure D is at one-half the depth of the liquid, and in the second case, at three-fourths the depth. The center of percussion is a point at which a blow would produce no shock upon the axis of suspension. (The distance from the axis of suspension to the center of percussion or the center of pressure equals the moment of inertia of the surface divided by the moment of the surface.)

EXAMPLES. —

1. A hole 20 cm. square is made in the vertical side of a ship, the upper edge of the hole being 2 meters below the surface of the water. What force must be exerted to keep the water out by holding a board against the hole, supposing one cubic centimeter of sea water to weigh 1.025 grams (Art. 42).

Ans. 86+ kg.

2. If (*a*) and (*b*), Fig. 16, are prismatic vessels of the same dimensions, filled with the same kind of liquid, how does the pressure against the end of one vessel compare with that against the end of the other?

Ans. Twice as great in (*b*) as in (*a*).

44. A Liquid at Rest in Communicating Vessels. — From Art. 42 it is seen that any number and variety in shapes of vessels containing a given liquid, and having the same size of

base, will have the same pressure on that base if the liquid is at the same height above the base; and if all are in communication with one another by any variety of size in the opening from one to another, the liquid will rise to the same height in all to come to equilibrium; for wherever any one connects with any other the *surface of union is a common area* to both, and will be equally pressed by the liquid in both only when there is an equal height of liquid above it on both sides.

Illustrate with various-shaped vessels, also with apparatus, to show equal pressure in all directions, laterally and vertically.

45. Archimedes' Principle. — A body wholly or in part immersed in a fluid loses a part of its weight equal to the weight of the fluid displaced. This statement, known as the principle of Archimedes, may be so extended as to apply to the case where the loss of weight is greater than the weight of the body, in which case the weight remaining to the body is a negative quantity, or a force greater than its own weight is required to keep the body submerged. This loss of weight is the buoyant effort of the fluid upon the body. If the body weighs more than its own bulk of the fluid, it will sink through the fluid under the action of gravity, but with diminished weight; if it weighs less than its bulk of the fluid, it will sink into the fluid until it has displaced as much fluid as has a weight equal to that of the body. The principle was originally applied only to solid bodies immersed in liquids, but it holds equally for all fluids, gases as well as liquids. (Hence the difference between the weight of a body in air and in a vacuum.)

Demonstrate for liquid by the hydrostatic balance with the cup and cylinder; also by the Cartesian divers.

Experiments Nos. 14 and 15, pages 92–94. — Demonstration and Illustrations of Archimedes' Principle.

46. Buoyancy. — This term expresses the resultant of the forces tending to sink a body in a liquid and those supporting it, and means that the total upward pressure upon the body may be more (or less) than the total downward pressure and weight combined.

A body such as wood or cork rises through and floats upon

water because, we often say, it is specifically lighter than water, and correspondingly we say that iron sinks because it is heavier; but a floating body if submerged will rise, not because it is specifically lighter than the liquid, but because of the vertical upward pressure acting upon it. That such pressure should be greater upward than downward is owing sometimes, it is true, to the less specific gravity of the body, but *it is that pressure that causes the body to rise.* In Fig. 17 the body C is pressed downward by the weight of a column of liquid of height EF plus the weight of C, and is pressed upward by the weight of a column of the liquid of the height GF. If C be pressed upon the bottom of the vessel so as to exclude the liquid from beneath it, it cannot rise, no matter how light the body nor how heavy the liquid; it is the more securely fixed to the bottom the greater the sp. gr. of the liquid.

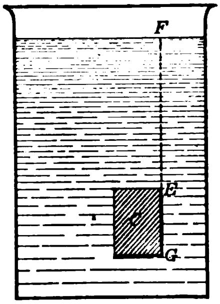


Fig. 17. Pressure upon a Submerged Body.

Experiment No. 16, page 94. — Cork Remains Submerged under Mercury.

The body, then, comes to the surface by reason of the upward pressure under it. (Swimming is no easier in very deep than in moderately deep water.) Ordinarily, in such discussions, the part played by the pressure of the air is not considered, as it usually counteracts itself, aiding the upward pressure of the liquid as well as pressing the body down. If such condition does not exist, if the air pressure acts only in one direction, the conditions of equilibrium would require that this be taken into account; and, to *generalize* the statement of the conditions of buoyancy, we would say: "A floating body sinks to a depth such that the buoyant effort which the liquid exerts upon it is greater than the pressure which the air exerts upon it from above downwards, by an amount equal to the weight of the body."

Now let us conceive of a vessel containing a liquid, and that a body fitting the vessel air-tight is placed upon the liquid. There is then air pressure upon the body from above, there is the weight

of the body, and there is liquid pressure from below, and if it is a floating body, the preceding statement exactly applies. Suppose the relative positions of the vessel, liquid and float to remain the same with the vessel inverted; then the proposition would read conversely: "The pressure which the air exerts upon the body from below upwards is greater than the pressure which the fluid exerts upon it from above downwards, by an amount equal to the weight of the body, and equilibrium results. In case the upward pressure is still greater, the body rises."

Experiment No. 17, page 94. — One Test Tube Falling Upwards into Another.

47. Specific Gravity. — This is a term to be applied to substances, not to bodies; for two bodies of the same material may differ in gravity, which is weight, but be alike in specific gravity. Whenever the term "specific" is applied to a quality, it means the measure of that quality in one kind of material as compared with the measure of it in a specified quantity of some specified substance taken as a standard. In physics, the substance oftenest used as a standard is water, and "specific gravity" of a substance means the weight of any portion of such substance compared with the weight of *an equal volume of water*. Its numerical value is the ratio of the weight of the substance to the weight of an equal volume of water, and therefore it is an abstract number. Obviously the size of the body used in the comparison makes no difference, since it is to be compared with a body of water of the same size. (See Barker, p. 162.)

Archimedes' principle is the basis of the methods of determining specific gravity by weighing, for if a body is weighed in air and then in water, the loss of weight in water is the weight of an equal volume of water, and therefore the specific gravity is the weight in air divided by the loss of weight in water.

$$\text{Sp. gr.} = \frac{\text{wt. in air}}{\text{loss of wt. in water}}. \qquad \text{(A)}$$

If we take as unit weight the weight of unit volume of water, as, e.g., one gram is weight of one c.c. of water, then the loss of weight in water is at once the weight (in grams) of its volume of

water and also the number of units of volume (c.c.) in the body, and the specific gravity then may be the ratio of the weight to the volume, or from (A),

$$\text{sp. gr.} = \frac{\text{wt. in grams}}{\text{vol. in c.c.}} \text{ and vol. in c.c.} = \frac{\text{wt. in grams}}{\text{sp. gr.}}. \qquad \text{(B)}$$

48. The Celebrated Story of "Eureka." — The following account of Archimedes' discovery of his "principle" is quoted by Mach (*Science of Mechanics*, pp. 86, 87), from Vitruvius, *De Architectura*.

"Hiero, when he obtained the regal power in Syracuse, having, on the fortunate turn in his affairs, decreed a votive crown of gold to be placed in a certain temple to the immortal gods, commanded it to be made of great value, and assigned for this purpose an appropriate weight of the metal to the manufacturer. The latter, in due time, presented the work to the king, beautifully wrought; and the weight appeared to correspond with that of the gold which had been assigned for it.

"But a report having been circulated that some of the gold had been abstracted, and that the deficiency thus caused had been supplied by silver, Hiero was indignant at the fraud, and, unacquainted with the method by which the theft might be detected, requested Archimedes would undertake to give it his attention. Charged with this commission, he by chance went to a bath, and, on jumping into the tub, perceived that just in the proportion that his body became immersed, in the same proportion the water ran out of the vessel. Whence, catching at the method to be adopted for the solution of the proposition, he immediately followed it up, leapt out of the vessel in joy, and, returning home naked, cried out with a loud voice that he had found that of which he was in search, for he continued exclaiming, in Greek, εὕρηκα, εὕρηκα (I have found it! I have found it!)."

49. Solution of the Problem of Hiero's Crown. — The method by which the principle was applied in this famous case is shown in the following example (from Goodeve's *Mechanics*):

"The crown of Hiero, with equal weights of gold and silver, were all weighed in water. The crown lost $\frac{1}{14}$ of its weight, the gold lost $\frac{4}{77}$ of its weight, and the silver lost $\frac{2}{21}$ of its weight. Prove that the gold and silver were mixed in the proportion of 11 : 9."

If $\frac{1}{x}$ = the part of the crown's weight that was gold, then $\frac{x-1}{x}$ = the part of the weight that was silver, and the ratio of gold to silver is $\frac{1}{x-1}$.

Let w = the weight of the crown.

If any weight of gold loses $\frac{4}{77}$ when weighed in water, its specific gravity is weight in air divided by loss of weight in water, or $\frac{77}{4}$, and from equation (B), Art. 47, the volume of any quantity of gold will be wt. ÷ sp. gr.

Weight of gold in the crown is $\frac{1}{x} w$, and $\frac{4 w}{77 x}$ = volume of gold in crown. Similarly, $\frac{2}{21} \frac{(x-1)}{x} w$ = volume of silver in crown, and $\frac{1}{14} w$ = volume of the crown itself. Equating the sum of the volumes of the two metals with the volume of the crown itself,

$$\frac{4}{77} \frac{w}{x} + \frac{2}{21} \frac{(x-1) w}{x} = \frac{1}{14} w;$$

This equation gives $x = \frac{20}{11}$.

Then $\frac{1}{x}$ is $\frac{11}{20}$ which is the gold, and $\frac{9}{20}$ is the silver, or the gold is to the silver as 11 : 9, Q.E.D. Also, the above ratio $\frac{1}{x-1} = \frac{11}{9}$. Q.E.D.

A similar method may be applied to the determination of the respective weights of gold and quartz in a nugget containing the two substances.

EXAMPLES. — (The buoyant effect of the air is neglected.)

1. A piece of cork of mass 300 g. and sp. gr. 0.25 is placed in a vessel full of water. How much water will overflow? How much would overflow if the float were a piece of wood of the same mass as the cork, and sp. gr. 0.92?
Ans. 300 c.c.

2. A body weighs 62 g. in air and 42 g. in water. What is its specific gravity? What its volume? *Ans.* 3.1; 20 c.c.

3. The sp. gr. of ice is 0.918 and that of sea water 1.03. What is the volume of an iceberg which floats with 600 cubic meters exposed?
Ans. 5518 cubic meters.

4. A solid which weighs 120 g. in air weighs 90 g. in water, and 78 g. in a solution of zinc sulphate. What is the sp. gr. of the solid and of the solution?
Ans. 4; 1.4.

5. A bottle weighs 25 g. when empty, 64.74 g. when filled with water, and 65.53 g. when filled with milk. What is the sp. gr. of the milk?
Ans. 1.02.

If a piece of rock of weight W consists of gold and quartz, let x and y be the weights of gold and quartz respectively, m, n, r, the sp. grs. of gold, quartz and the mixture. Then

$$W = x + y. \tag{1}$$

Also, volume of specimen = vol. of gold + vol. of quartz, or, from Art. 47,

$$\frac{W}{r} = \frac{x}{m} + \frac{y}{n}, \tag{2}$$

and from Eqs. (1) and (2) the values of x and y may be determined.

6. A diamond ring weighs 65 grains in air and 60 grains in water; find the weight of the diamond if the sp. gr. of gold is 17.5 and that of diamond 3.5.

Ans. 5.625 grains.

7. The sp. gr. of milk is 1.02. A quantity adulterated with water is found to have a sp. gr. of 1.015. What proportion of water has been used? (Suggestion: 1 c.c. of the mixture weighs 1.015 g., 1 c.c. of water weighs 1 g.) *Ans.* Volume of water equals one-third of the volume of milk.

50. **Density.**—This term, like specific gravity, is to be used not so much to distinguish one body from another as to distinguish the kinds of material of which the bodies are composed. It is the relation of the mass of a body to its volume, and would, therefore, be the same for all bodies of the same material, whatever their size.

When so used, density must be expressed as so many units of mass per unit volume; e.g., grams per cubic centimeter, or pounds per cubic foot, etc.; and the number thus obtained is called the *absolute density* of the substance.

Absolute density of any substance, then, may have different numerical values according to the units in which it is expressed.

When density is expressed in comparison with that of some particular substance taken as a standard, the number expressing it is an abstract number, and this is called the *specific density* of the substance.

The specific density of any substance, then, may have different numerical values *according to the substance that is chosen as a standard.* If the standard substance have unit mass in unit volume, the specific density will be numerically the same as the absolute density. This is the case with water measured in c.g.s. units, for a mass of one gram of water at its maximum density has a volume of one cubic centimeter.

The mass of any body in grams divided by its volume in cubic centimeters is its absolute density in grams per cubic centimeter, and the numerical value of this ratio is the specific density, compared with water.

In physical comparison of densities water is usually taken as the standard, but sometimes gases are compared with air, or, more frequently, with hydrogen as a standard. Then the absolute density might still be expressed as gms./c.c., while the specific

density would be a different number. For example, the absolute density of air would be 0.001293 g./c.c., and if hydrogen were the standard for comparison the specific density would be 14.43, but if water were the standard the specific density would be 0.001293.

Since the weight of any body is directly proportional to the mass of it, the weight per unit volume of various substances will be in the same proportion as the mass per unit volume of the same substances.

The ratio of the weight of, say, one cubic centimeter of a substance to that of the same volume of water is its specific gravity. The ratio of the mass of one cubic centimeter of a substance to that of one cubic centimeter of water is its specific density; and this *ratio* is the same for masses and for weights; therefore, when the same substance is chosen as the standard for specific gravities and for specific densities, the specific gravity of a substance and its specific density are the same number, and if the standard substance is water, the specific gravity, the specific density, and the absolute density in grams per cubic centimeter will all be the same number.

Note.— Density should not be defined as the closeness of particles in a substance. While it is true that if a given mass of a substance, say, air, for example, have its particles closer (under compression) at one time than at another, it will be denser, it does not follow that of two unequally dense substances the particles of the denser one are closer together than those of the rarer. Especially is this noticeable with gases, in which, as will be shown (Arts. 77, 78), hydrogen, containing the same number of molecules in a given volume as oxygen, is only about one-sixteenth as dense.

51. Determination of Density.—Any contrivance to find the number of grams in a cubic centimeter of a substance will determine its density, and any contrivance to compare the weight of a body with the weight of an equal volume of water will determine the specific gravity of the material of the body. This is with the understanding that water is the standard for densities as well as for specific gravity, so that the same number expresses the specific gravity and the specific density.

The forms of apparatus and the methods of using them are various, the following being the principal ones:

(*a*) The hydrostatic balance.
(*b*) Hydrometer, constant immersion.
(*c*) Hydrometer, constant weight.
(*d*) Mohr's (or Westphal) balance.
(*e*) Jolly's balance.
(*f*) Specific-gravity bottle (pycnometer).
(*g*) Volumometer (stereometer) (can be explained only after Boyle's law, and barometer).
(*h*) Hare's apparatus.

(Any convenient selection of these may be used for illustrating the lectures.) The principle of the last-named needs explaining.

In any column of liquid the downward unit pressure under gravity varies as the height, or $p \propto h$; also for a given height of column, the pressure varies as the density, or $p \propto \delta$ (density); therefore, in general, $p \propto \delta h$. In the apparatus, Fig. 18, the pressure due to each column is equal, being that of the atmosphere minus the pressure of the air in the common space above the two liquids. Therefore, calling the pressure, density and height p, δ and h respectively, $p_1 = k\delta_1 h_1$, $p_2 = k\delta_2 h_2$; but $p_1 = p_2$, therefore, $\delta_1 h_1 = \delta_2 h_2$, or

$$\frac{\delta_1}{\delta_2} = \frac{h_2}{h_1}. \qquad \text{Q.E.D.}$$

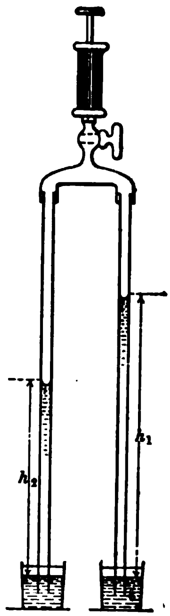


Fig. 18. Comparison of Densities by Liquid Columns.

For further consideration of specific gravity and density, see Lodge, *Elementary Mechanics*, Arts. 182 to 187; and Glazebrook and Shaw, *Practical Physics*. (Alcoholometers, lactometers, etc., are only specially graduated forms of hydrometers.)

Experiments Nos. 18 and 19, page 95. Specific Gravity and Densities.

EXAMPLES. —

1. A solid iron cylinder 5 cm. in diameter and 10 cm. long has a mass of 1531.5 g. What is its density? *Ans.* 7.8 g. per c.c.

2. If the specific density of mercury is 13.6, how many grams will be required to fill a tube 76 cm. long and 4 mm. in internal diameter?

Ans. 129.9 g.

3. The volume of a balloon filled with coal gas is 1000 cubic meters, and its weight 400 kg. If the density of the gas is 0.0007 g. per c.c. and that of the air 0.001293 g. per c.c., what additional weight can the balloon sustain in the air? *Ans.* 193 kg.

4. In Hare's apparatus, Fig. 18, a solution of zinc sulphate stands in one tube at a height of 43 cm., while the level of the water in the other tube is at a height of 60.2 cm. What is the specific density of the solution? What its absolute density? *Ans.* 1.4; 1.4 g. per c.c.

52. **Surface Tension.**— Up to this point, solids, liquids and gases have been considered in mass (molar physics); certain features, however, especially of liquids and gases, must be regarded in their intermolecular actions (molecular physics).

The action of molecular forces is more easily examined in fluids than in solids. The form of a liquid which is separated from adjacent surfaces by force between its molecules is globular. In a small quantity of the liquid the molecular attraction is largely in excess of the effect of gravity, and the body is almost strictly spherical,— perfectly so, if it is in rotation about various axes. But when the mass is large it becomes flattened into a spheroidal form under the influence of gravity. The direct effect of molecular attraction (cohesion) in such a liquid is to put the surface layer in a state of tension. This will bring the liquid mass into such a form as to include the mass with the least extent of surface possible; i.e., the potential energy of the surface film will come to a minimum.

Experiments Nos. 20 and 21, page 95. — Surface Tension.

It may be shown that the energy of a particle of a fluid is greater when the particle is very close to the surface of that fluid ($\frac{1}{1000}$ mm.) than when it is at a greater distance from the surface. An effect of this is that the particles near the surface are drawn inward toward the mass of their own fluid, and the surface is put into a state of tension, called "superficial tension." The energy of the molecules constituting the surface layer, due to the action of molecular forces, is superficial energy, and it is upon this

energy that capillary phenomena depend. Superficial tension, or the tension in a film, whether single or double, is seen in the contractile force exerted by a soap bubble, and the measure of the surface tension is the force per centimeter exerted by the film *across* a line in the surface.

Superficial energy depends upon the nature of both media of which the surface is a boundary. The media must be such as do not mix or we should have diffusion, but there is a coefficient of superficial energy for every surface separating two liquids that do not mix, a liquid and a gas, a gas and a solid, or two solids — none for two gases as any two gases diffuse into each other.

Suppose we have a glass beaker partially filled with water. We then have the three media—glass, water and air—meeting along the circular line around the glass. Denote the tension of the surface separating the glass and water by T_w^g, of that between glass and air by T_a^g, and that between air and water by T_w^a. These three tensions must be in equilibrium along the common line of junction, and therefore may comprise a triangle of forces. Furthermore, the angular positions of these surfaces, relatively to each other, will depend only upon the superficial tensions separating them, and therefore will always be the same for the same substances.

But it is not always possible to construct a triangle of three given lines. One must not be greater than the sum or less than the difference of the other two, and so not every three substances will take a position of equilibrium with distinct surfaces of separation. For example, if the tension of the surface separating air and water is greater than the sum of the tensions between air and oil, and oil and water, then a drop of oil on water will not come into equilibrium but will spread out, inserting itself continually between the more urgently separated fluids, and following the shrinking surface of separation between water and air, the oil getting thinner indefinitely.

When the superficial tension of a liquid is given, it is always understood that the liquid is in contact with air as the other medium, unless stated otherwise.

Suppose a horizontal surface of a solid, as glass, is in contact with two fluids, f_1 and f_2; if the tension of the surface separating the solid from the first fluid is greater than the sum of the tensions between glass and f_2 and between f_1 and f_2, the first fluid, f_1, will gather into a drop, and f_2 will spread over both f_1 and the solid (glass). If one of the fluids is air (usual case), and the other is a liquid, then the liquid, if it corresponds to f_1, will gather into drops. This would be the case with mercury; but if the liquid corresponds to f_2, it will spread over (i.e., wet) the surface of the glass; this would be the case with water.

If the above inequality does not apply, the tension of the surface separating two of the substances is greater than the

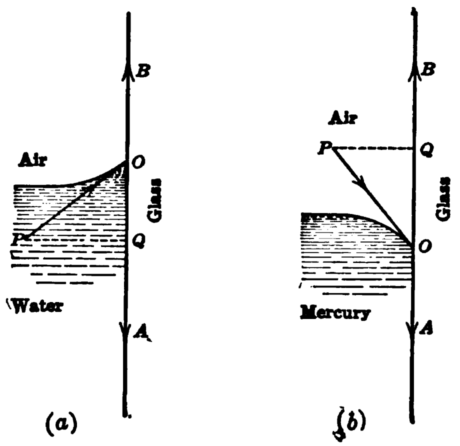


Fig. 19. Surface Tension.

difference of the other two tensions. Suppose, for illustration, in the vessel before us (a beaker partly filled with liquid), $T_w^a > T_w^g - T_a^g$. The side of the vessel being vertical, the tension between the glass and each fluid will be vertical. Suppose O, Fig. 19 (a), represents the upper edge of the liquid. Let $OB = T_a^g$, $OA = T_w^g$, and $OP = T_w^a$. Make $OQ = T_w^g - T_a^g$; then, whatever be the value of the tension between air and water ($= OP$), P cannot be higher or lower from O than Q, and, the tension being greater than OQ, takes a position like OP, the sur-

face of the liquid making an angle with that of the solid, POQ, which is called the "angle of capillarity" α. (In fact, with water and glass, $\alpha = 0$, and the liquid film is spherical.) In cases where the resultant of the three tensions acts downwards, the liquid is depressed instead of elevated as in the case of mercury in glass, where $\alpha = 45^\circ \pm$, Fig. 19 (*b*).

Note. — The film, being in a state of stress, possesses potential energy, and its tendency at any point is so to readjust itself as to reduce this energy to a minimum. It will therefore contract to its smallest possible area. If it incloses a given volume of substance, the form of least surface is a sphere.

As to the pressure of the film upon the figure which it incloses, if it has a curved surface, the normal pressure, due to tension T along a line of section whose radius of curvature is R, is proportional to $\frac{1}{R}$, and if R' is the radius of curvature of a section perpendicular to the first, the pressure per unit of area at their intersection is as $\frac{1}{R} + \frac{1}{R'}$. In a sphere $R = R'$, and the pressure is proportional to $\frac{2}{R}$. The sum of $\frac{1}{R} + \frac{1}{R'}$ represents actually the difference of pressure on the two sides of the film, and the film will always take a form such that at any point $\frac{1}{R} + \frac{1}{R'}$ shall be a minimum. If the film has the same pressure on both sides, as, e.g., a soap film on a wire frame and exposed to atmospheric pressure on both sides, then $\frac{1}{R} + \frac{1}{R'}$ must equal zero; and if one radius is positive, the other is negative; if the surface is warped in shape, then at any point where the curve of section is concave along one line there must be, along a direction at right angles to this line, an equal curvature in the opposite sense, or convex. If the figure is plane, R and R' are both infinite.

Water has a greater surface tension than any other liquid except mercury. It decreases rapidly with rise of temperature, and is much lessened by impurities.

Experiments Nos. 22 to 24, page 96.— Surface Tension.

53. Capillarity. — When a tube of fine bore, open at both ends, is dipped into a liquid, the surface tension not only raises or depresses the liquid in contact with the material of the tube on the outside, but may raise or depress the column of liquid inside the tube to a considerable distance as measured from the general level of the liquid at a distance of a centimeter or more from the tube. The action on the liquid in the tube is known as capillarity. It is due to surface tension and applies also to a

liquid between two plates. If, in Fig. 20 (1), b is the distance between the plates, h ($= BC$) the height which the liquid rises above the outside level, and l the length of the prism thus lifted, the volume thus supported is bhl; if w is the weight per unit volume, the total weight is $wbhl$. This liquid is drawn up by the tension of the surface film along the glass. If the tension is

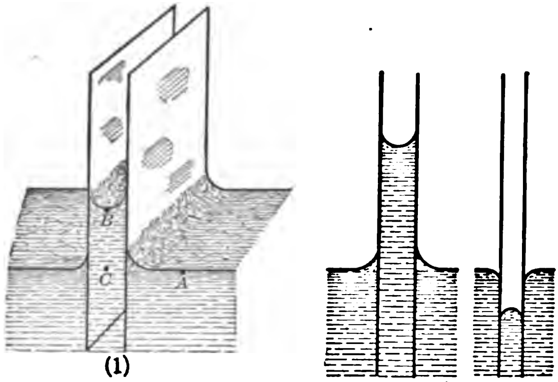


Fig. 20. Elevation or Depression of Liquids by Capillarity.

T dynes per centimeter of its length, and the liquid joins the glass at an angle α, the vertical component of the force would be $T \cos \alpha$ dynes per centimeter. In the case of water, $\alpha = 0$, and the lifting force is T dynes/cm. As there is such a line along each plate, the total lifting force is $2\,Tl$. Equating this with the weight supported, we have $2\,Tl = wbhl$,

whence, $$h = \frac{2\,T}{wb}. \qquad \text{(A)}$$

T and w should be expressed in like units. If T is dynes, and w is grams, the latter must be multiplied by 980. If, instead of plates, a tube of inner radius r (Fig. 20 (2)) be put into the liquid, the length of film exerting tension is $2\,\pi r$ and its force is $2\,\pi r T$; the volume of liquid raised is $\pi r^2 h$, and its weight $\pi r^2 h w$; equating these, $2\,\pi r T = \pi r^2 h w$,

whence, $$h = \frac{2\,T}{wr}. \qquad \text{(B)}$$

This height would be the same as with the plates if $r = b$; if the diameter of the tube is equal to the width between the plates, $r = \frac{1}{2} b$, and the height of rise in the tubes is twice as great as between the plates. Both with tubes and plates the rise of the liquid is inversely proportional to the width of the column.

(Shown by capillary tubes as in Fig. 21.)

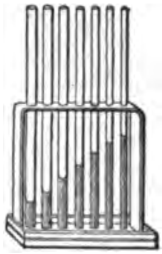


Fig. 21.—Rise of Liquids by Capillarity.

In the case of the liquid raised between the two plates, the fluid pressure at the level surface outside the plates, as at A, Fig. 20 (1), is equal to the atmospheric pressure upon it, and it is the same as this at all points in the liquid at the same level, as at C. Above C there is a column of liquid to B, and upon that the atmosphere. The pressure in the liquid at B is less than at C to the extent that the surface tension at the top of the column has opposed the weight of the atmosphere; i.e., to the extent of the pressure due to the height BC of the liquid. (See Art. 52, Note.) Upon that part of the plates between which there is a liquid column, the pressure of the liquid outward against the plates is less than the pressure of the atmosphere on the outside forcing the plates together. Accordingly, if two clean glass plates are in contact except for a very thin layer of water which has a sharp curvature concave outward all along the edge of the stratum of water, the pressure of the liquid outward is less than that of the atmosphere, and the plates require a considerable force to separate them. The spread of a drop of water between the plates of itself pulls them together. If the water is thin enough to sustain a column 30 cm. in height, this would be approximately 0.03 of an atmosphere in pressure, or the excess of pressure holding the plates together would be 30 grams per square centimeter. On plates 10 cm. square this would be a total pressure of 3 kg.

On the other hand, if two such plates are forced together with a layer of mercury between them, the resistance becomes greater as the layer becomes thinner, forming an exceedingly elastic

cushion. (See Tait, *Properties of Matter*, Chap. XII; also Glazebrook, *Laws and Properties of Matter;* also article on "Capillarity" by Clerk Maxwell in *Encyclopædia Britannica*.)

Experiments Nos. 25 and 26, page 97, showing capillary sustaining force in tubes and between plates.

EXAMPLE.—If the surface tension of water is 81 dynes per centimeter, how high will water rise by capillarity in a glass tube one-tenth of a millimeter in diameter? To what depth would mercury be depressed in the same tube if the surface tension is 540 dynes per centimeter?

Ans. Water, 33 cm.; mercury, 11.43 cm.

54. Pneumatics.—The science of gases. As gases have a tendency to indefinite expansion, so, upon increase of pressure, their volume may be reduced.

Expansion on removal of pressure may be shown by rubber balloon or other thin closed vessel under receiver of air pump.

Experiment No. 27, page 97.—Expansive Force of a Gas.

55. Atmospheric Pressure.—The downward pressure of the atmosphere may be shown by the pressure of a receiver upon the plate of an air pump. (Action of the pump to be explained later.)

Upward pressure may be shown by card under an inverted beaker full of water.

Experiment No. 28, page 97.—That the water is not retained in the vessel by any effect of the card acting as an ordinary stopper may be shown by using a beaker with the open end covered with mosquito netting. Placing the card in position as before, it is held up and the water does not escape; but while the vessel is thus inverted the card may be taken away and still the water does not run out if the under surface is kept horizontal. The card may be dispensed with by simply placing the hand over the netting when inverting the beaker. (The surface film helps somewhat in supporting the liquid.) As soon as the beaker is inclined slightly, the pressure is different at different parts of the lower surface and the water escapes. (See Weinhold, *Physikalische Demonstrationen*, p. 203.)

Experiment No. 29, page 98.—Pressure in all directions is shown by the fact that the Magdeburg hemispheres hold alike in every position,—horizontal, vertical, or any other.

To calculate pressure holding the hemispheres together, compute pressure upon area of a great circle, not of the surface of a hemisphere.

The original experiments with the air pump were conducted by Otto von Guericke at Magdeburg (1650–1672).

Prior to the time of Galileo, the rise of water in a common pump, and similar phenomena, were ascribed to Nature's horror of a vacuum, and as early as 1640 Galileo had made attempts to weigh air. In 1643 Torricelli, a pupil of Galileo, measured the extent of the *horror vacui* by means of the mercury column in a long tube, forming the ordinary barometer column, with what is known as a Torricellian vacuum above it. He thus showed that Nature's horror of a vacuum did not extend above about 76 cm. of mercury, and he was able to determine the weight of the atmosphere. At the earth's surface the pressure of the air is equal to the weight of a column of air, everywhere as dense as at the earth, five miles high. If a vacuum were formed, air would rush in to close it with a velocity equal to that of a body that has fallen from a height of five miles under gravity, or about 1300 feet per second. (See *infra*, Art. 61.)

Torricelli's work was confirmed and extended by Pascal, with columns of various kinds of liquids, 1647–1653. Pascal showed (1648) that the height of the mercury column, or any liquid similarly supported, is due to *air pressure*. After the experiments of his brother-in-law, Perier, at Puy de Dôme, Clermont (a mountain about 4300 feet high), in which experiments the barometer showed the (to Pascal) astonishing difference of over three inches at the foot and at the top of the mountain, Pascal made various further experiments at lesser heights in the tower of the church of St. Jaques, Paris, about 150 feet high.

Investigations of air and air pressure were made by Robert Boyle, and by Mariotte, about 1660.

56. Barometers. — Mercurial, aneroid; exhibit and describe. Standard barometer height is 76 cm. height of mercury column at temperature of 0° C., at sea level in latitude of 45°. Lowest barometer record for New York was 28.61 inches, January 3, 1913.

with an increase of P for a while, the curve sloping downward; and then as P is further increased, PV also increases, giving a curve sloping up. If PV were constant, the curve would be a horizontal line. The significance of this will be brought out more fully under the subject Heat.

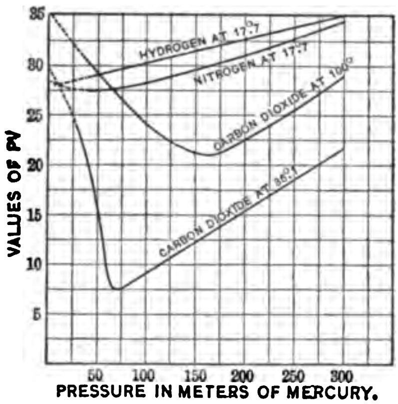


Fig. 23. Departure of Gases from Boyle's Law at High Pressures.

It is seen, however, that Boyle's law is very nearly true for gases in a high state of attenuation. The particles of a gas may then be thought to be in some sense analogous to those of a substance dissolved in water, if the solution is very dilute. (See article "Osmosis.")

59. Elasticity of a Gas at Constant Temperature.—If the gas conforms to Boyle's law, suppose we have a volume V at pressure P. Then the volume will be diminished in just the same proportion that the pressure is increased. If to P is added a small pressure p, this is $\frac{p}{P}$ part of the original pressure, and the volume will be diminished $\frac{p}{P}$ part of the original volume, or the decrease in volume is $\frac{p}{P}V$. Now the strain is the ratio of the change of volume to the original volume, or $\frac{p}{P}V \div V$, or the strain equals $\frac{p}{P}$. The stress is p, and

$$\text{elasticity} = \frac{\text{stress}}{\text{strain}} = \frac{p}{\frac{p}{P}} = P.$$

That is to say, for a gas at constant temperature, the elasticity is equal to the pressure, a result that we shall recur to in the study of sound.

Note. — This may be more simply shown by the calculus. If $PV = \text{const.}$, P and V being both variable, then by differentiating, $P\,dV + V\,dP = 0$; whence $\frac{dP}{dV} = -\frac{P}{V}$, or $-V\frac{dP}{dV} = P$. Now $dP = \text{stress}$ and $-\frac{dV}{V} = \text{strain}$ and $E = \frac{\text{stress}}{\text{strain}} = -\frac{V\,dP}{dV} = P.$

60. Air Pumps. — Exhibit and operate: gauges, barometer column, siphon gauge, McLeod gauge (see Barker, p. 199); the Fleuss pump sold under the name "Geryk" vacuum pump, described in Watson, p. 155; mercury pumps, — Sprengel, Geissler (see S. P. Thompson, *Development of Mercurial Air Pump*).

61. Velocity of Efflux. — If a liquid is at rest in a vessel, as in Fig. 24, the pressure outward, due to the liquid, at a point B is proportional to AB, the depth of B below the free surface of the liquid. The total pressure is that due to the weight of the liquid and of the atmosphere above it. If an orifice is opened at B, there will be a back pressure of the atmosphere at B. The excess of pressure, then, to force the liquid out at B, is that due to the weight of the liquid from B to A. This excess of pressure with the liquid at rest represents potential energy at B equal in amount to the work of raising the liquid from B to A. If the liquid at B is not held so as to exert pressure against the vessel, but is free to escape, its potential energy is at once changed into kinetic energy, and the velocity of any mass m escaping is given by the equation TE = KE

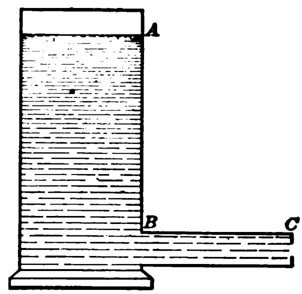


Fig. 24. Efflux of a Liquid.

$$mgh = \tfrac{1}{2}mv^2, \text{ whence } v^2 = 2gh.$$

This is independent of the density of the fluid. If, instead of a simple orifice at B, there is a pipe for outflow with a smaller orifice at the extremity C, calling v the velocity of efflux at C and v' the velocity of flow within the pipe BC, v' will be smaller than v and there will be a pressure p' upon the sides of the pipe to

correspond to a head h' smaller than h, so that $mgh' + \frac{1}{2} mv'^2 = mgh$, and $v'^2 = 2g(h - h')$.

62. Aspirating Action of Flow. — When a fluid is flowing through a pipe of uniform bore, if we neglect the additional force required merely to overcome the friction of the walls of the tube, the static pressure due to a given head will be the same at all places where the velocity is the same, as at *A* and *B*, Fig. 25 (*a*). But if at a point, as *C*, there is a narrowing of the pipe, the velocity in the throat at *C* will be increased, and the outward pressure decreased, since the energy of the fluid itself is not altered. If *C* is connected with the air or with another vessel, there will be a suction into the pipe *AB* at *C*.

If *BB*, Fig. 25 (*b*), is a perforated disk attached to the pipe *A*, and *DD* is a light disk close beneath it, by blowing through *A* the air escapes through the small space *C*, with a correspondingly increased velocity and, therefore, diminished pressure against the plate *DD*, and the latter rises and is held up against *BB*, and harder the harder it is blown against. It might seem that if the air between *BB* and *DD* has less pressure than the external air it would not escape, but it is the pressure transverse to the direction of motion whose diminution permits the rise of *DD*.

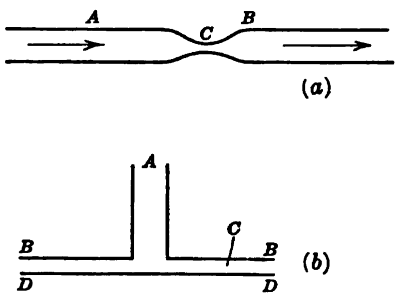


Fig. 25. Aspirating Action of Flow.

The added effect of the velocity gives the escaping air at the rim a total of energy due to the pressure and velocity greater than the energy equivalent to the pressure of the external air. This might be true even if the pressure within the space *C* to hold the plates apart is very small. Of course the velocity of the escaping air would have to be very great. (See Hastings and Beach, Art. 100; also article on "Bernoulli's Principle," by W. S. Franklin, in *School Science and Mathematics* for January, 1911.)

63. Siphons: Safety, Intermitting. — To explain the action of the siphon, consider the forces acting to drive the liquid past the highest point *B* in Fig. 26.

The force to drive the liquid from right to left is the atmospheric pressure at *A* minus the pressure of the liquid column of vertical height from *B* to *A*; the force to drive from left to right is the atmospheric pressure at *E* minus the pressure of the liquid column of vertical height from *B* to *E*. Pressure to left exceeds that to right by pressure of liquid column of vertical height equal to the difference between levels of *A* and *E*.

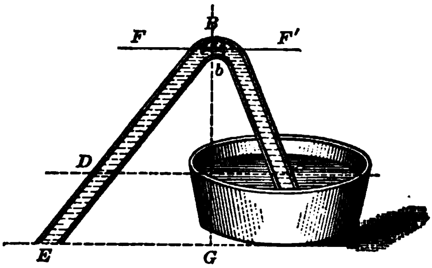


Fig. 26. The Siphon.

The intermitting siphon is sometimes arranged as a Tantalus vase (Fig. 27 (*a*)), the siphon being concealed within the body of the figure of a man of such height that the water rises nearly to his lips and then recedes.

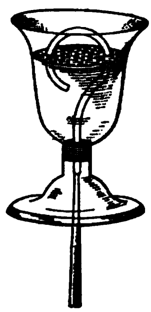


Fig. 27 (*a*). Tantalus' Vase.

Tantalus, having offended Jupiter by intemperate language, was consigned to Tartaros, and there punished as described by Homer, who makes Odysseus say: "And I saw Tantalus suffering grievous torments, standing in a lake, and the water dashed against his chin, but he resembled one thirsty, and could not take any to drink, for as often as the old man stooped, eager to drink, so often the water disappeared, being absorbed," etc. He only forgot his thirst in hearing the music of Orpheus. Origin of the word "tantalize."

The explanation of intermittent springs by a large cavity within a hill or mountain, from which a siphon-shaped channel leads, is discredited by geologists and physiographers as highly improbable, though it is possible that a siphon might be formed and its action occur intermittently by fissures and strata, somewhat

as roughly indicated in Fig. 27 (*b*), where *BCDE* is fed by several veins as *AA* and discharged through the longer leg *EF*, the water

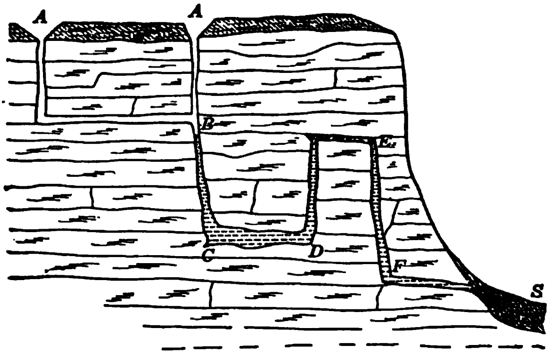


Fig. 27 (*b*). Intermittent Spring.

issuing from the soil finally at *S*. If efflux from *S* is more rapid than influx at *B*, the flow at *S* ceases when *CDEF* is emptied and resumes when it is filled. (Martonne, *Traité de géographie physique*.)

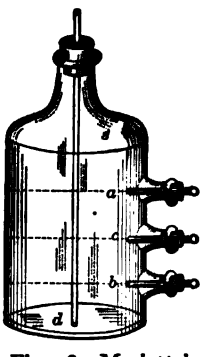


Fig. 28. Mariotte's Bottle.

Since the velocity of efflux from a siphon is determined by the height from the orifice of efflux to the surface of the liquid, it is independent of the nature of the liquid.

64. Mariotte's Flask. — This is a flask (Fig. 28) with side tubulures *a*, *b*, *c*, etc., for efflux of liquid, and a tube fitted air-tight, but which can be slid up and down through the stopper, thus fixing the height of the lower extremity *d* at pleasure. If *d* is higher than *b* the liquid escapes at *b* and the pressure in the space *s* above the liquid is slightly diminished, air entering at *d*. At *d* the pressure is just that of the atmosphere; from *d* upwards through the liquid the pressure diminishes and the air in bubbles rises, maintaining an air pressure in *s* which, added to the pressure of the liquid column *sd*, equals the pressure of the atmosphere. From *d* downwards the pressure increases and at *b* its

excess over that of the atmosphere is equal to that due to the height from *b* to *d*. This being kept constant, the velocity of efflux is constant, no matter what the depth of the liquid in the flask, so long as its surface is higher than *d*. The velocity can be regulated by the position of *d*. If *d* is below the level of the tubulure that is open, the liquid will not escape from that opening, and it will stand in the tube *cd* at the level of that opening.

The intermittent fountain (Fig. 29) illustrates the failure of a liquid to flow out of a vessel if the pressure of the air and liquid within the vessel is less than that of the atmosphere.

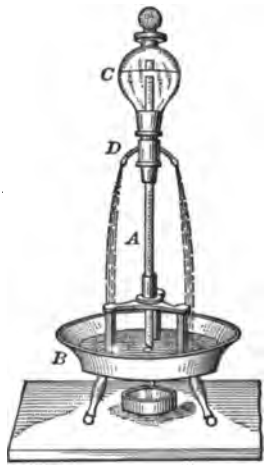


Fig. 29. Intermittent Fountain.

A large vessel *C* is closed air-tight at top and bottom but has one or more small orifices *D* through which water can flow out. A tube *A*, open at both ends, extends from the top of *C* nearly to the level of the basin *B*. Through a small orifice in the bottom of the basin liquid can escape, but not so rapidly as from the orifices *D*. As the water flows out of *C* air continually enters below, but the basin gradually fills up until the lower end of the tube *A* is closed. The action of the fountain then ceases until the water escaping from *B* opens the tube at the lower end, and the operation repeats itself.

65. Pumps. — Illustrate by model and diagram. An application of Boyle's law. Enlargement of air space under piston decreases the air pressure in it, and water rises under the excess of atmospheric pressure outside of the pump. Suction not a force but only a permission for excess of pressure to act.

66. Kinetic Theory of the Structure of Matter. — (Preliminary to Heat.) (See Watson's *Physics*, Arts. 140, 141; then Arts. 162, 163 and 164.)

This theory has nothing to do with the ultimate nature of matter, but sets out with the idea of atomic structure, and

proceeds with the combinations of atoms in the form of molecules, which are separate from one another and are perfectly elastic. The molecule is the elementary form, though the molecule may be a combination of several atoms, and the atom an aggregation of many corpuscles (or electrons). The theory applies to liquids and solids as well as to gases, but it is most readily exploited in connection with the gaseous form of matter, the essential ideas being embraced in the following seven propositions (see Wormell's *Thermodynamics*, pp. 154 to 160).

(1) The molecules of the same gas must be alike, but those of different gases must differ in properties or in structure. They must be separated by intervals which are very great compared with the size of the molecules.

(2) The molecules of a gas move in straight lines.

(3) When the molecules come into contact, they impinge so that their directions of motion change.

(4) All the molecules of the same gas have the same mass, and when they impinge they always rebound.

(5) In the same gas or mixture of gases the mean energy for each particle is the same.

(6) The pressure of a gas per unit of area is proportional to the number of molecules in a unit of volume and to the average energy with which each strikes this area.

(7) The pressure per unit area is proportional to the density of the gas and to the average square of the velocity.

The first five of these propositions being postulated, or accepted as not inconsistent, the last two call for demonstration.

For proposition 6, concerning the pressure exerted by a gas: Conceive of a cubical vessel, with one centimeter length of edge, filled with a gas of which each molecule has a mass m. Suppose the directions of the edges to be W.-E., N.-S., up-down. If the number of molecules is very great, the effect of their movement at any instant in all directions will be the same as if one-third of them were moving parallel to the W.-E. edge, one-third parallel to the N.-S. edge, and one-third parallel to the up-down edge. Also we may assume an average velocity for a particle

in each of these directions, and call it $\overline{V}$. When a molecule of mass m impinges upon a side of the vessel with a velocity $\overline{V}$ and rebounds with an equal velocity in the opposite direction, its change of momentum is $2\,m\overline{V}$. This will occur against the same face of the cube for one molecule as often as the molecule goes across the cube and back, or a distance of 2 cm. The time to travel 2 cm. is $2/\overline{V}$. Therefore, for one molecule a change of momentum of $2\,m\overline{V}$ is made in $2/\overline{V}$ seconds. Since the force thus exerted is expressed by the relation

$$\text{force} \times \text{time} = \text{change of momentum},$$

the force here is the rate at which momentum is changed; i.e., it is for each molecule $2\,m\overline{V}$ divided by $2/\overline{V}$ or $m\overline{V}^2$.

If N is the total number of molecules in this cubic centimeter of gas, then $\frac{1}{3}N$ may be considered the number moving parallel to one edge at any time, and therefore the number whose rebound causes the pressure on one face of the cube (or vessel containing the gas). The total pressure, therefore, on one face of the cube, or the pressure per square centimeter in any direction, is $p = \frac{1}{3}Nm\overline{V}^2$. The total kinetic energy of N molecules is $\frac{1}{2}Nm\overline{V}^2$. p, therefore, is proportional to the kinetic energy, being equal numerically for unit cube to two-thirds of it. Q.E.D. (Cf. Exs. (*a*) and (*d*), Art. 21.)

67. Mean Velocity of the Molecules of a Gas. — Let $\rho =$ density of a gas; this is the mass per unit volume, and if m is the mass of one molecule and N the number of molecules in unit volume, $\rho = Nm$. From the preceding article $p = \frac{1}{3}Nm\overline{V}^2$, and if for Nm we put ρ, then $p = \frac{1}{3}\rho\overline{V}^2$. This proves proposition (7). Then $\frac{p}{\rho} = \frac{1}{3}\overline{V}^2$. If $M =$ the entire mass of any quantity of gas, and v its volume, then $\rho = \frac{M}{v}$, and since $p = \frac{1}{3}\rho\overline{V}^2$, substituting in this the value of ρ we get

$$pv = \tfrac{1}{3}M\overline{V}^2 = \tfrac{2}{3}\cdot\tfrac{1}{2}M\overline{V}^2.$$

That is, if M is the mass of a given volume, the product of the pressure by the volume is two-thirds of the energy of translation of the molecules of that gas.

With a gas that conforms to Boyle's law, for any given temperature pv = const., and since the density is the ratio of mass to volume, if the total mass of the gas is M its density $\rho = \frac{M}{v}$, or $v = \frac{M}{\rho}$, and $pv = \frac{p}{\rho}M$. Therefore, for a given mass M, $\frac{p}{\rho} = \frac{pv}{M}$ = const. But we have just seen that $\frac{p}{\rho} = \frac{1}{3}\overline{V}^2$, therefore, $\frac{1}{3}\overline{V}^2$ = const. or $\overline{V}^2$ = const. Hence, if Boyle's law holds, the mean velocity of the molecules is constant. From the equation $\frac{p}{\rho} = \frac{1}{3}\overline{V}^2$, we have $\overline{V} = \sqrt{\frac{3p}{\rho}}$. Therefore, if we know the pressure and density of a gas, we can compute the mean velocity of its molecules. The formula shows that the mean velocity is inversely proportional to the square root of the density. Under a pressure of one atmosphere, 1,013,250 dynes per sq. cm. at the temperature of 0° C.,* we have:

	ρ, in g.	$\overline{V}$, cm. /sec.	ρ, compared to H.
Hydrogen	.0000896	185,000	1
Nitrogen	.001257	49,400	14.03
Air	.001293	48,700	14.43
Oxygen	.001430	46,500	15.96
Carbon dioxide	.001974	39,600	22.03
Chlorine	.003133	Students Compute	34.97

68. Diffusion. — (*a*) Gases: Since it is characteristic of a gas to fill any space that is open to it, the kinetic theory readily prepares us for the fact that two different gases in communicating vessels diffuse into each other; and this will take place even through a septum between them if the partition is porous, like a coarse membrane or unglazed earthenware. It is found that

* This is the value of one atmosphere by taking the exact figures for density of mercury = 13.596, the standard barometer height = 76 cm., and the acceleration of gravity in latitude 45° = 980.6 cm./sec.². For any other latitude λ at sea level, $g = 980.506 - 2.5028 \cos 2\lambda$ (See Hastings and Beach, *General Physics*, Art. 30).

the rate of passage through such a partition, which is really passage through very small orifices or tubes, is inversely proportional to the square root of the density. If r is the actual velocity with which the gas moves through the partition, $r = \sqrt{\frac{2p}{\rho}}$. (For demonstration, see Watson, Art. 138.) If two bottles, one containing, say, hydrogen, and the other carbon dioxide, both at atmospheric pressure, are thus placed in communication through a porous septum, each will at once begin to diffuse into the other at a rate inversely as the square roots of their densities; that is, the hydrogen will enter the CO_2 at a much greater rate than the CO_2 enters the hydrogen, the pressure in the CO_2 bottle increasing and that in the hydrogen decreasing. Eventually this disparity in the rate will change with change of pressure, and diffusion will cease when the gases have become intimately mixed.

Experiment No. 30, page 98. — Diffusion of Illuminating Gas through Porous Cup.

(*b*) Liquids: This phenomenon, in connection with the pressure produced by the more rapid diffusion in one direction than in the other, has opened up some interesting facts with liquids. Here diffusion is more to be wondered at than with gases, since liquids have no tendency to expansion, and under gravity they rest on their base and exert a pressure downward. And yet, if, of two liquids that are miscible, the lighter one rests directly upon the heavier one in the same vessel, not only does the upper one gradually sink into the lower, but the lower, heavier one rises, contrary to gravity, into and through the upper one.

Experiment No. 31, page 99. — Diffusion of Liquids.

69. Osmosis. — This process of diffusion will take place even if one liquid, say, a dilute solution of sugar, is inclosed in a porous sack and dipped into the other, say, water. In such case the pressure in the sack is materially increased by the greater rate of diffusion of water into the (denser) solution. This pressure, called osmotic pressure, varies with the density of the solution

and with the nature of the substance dissolved. The explanation of osmotic pressure by the kinetic theory is that "the semi-permeable membrane is struck on both sides by water molecules, but since there are fewer water molecules per unit volume, some of the space being occupied by sugar molecules which cannot traverse the membrane, more water molecules will, in a given time, strike the outside than the inside of the membrane, and hence, as the water molecules can pass through the membrane, more water molecules will enter than leave." (Watson, Art. 164.)

Now, when the substance is dissolved, it may be considered as distributed throughout or occupying the entire volume, and if we take a definite mass, say one gram, of sugar and dissolve in various quantities of water, thus giving to this gram various volumes, we find its osmotic pressure varying inversely as the volume, or pv = const., the substance when in solution conforming to Boyle's law for gases, provided the solution is dilute, as illustrated in the following table:

Percentage of sugar in solution.	Osmotic pressure P, in cms. of Hg.	Vol. V, of solution containing one gm. of sugar.	PV.
		c.c.	
1	53.5	99.6	5239
2	101.6	49.6	5039
4	208.2	24.61	5124
6	307.5	16.34	5025

A peculiar consequence of the incessant movement of the molecules of a liquid or a gas is seen in the so-called *Brownian movement.* A body very large relatively to the molecules, when placed among them, may be at rest under the promiscuous bombardment which it receives on all sides, and any but very minute particles of matter are thus relatively large. But when they are so small as to be comparable with the molecules they are no longer in equilibrium, and are found to be incessantly in motion, irregularly driven hither and yon, so that the molecules themselves are unquestionably constantly shifting their position. (See Soddy, *Matter and Energy*, pp. 78, 91, and 103.)

EXPERIMENTS TO ILLUSTRATE CHAPTER I.

Experiment No. 1, Art. 13. Vortex Motion.

A box of ordinary cardboard, Fig. 30, from six to ten inches on a side and six inches in depth, will serve. Laying the box on its side, with a circular hole about three inches in diameter in the front (i.e., bottom of the box), the circular steady vortices are formed. By having a card as a flap that can be turned over the front, with an elliptical hole with axes about one and a half inches and three inches, the vibrating vortices are produced. The drumhead at the back of the box may be a light frame fitting the box and having a piece of chamois skin stretched over it. A slight tap on the chamois in the line of the orifice is sufficient to expel a ring.

Fig. 30. Vortex Rings.

Experiment No. 2, Art. 17. Nos. 2 to 5, incl., Illustrate Inertia.

To a body weighing four or five pounds attach a portion of string, both above and below, that is just strong enough to carry a little more than the given weight. If the body is lying on the. table, with deliberation it may be lifted by means of the upper string, but if the attempt is made to lift it suddenly the string breaks. If the weight is suspended, as in Fig. 31, a steady pull on the lower cord is transmitted to the upper one, which has to sustain the weight and the added pull. If the latter is gradually increased, the upper string breaks; if, however, the lower string is sharply jerked, it is broken, while the upper one is not affected. Before the force that is suddenly applied to one cord has time to overcome the inertia of the large mass (or to accelerate it) sufficiently to increase the tension in the other cord, it is itself broken.

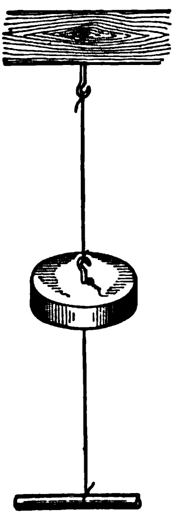


Fig. 31. Time-element in Inertia.

Experiment No. 3, Art. 17.

Place a coin on a card; if it is not moved too rapidly the coin can be carried around on the card without slipping. Put the card and coin over the mouth of a narrow beaker or wide-necked bottle; flip the card sharply horizontally and it flies out without moving the coin, which drops into the vessel. The experiment admits of innumerable variations.

Cut a strip of ordinary writing paper about four centimeters wide and twenty-five centimeters long. Let about one-third of its length lie on the table, while the unsupported end is held by the thumb and finger of the left hand. A disk or coin, or even a pile of a dozen of them, laid on the part of the strip that is over the table moves readily about with the paper as this is pushed or pulled around, but if the strip is struck downward sharply between the left hand and the table, it is jerked out, while the disk remains undisturbed. The less surface of contact there is between the disk and the strip the better. With a nickel five-cent piece standing on edge, with its faces parallel to the length of the strip, the effect is more striking.

Experiment No. 4, Art. 17.

On the shaft of an electric motor making 1000 to 1500 revolutions per minute, mount a hub of four to six inches in diameter. A light endless chain, of such length that when placed on the hub it hangs several inches below it, is so suspended and the motor started. When the disk has brought the flexible chain to its own speed, the latter will be found to have acquired considerable rigidity, and if it be carefully slid from the hub, and allowed to drop a few inches to the table, it will rebound and skip and roll some distance, like a hoop, before it collapses.

While the chain is in motion on the hub, it may be made to take various positions which persist apparently in opposition to gravity. By pressing it with a light roller that does not greatly impede its motion, it may be made to take an inclined or nearly horizontal position, which it maintains with the free end of the loop unsupported. (See Goodeve's *Principles of Mechanics*, Art. 36.)

Experiment No. 5, Art. 18. Foucault's Experiment.

In a place as free as possible from drafts and jars, suspend a heavy ball (ten pounds or more in weight) by a fine wire twenty feet or more in length. The upper support should be rigid, and under the bob should be fixed a bristle or fine wire stylus. A board several feet in length and hinged at one end can be adjusted at such a height that when it is level the stylus will just touch a smoked-glass plate on the board, and when the free end is lowered, the stylus will pass clear of the glass.

The bob is drawn back about a foot or eighteen inches from the vertical and secured in that position until all sidewise motion has ceased. It must

also be carefully adjusted so that when released it will not take on any rotary motion. A good plan for releasing it without shock is to draw it back by means of a cord attached to a ring of spring wire somewhat larger in diameter than the bob. When the cord is burned off by a match, the wire loop springs open and drops off. After the pendulum has thus been set swinging, at an observed instant the board is raised to the horizontal and a mark is made by the stylus as it passes over the plate. The board is at once lowered and the pendulum is left swinging. At the end of a half-hour another record may be thus taken, and again in another half-hour. By this time the pendulum will probably be swinging in a narrow ellipse, so that the three lines may not intersect in a common point; but the angular deviation of the plane of oscillation in each half-hour can be readily measured when the smoked glass is removed from its table, and the records may be projected upon the screen.

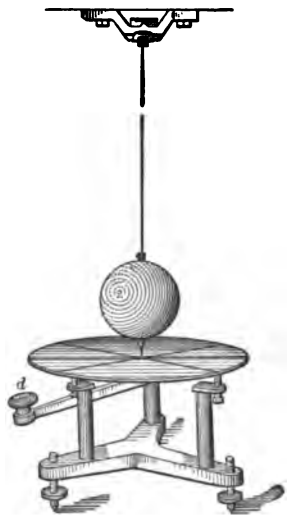


Fig. 32. Foucault Pendulum.

More elaborate methods both of suspension and of recording may be used to advantage.

The elliptical path into which the pendulum is almost certain to fall is probably due to the fact that no wire for suspension can be obtained that is everywhere round; the cross-section will have unequal diameters, and the wire will be stiffer in one direction than in a direction at right angles to that.

The fewer the oscillations made by the pendulum the less will be the disturbance in this respect; hence the advantage of a long wire, and correspondingly long period. Also, the heavier the bob the smaller the disturbance from air currents or other slight causes.

Experiment No. 6, Art. 20. Independent Action of Simultaneous Forces.

If two balls at the same height above the floor are so arranged that when one is violently projected horizontally the other drops vertically at the same instant, they reach the floor at the same time.

Numerous contrivances have been devised, of which Fig. 33 shows one that is simple and convenient for frequent use. On a base board *AB* is pivoted by a screw another board *CD*, about 40 cm. long and 5 cm. broad. The edge of *CD* is about two centimeters from the edge of *AB*, and under

one end of CD is fastened a piece of sheet tin about two by four centimeters. This board can be placed upon any table, and with balls as in the figure, a

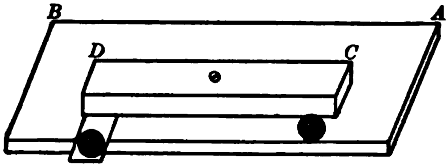
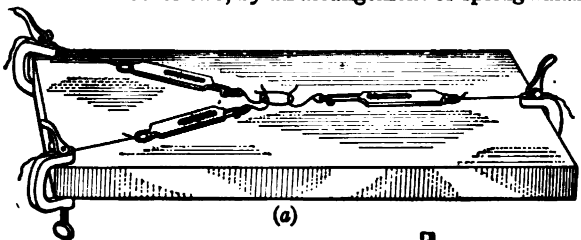


Fig. 33. One Ball is projected horizontally while another is dropped vertically.

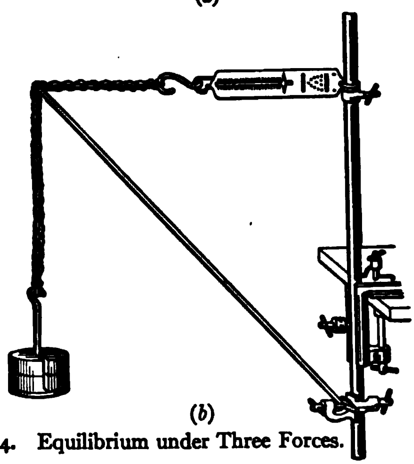
blow against CD at C projects one ball and drops the other at the same instant. Both strike the floor at the same time.

Experiment No. 7, Art. 23. Concurrent Forces.

Since when three forces are in equilibrium either one is equal and opposite to the resultant of the other two, by an arrangement of spring balances as in



(a)



(b)

Fig. 34. Equilibrium under Three Forces.

Fig. 34 (a) the three forces can be adjusted to any variety of magnitude and position (but passing through a common point), and one will be represented by the diagonal of the parallelogram constructed upon the other two, or it will form the third side of a triangle in sequence with the other two.

An arrangement like Fig. 34 (*b*) serves the same purpose. In fact, in either case it is not necessary to know the actual magnitude of more than one of the three forces, except to prove the correctness of the experiment.

Experiment No. 8, Arts. 17 and 25. Centrifugal Force.

If a looped chain be suspended by a thread from the spindle of a whirling table, Fig. 35, and be set rotating about the vertical, it will widen out and

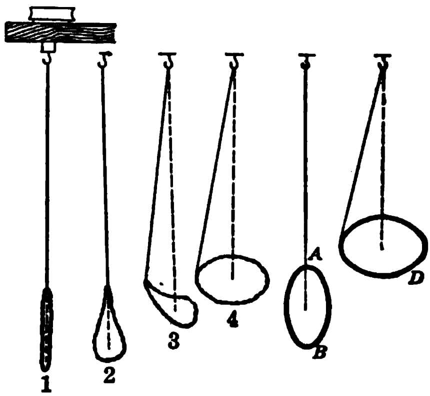


Fig. 35. Whirling Chain, etc.

at first produce a pear-shaped figure, but as the speed increases it passes through the forms 1, 2, 3, and finally places itself in the position 4, in which every link is as far from the axis of rotation as possible. The large gain of energy by the chain is now partly kinetic, corresponding to the rotation, and partly potential, corresponding to the higher position which the chain now occupies as a whole, compared to the position 1. If instead of a chain a solid ring be suspended as AB, on rotating it the ring takes the position D; always the position which most nearly permits every particle to move on in a straight line.

If a round flask be suspended instead of the chain or ring, and liquids of different densities, say mercury and water, be placed in it, on rotating, the liquid of greater mass displaces the other, showing that the centrifugal force is greater with the greater mass for any given speed and radius.

Experiment No. 9, Art. 27. Determination of g.

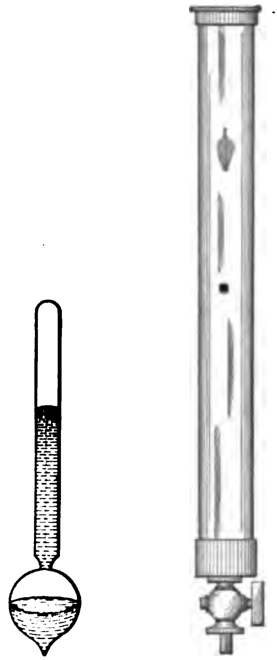
From a helical spring which vibrates rapidly without any added weight, suspend as great a weight as the spring will permit without being permanently distorted. Measure the extension produced by the weight, and time its oscillations, taking the mean of several observations. If elongation is e and period T,

$$g = \frac{4\pi^2 e}{T^2}.$$

Experiment No. 10, Art. 28. Bodies Falling in a Vacuum.

The water hammer, Fig. 36 (*a*), is a tube and bulb containing water and vapor at very low pressure; when suddenly inverted the water column falls like a solid, striking the bottom of the tube with a blow like that of a hammer, and the drops through the throat from the bulb fall into the water with a sharp metallic click.

(*b*) is the common guinea and feather tube. It usually contains a few leaden shot and irregularly shaped pieces of paper. When the stopcock is open there is full air pressure in the tube, and in a vertical position the lead balls drop quickly, while the paper saunters after them lazily. When the air has been exhausted from the tube, and the latter is turned into a vertical position promptly, the paper and the lead dart to the bottom of the tube together.



(*a*) (*b*)

Fig. 36.
Water Hammer. Guinea and Feather Tube.

Experiment No. 11, Art. 34. Mechanical Paradox.

Both the double cone and the cylinder, Fig. 37, illustrate a body rolling up hill under gravity, and also the tendency of potential energy to reduce to a minimum.

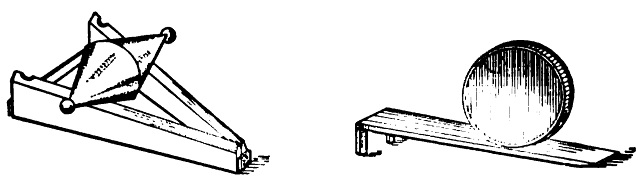


Fig. 37. Bodies Rolling up Hill.

Experiment No. 12. Art. 35. Galileo's Method of Determining g.

A piano wire AB (Fig. 38), say, ten meters long, is stretched across the room, with a descent AC of about one meter. A light grooved pulley

carries a weight of 100 g. to 200 g., and is released from A at a signal when a stop watch is started. When the pulley reaches B the watch is stopped and

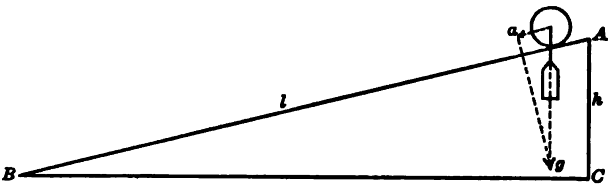


Fig. 38.

the time of descent noted. With uniform acceleration a, the distance l that is traversed in time t is $l = \frac{1}{2} at^2$,

whence $$a = \frac{2l}{t^2}. \qquad \text{(A)}$$

Theoretically, on the plane BAC,

$$\frac{a}{g} = \frac{h}{l}. \qquad \text{(B)}$$

Substituting the value of a from Eq. (A) in Eq. (B), we have $g = \frac{2}{h}\left(\frac{l}{t}\right)^2$.

Instead of the wire and pulley, Galileo used a grooved board down which balls were rolled. This avoids the sag in the wire.

Experiment No. 13, Art. 35. Forces and Displacements.

Besides the ordinary illustrations of levers, inclined plane, pulley, etc., the following well illustrates the principle of this article.

Fig. 39 shows a model that may be turned out of wood, or constructed of cardboard or heavy manila paper. The diameters, and consequently the circumferences, of the end and the middle portions of the roller have a definite simple ratio, say 4 : 5.

Two threads are attached to the end portions, wound round a few times, and then fastened to the hooks in the frame above. Two other threads are attached to the thicker part of the roller, given a few turns round it in the reverse direction to the first ones, and their ends connected by a stick whose weight is negligible. The weight necessary to put upon this cross-bar to prevent the roller from descending is determined by these considerations: Suppose the roller makes one complete turn in descending; it unwinds the

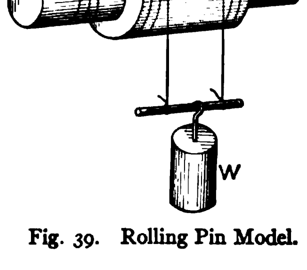


Fig. 39. Rolling Pin Model.

outer strings and thus lowers the whole system a distance equal to the circumference of the smaller cylinders, but it winds up the cords on the larger cylinder and so raises the bar and weight a distance equal to the larger circumference. Suppose the smaller circumference to be, say, 12 cm. and the larger 15 cm.; then a descent of the roller and weight a distance of 12 cm. is attended by a rise of the weight a distance of 15 cm., so that the actual rise of the weight is 3 cm., or one-fourth as far as the roller descends. For equilibrium of roller and weight, the work of the cylinder in descending must equal that of raising the weight, or weight of roller $\times$ distance it descends = wt. of $W \times$ height it ascends; i.e., with the above dimensions, $\frac{W}{R} = \frac{4}{1}$ or $W = 4\,R$. A convenient size is about 15 cm. in length for the roller altogether, and the diameters of the two parts 10 cm. and 12 cm.

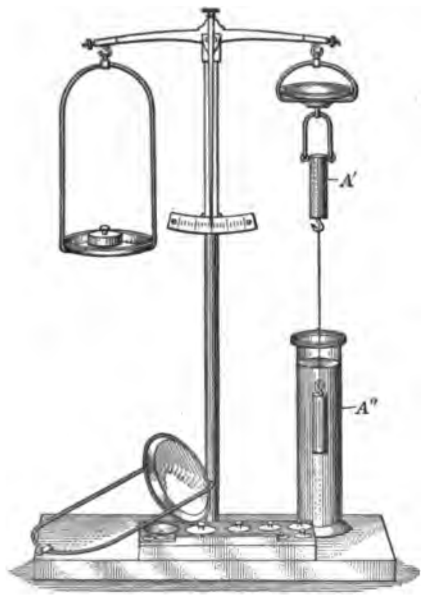


Fig. 40. Cup and Cylinder Apparatus.

Experiment No. 14, Art. 45. The Principle of Archimedes.

The original Archimedean demonstration is the one best suited for lecture purposes. In the figure (Fig. 40), the cylinder A'' underneath just fills the cylindrical vessel A' from which it is suspended. The cup and cylinder as suspended from one arm of a balance are exactly counterpoised by weights in the pan at the end of the other arm, before the vessel of liquid is used. When the cylinder is immersed in any liquid as in the figure, it no longer matches the weight in the other pan, but the balance is restored when the cup A' is exactly filled with the same kind of liquid as that in which the cylinder is immersed.

Experiment No. 15, Art. 45. The Cartesian Diver.

The figure of glass or porcelain, Fig. 41, is hollow, the cavity being open at the bottom, so that, when partly filled with water, air is inclosed in the upper space. The figure may be regarded as consisting of glass and the inclosed air. When it is submerged it displaces (or occupies the place of) a definite volume of water. The *weight* of this volume of water is then the

measure of the lifting or buoyant effort upon the figure. If this is greater than the weight of the figure the latter rises, and a small portion of it will project above the surface of the water in the vessel. If now the air in the upper part of the vessel is compressed, the pressure upon the water in the vessel forces more water into the figure. The figure now *displaces* (or occupies the place of) *less water* than before, and is accordingly buoyed up with *less force*. The figure itself is neither heavier nor lighter than before, but, because of the reduction in volume of the air in it, less water is displaced, and a smaller upward force acts upon it. If the force is less than the weight of the figure, the latter sinks by gravity. When the pressure at the top of the vessel is removed, the pressure of the air within the figure forces water out of it (displaces water), the buoyancy is increased, and the figure rises.

Fig. 41. The Cartesian Diver.

(Instead of a wide-topped jar with a rubber sheet, a bottle with a tube through a rubber stopper may be used. An ordinary atomizer bulb on the tube makes it easy to apply or remove pressure.)

The accompanying figure of a longitudinal section of a Holland submarine boat shows that the action of this is like that of the Cartesian diver. Compressed air is stored at high pressure in strong receivers. Low in the hull of the vessel is a series of chambers that can be opened to the water

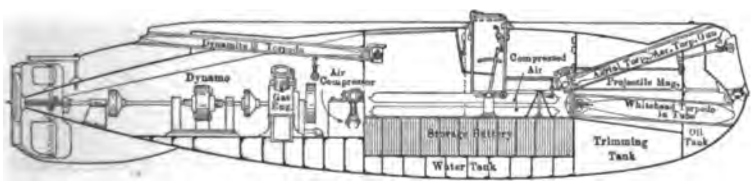


Fig. 42. Longitudinal Section through Holland Submarine Boat. (Early Form.) By courtesy of *The Scientific American*.

outside. When they are full of water the vessel is heavier than the water it displaces and it sinks. By admitting air under pressure into one or more of these water tanks, water is driven out (displaced), and the buoyancy increases so as to float the vessel.

In the more modern type of submersible boat the same principles are applied, but with the admission of water into the compartments the total water displaced nearly but not quite equals in weight the weight of the boat, and so the vessel remains afloat with a very small portion exposed.

The vessel is driven completely beneath the water by having rudders so placed as to direct the boat downward when the propeller is in motion.

Experiment No. 16, Art. 46. Cork Remains Submerged under Mercury.

Select a strong glass tumbler with a bottom that is flat or slightly concave on its upper surface, and a cork stopper, about two centimeters in diameter and two or three centimeters in height, which is quite smooth and flat on its larger end. By means of the fingers, or a piece of wood or iron, hold this smooth face firmly against the bottom of the tumbler and pour mercury into the vessel until the cork is completely covered. The mercury will not readily pass under the cork, there is therefore no upward pressure against the cork, and it remains submerged when its support is removed. Though hidden from view, if the glass is raised it is plainly seen from beneath. With the least twitch to the cork, admitting mercury under it, it leaps to the surface.

Experiment No. 17, Art. 46. One Tube Falls Upward into Another.

Select two test tubes of such size that one may be inserted in the other with very little play but without much friction. Holding the larger one vertical, fill it with water and push the smaller one, closed end down, into the larger, an inch or two. Then, holding both in place, invert the two and release the lower (smaller) one. Instead of falling out, it rises into the upper one, displacing the water above it, and continuing with an accelerated motion as far as the length of the tubes will permit.

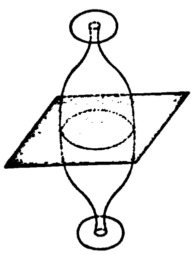


Fig. 43. Transfer of Liquids.

Experiment No. 18, Art. 51.

Determine the density of several liquids by Hare's apparatus, in each case taking the height of liquid columns at several different levels, and compare results with readings of hydrometer.

Experiment No. 19, Arts. 50 and 51. Two Vessels Filled with Liquids Exchange their Contents.

Two glasses, Fig. 43, of the same capacity and size of rim, are filled to the brim with liquids that do not readily mix, say, one with water and the other with kerosene.

Hold a card over the glass containing the denser liquid, invert it, and place it exactly above the other glass. Carefully slide the card to one side until a small segment of circle affords communication between the two vessels. The denser liquid passes down at one end of the segment and the rarer rises at the other end until the two liquids have completely changed glasses. By careful manipulation the card may be slid back to

its place and the upper vessel replaced upon the table, the liquids having been interchanged without a third vessel.

Experiment No. 20, Art. 52. Nos. 20 to 24, incl., Illustrate Surface Tension.

In a bottle (Fig. 44), put a saturated solution of zinc sulphate, sp. gr. about 1.4, to the depth of three or four centimeters; on this, by means of a pipette, place about one cubic centimeter of carbon bisulphide, sp. gr. 1.29, and carefully add water a few centimeters more in depth. The carbon bisulphide will assume a spheroidal form (if the globule is small it will be almost spherical), and will float between the solution and the water.

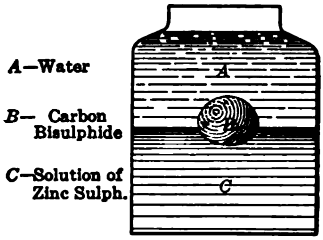


Fig. 44.

Experiment No. 21, Art. 52.

Moisten a clean plate of glass with a thin film of water. If this film is touched with the end of a straw that has been dipped in sulphuric ether, the tension of the film is so weakened that the water is drawn back in every direction, leaving a circular spot comparatively dry. This action occurs owing to the vapor of ether even before the straw touches the water.

Experiment No. 22, Art. 52.

A wire ring (Fig. 45) about five centimeters in diameter has suspended from it a thread with a loop at the end. If the ring be dipped in a soap

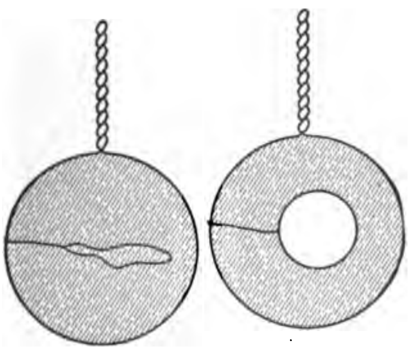


Fig. 45. Tension of Soap Film.

solution, it will be overspread by a film that supports the thread. By a hot wire puncture the film within the loop. The film without the loop contracts to its smallest area, which is reached when the loop incloses the largest area; the figure of largest area inclosed by a line of given length is a circle.

Moreover, since the tension across the thread is everywhere equal and everywhere perpendicular to the bounding line, the figure would be a circle.

If soap films are formed on any wire frame and broken at one or more places, the remaining figure will always have that surface, however peculiar its shape, which has a smaller area than any other surface everywhere equally taut and bounded by the same edges. Nos. 21 and 22 are readily projected on the screen by a lantern.

Experiment No. 23, Art. 52.

On the surface of clean water drop a few small pieces of gum camphor. The camphor dissolves unequally rapidly at various points and the tension of the film of water is weakened. The contracting film twitches the camphor around in an erratic manner. Like the water, the camphor must be clear of grease, — it must not be fingered. With a shallow vessel, this experiment is readily shown on the screen by a vertical projection apparatus.

Experiment No. 24, Art. 52.

In a beaker containing a small quantity of water place a few drops of oil. The addition of a large proportion of alcohol, with stirring, will reduce the sp. gr. until the oil will rest suspended in the body of the mixture. The oil will then form globules, each surrounded by a film in a state of tension, the film contracting to the smallest possible size to contain the oil inclosed by it. For a given volume the form with the smallest superficial area is that of a sphere; and, moreover, the coming to this form is an example of potential energy in the film reducing to a minimum.

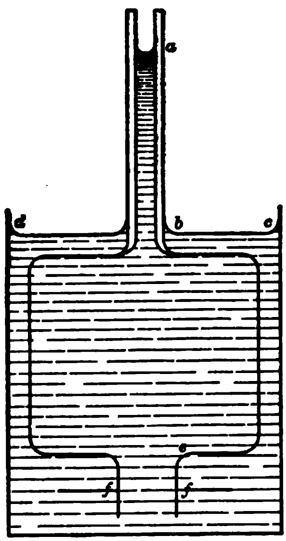


Fig. 46. Supporting Action of Capillarity.

Experiment No. 25, Art. 53. Nos. 25 and 26 Illustrate Capillarity.

A fine tube *ab*, Fig. 46, with a bore, say, 0.5 mm. in diameter, widens at *b* to a large bulb, say, five or six centimeters in diameter, and of length *be* about five centimeters, the lower continuation being too wide for capillary action, say, one centimeter or more in diameter. On dipping this tube into a vessel of water the latter rises in the fine tube about six centimeters by capillarity. If, now, the tube be gradually raised, the water at *a* sinks in the tube but continues

at the same height above *dc*, and presently the very narrow column in the tube is supporting the wider portion of liquid in the bulb between *b* and *e*, and will sustain it entirely above the level of *dc* if the height *be* is less than the original capillary column *ba*. When the apparatus has been raised until a line as *ff* reaches *dc*, then if *bf* is greater than the length of the column sustained by capillarity in *ab*, the liquid in *ab* sinks into the large bulb, and the contents of the latter return to the vessel until the liquid is at the same level within the bulb and without.

Experiment No. 26, Art. 53.

Exhibit or project upon the screen the form of liquid between two plates in wedge shape, Fig. 47.

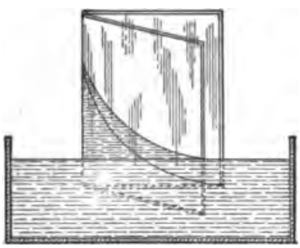


Fig. 47. Rise of Liquid between Two Plates.

Experiment No. 27, Art. 54. Expansive Force of a Gas.

If an air-pump receiver with open top have a thin rubber sheet tied over the mouth, and the air be exhausted, the downward pressure of the atmosphere is shown by the depression of the rubber cover until it may burst. On the other hand, if a wide-mouthed bottle be closed by such a rubber membrane, and placed under a bell-jar receiver, when the air is extracted from the latter the pressure within the bottle forces the rubber membrane outward. The point especially to notice in this is that the internal pressure was there before the removal of the external air as well as afterwards; the exhausting of the air from the receiver did not put any force into the bottle that was not there before.

Experiment No. 28, Art. 55. Nos. 28 and 29 Illustrate Pressure of the Atmosphere.

Fill a tumbler to the brim with water; hold a card closely upon the top of the glass and invert it. The card will be held against the glass by atmospheric pressure, upwards or sidewise. Continue the experiment as described in Art. 55.

Experiment No. 29, Art. 55.

Magdeburg hemispheres (Fig. 48) are two hollow hemispheres neatly fitted together, one of them having a stopcock by which it can be attached to an air pump. When put together with full air pressure within, they are easily separated, but when the air is exhausted they withstand a strong effort to pull them apart. The fact that this is true, no matter what may

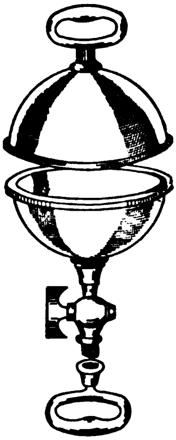


Fig. 48. Magdeburg Hemispheres.

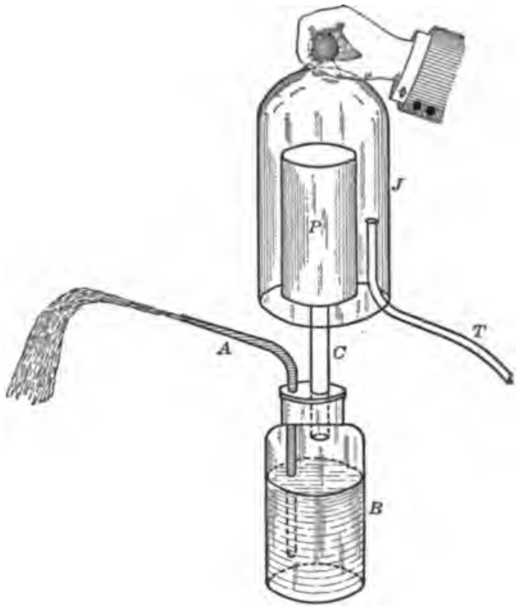


Fig. 49. Osmosis of Gases.

be the position of the hemispheres, again illustrates the equality of fluid pressure in every direction. If, in this condition, they are placed under the receiver of the air pump and the air is exhausted from around them, they fall apart of themselves.

Experiment No. 30, Art. 68. Osmotic Pressure.

A closely sealed bottle B, Fig. 49, partly filled with colored water has an efflux tube A and another tube C leading from the upper part of B into the sealed porous cup P (a small new battery jar with a cork stopper that fits tightly and is well covered with sealing wax). The bell jar J is filled with illuminating gas by the tube T and is then placed to envelop P. The pressure due to the osmosis of the illuminating gas causes the liquid in B to

flow out through A. On the removal of J, the rare gas in P issues through the porous cup into the denser atmosphere, and the liquid in A runs back into the bottle.

Experiment No. 31, Art. 68: Diffusion of Liquids.

Put a solution of copper sulphate to the depth of five or six centimeters in a narrow glass vessel, and carefully add an equal depth of alcohol on top of the solution by letting it trickle down the side of the vessel from a pipette. Cover the vessel and set aside for several days where it will not be shaken. Observe the progress of diffusion.

CHAPTER II.

HEAT.

70. Temperature. — Temperature is a term which is applied to a body to express that condition to which we ascribe the sensations which we indicate by the terms hot, cold, warm, cool; and differences in which we signify by the various degrees of these adjectives, as hotter, colder, etc. But it is found that our sense of touch, or even our "temperature sense" as distinguished from the sense of touch, is often inadequate to distinguish correctly the changes in the condition of a body, or the difference of condition of different bodies. And besides, we should have some way of expressing temperature in physical terms rather than physiological; for our senses may so far deceive us as to make us feel cold and shiver when our temperature is above normal. So we can only proceed with the above idea of temperature temporarily.

With a change of temperature in a body, other changes are observed to take place, the most common being, perhaps, a change in size; and it is found, furthermore, that under certain conditions a body always has the same size, and changes in size by the same amount between two of these so-called standard conditions. For example, any given mass of mercury, if placed in melting ice, will always increase in volume in the same proportion when transferred into water that is boiling under a pressure of one atmosphere. And so will any other substance which under these circumstances is continuous in its physical state, i.e., solid, liquid or gaseous.

The temperature of melting ice, then, is considered constant in nature, and also that of water boiling under pressure of one atmosphere. The difference between these might be considered a unit difference of temperature, and the difference in size of a body, corresponding to this difference of temperature, might be

taken as the means of measuring various other differences of temperature. It is convenient, however, to use a smaller unit of difference of temperature, a fractional part of the difference between the above two standard conditions of freezing and boiling water. If the expansion of a substance, as mercury, between these temperatures, is divided into, say, 100 equal parts, one of these parts is taken to represent a unit change in temperature, which is called one degree (1°) Centigrade. It is at first only assumed that equal successive changes in the physical condition of the substance whose temperature is to be examined will be attended by equal changes of volume in the substance by which the temperature is being examined; but subsequent study of various substances shows that the assumption is correct within determinate limits, and what those limits are for different substances. A substance, then, so prepared as to show its changes of volume exactly, becomes an instrument to measure changes of temperature, and is called a thermometer. But observe, it can only show *differences* in temperature, and in no wise tells temperature in an absolute sense; nor have we, up to this point, any meaning for temperature in that sense.

71. **Temperature Scales.** — While the standard temperatures, and therefore the difference between them, are fixed in nature and are independent of our means of examining them, a degree of temperature may mean any fraction we please of this fixed difference; and, also, the same fraction may be apparently of very different size on different thermometers.

72. **Thermometer Scales.** — The common scale for scientific use is that in which the temperature of melting ice is marked zero, that of boiling water 100, and the range of temperature from freezing to boiling water is divided into one hundred equal parts or degrees. It is the centigrade scale.

The common scale in domestic use in England and America, and also for engineering use in those countries, is the Fahrenheit, on which the temperature of melting ice is marked 32 and that of boiling water 212; while in Russia, and to some extent in other European countries, the Réaumur is used. For these refer fur-

ther to elementary physics. (Exhibit the thermometer with no scale marked; is it then a Fahrenheit thermometer or a Centigrade? How, when it has *two* scales?)

All thermometers except electrical ones depend for their indications upon expansion of bodies with rise of temperature and corresponding contraction on cooling. They may, therefore, be of solid, liquid or gaseous material. The actual measurement that is made in determining temperature is, then, of a *length*. Two fixed temperatures in nature are needed to get a correct scale, which is divided arbitrarily and the numbers themselves counted from an arbitrary zero.

(Exhibit thermometers, solid, liquid, gaseous; maximum and minimum thermometers. Conditions of a good thermometer: in general, large change of volume per degree, promptness to change, wide range. Deep-sea thermometry, pyrometry, electrical determination of temperature change.)

There are a few substances that act directly at variance with the law of expansion by heat; such are iodide of silver, India rubber, and some alloys. As such conduct, however, is anomalous, we need more particularly to examine that which is normal.

Experiment No. 32, page 147. Gravesande's Ring.

A bulb containing colored liquid shows expansion of liquid; and one containing air, the expansion of gas; the latter sensitive.

After settling upon the freezing and boiling points of water for the fixed temperatures, and therefore definite points on the thermometer scale, the division of the scale intervening into one hundred parts seems natural enough; but it may seem singular that Fahrenheit should have marked those points 32 and 212 respectively, dividing the intervening distance into 180 degrees; and that Réaumur should have used 80 divisions for the same range of temperature.

Note. — Daniel Gabriel Fahrenheit, Danzig, Holland, 1686–1736. His early thermometers were of alcohol; he first solved the problem of temperature comparisons by making thermometers that agreed with one another. Previously all apparatuses for the purpose were independent of one another and wholly arbitrary.

Fahrenheit made thermometers of alcohol as early as 1709 and did not know of the large coefficient of expansion of mercury and its suitability for such purposes until 1714. He employed various scales on his early instruments, his latest being the one now in common use and known by his name. Various statements have been made as to his choice of divisions, but all agree in his determination of the zero by a mixture of ice and salt, and which he called zero, as it was the lowest temperature reached in Holland in the unprecedentedly cold winter of 1709. He then marked on his thermometer the point of melting ice as 32° and that of the normal temperature of the human body as 96°. (Rosenberger, *Geschichte der Physik*, Vol. II, pp. 280, 281, no explanation being given as to the choice of numbers.) Fahrenheit simply determined the relative volumes of the mercury in his thermometer at his lowest temperature (zero), at the freezing of water, and at the boiling of water. These were found to be as the numbers 11,124, 11,156, 11,336, showing at the same time that the mercury within that range expanded at uniform rate. Then, of his arbitrary unit volume, at freezing there were 32 more than at zero, and at boiling 212 more.

Réaumur introduced his scale in 1731, using spirits of wine of such strength that for a range of temperature as large as between the freezing and boiling of water 1000 parts should expand into 1080 in bulk, dividing the interval into 80 parts. This is still used in Spain and Germany, and was the only one used in France prior to 1789. (Rosenberger, *loc. cit.*)

The centigrade scale was introduced in 1742 by the Swedish philosopher Celsius at Upsala, and was established in France along with the metric system during the French Revolution, 1790.

It is interesting to note that Fahrenheit in his experiments on thermometry learned of the variation of the boiling point of water, and recognized its connection with the atmospheric pressure, so that he proposed this use of the thermometer as a barometer. He thus anticipated, by a hundred years, Dr. Wollaston's hypsometer. (Rosenberger.)

Mercury is applicable only between the temperatures −34° C. and +350° C.; alcohol may be used as high as 78° C., and as low as −100° C. Alcohol begins to harden by getting pasty, but only at very low temperature (−129° C.). Maximum and minimum thermometers; Bréguet's metallic thermometer, coiled spiral of triple layer of metal, inner coil of silver (most expansible), then gold, and outside is platinum (least expansible). Other common metal thermometers of two metals, the dial thermometers common in cars. Differential thermometer to show the fact of difference in temperature, but not convenient for measuring.

That solids have different rates (coefficients) of expansion is shown by compound expansion bar of brass and iron.

Experiment No. 33, page 147. Compound Metal Bar.

Gridiron pendulum, coefficients of steel and brass are as 11 : 19. Attachments in gridiron are shown in Fig. 50, where the solid lines are steel and the open ones brass.

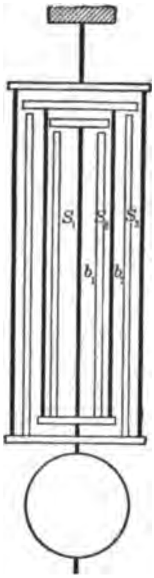


Fig. 50. Gridiron Pendulum.

Elongation downwards, $(s_1 + s_2 + s_3)k_s$.

Elongation upwards, $(b_1 + b_2)k_b$; putting these elongations equal to each other, we derive

$$\frac{s_1 + s_2 + s_3}{b_1 + b_2} = \frac{k_b}{k_s} = \frac{19}{11}.$$

Mercurial pendulum, expansion of steel: apparent expansion of Hg as 1 : 14.

Experiment No. 34, page 147. Trevelyan's Rocker.

Experiment No. 35, page 148. Contraction of India Rubber.

The promptness of expansion of metals is shown by Trevelyan's rocker.

If a metal be compressed heat is developed, i.e., given out, just as the application of heat to the metal resulted in its expansion. Also, if a wire be stretched and in consequence *enlarged,* it is more cold. The same applies to all substances which expand by being heated. It would then seem only consistent that a body which contracts by being heated should, in its turn, rise in temperature *when expanded*; and as India rubber does thus contract, it is interesting to note that it does give out heat when suddenly stretched.

Electrical determination of temperature change.

Coefficient of expansion:

For solids: linear (α), superficial ($2\ \alpha$), cubical ($3\ \alpha$);

For fluids: cubical (or voluminal) only.

Correction of balance wheel of watch or chronometer.

Difference between one degree Centigrade and one centigrade degree.

EXAMPLES. —

1. What temperature does a Fahrenheit thermometer indicate when a Centigrade thermometer reads 0°, −5°, 15°, 37.8°, 100°?

Ans. To third, 59°.

2. What is the temperature by the Centigrade scale when a Fahrenheit thermometer reads 0°, −5°, 32°, 100°, 392°?

Ans. To second, −20$\frac{5}{9}$°.

3. Show that −40° represents the same temperature on both the Centigrade and the Fahrenheit scale.

73. Nature of Heat (sketched briefly) — "The theory supposes that the molecules of every body are in a state of perpetual agitation, and this may consist in the motion of the molecule as a whole, or as a vibration or rotation of its constituent parts, or both. This molecular motion is supposed to depend upon the temperature: the hotter a body is, the greater the intensity of its molecular agitation. In a solid the molecules are supposed to oscillate around mean positions. Each is confined to a very small space which it never leaves. As the temperature rises the molecular agitation increases, and at length becomes so violent that the molecules break away from their imprisonment and wander about indiscriminately amongst each other. In this state the substance is said to be in the liquid form. . . . In order to endow the molecules with this extra motion, and also to overcome the forces which hold the molecules confined in the solid state, work must be done, and this work is the equivalent of what is known as the latent heat of fusion." Work is transference of energy. The body received energy in the form of heat, represented mechanically by the kinetic energy of all its molecules in the aggregate. The absolute energy is not determined, but only the change of energy, as indicated by change of temperature when that is the only effect. Further heating raises the temperature, increases energy of molecules; they finally break their bonds, and in gaseous form the energy due to gasification shows as pressure and temperature. (See Preston's *Theory of Heat*, p. 66 and further.)

Effects of heat (see Daniell's *Physics*, 1st ed., p. 322 to foot of p. 324, and from foot of p. 327 to middle of p. 328).

74. Anomalous Behavior of Water. — While substances usually change continuously in the same way with change of temperature from the point of liquefaction to that of gasification, water is exceptional, in that it first contracts on the application of heat until it reaches its smallest volume, and therefore its greatest density, and upon further rise of temperature it continually expands until it reaches a temperature where further application of heat converts the liquid into a gas. This temperature of maximum density is 4° C. or 39.2° F.

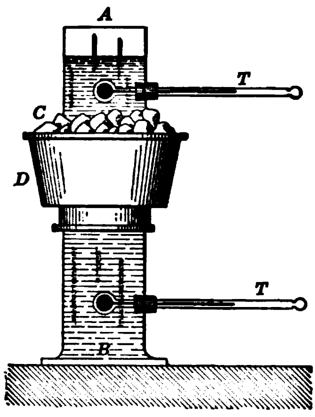


Fig. 51. Apparatus to show Temperature of Water at its Maximum Density.

(Apparatus as in Fig. 51 requires constant filling up with freezing mixture ($\frac{1}{3}$ coarse salt and $\frac{2}{3}$ ice shavings), and begin with water at about 45° F. It is well to have apparatus filled and fitted before the lecture begins, as the progress of the experiment is slow.)

All constants or peculiarities of water are important on account of its prominence in the economies of our daily lives.

The *consequence* of the conduct of water as it cools to the freezing point is important and apparent to every one; the reason of it may excite very different feelings, as is strikingly presented in Professor Tyndall's *Heat a Mode of Motion* (pp. 109–111), here quoted.

"Count Rumford was so impressed with the anomalous action of water that he devoted a whole chapter to speculations regarding it.

"'It does not appear to me,' he writes, 'that there is anything which human sagacity can fathom, within the wide extended bounds of the visible creation, which affords a more striking or more palpable proof of the wisdom of the Creator, and of the special care He has taken in the general arrangement of the universe to preserve animal life, than this wonderful contrivance.' Rumford's enthusiasm was excited by considerations like the following: Suppose a lake exposed to a clear wintry sky. The superficial water is first chilled; it contracts, becomes heavier, and sinks by its

superior weight, its place being taken by the lighter water from below. In time this is chilled and sinks in its turn. Thus a circulation is established, the cold dense water descending, and the lighter and warmer water rising to the top. Supposing this to continue even after the first pellicles of ice have been formed at the surface, the ice would sink, and the process would not cease until the entire water of the lake would be solidified. Death to every living creature in the water would be the consequence. But just when matters become critical, Nature, speaking poetically, steps aside from her ordinary proceeding, causes the water to expand by cooling, and the cold water to swim like a scum on the surface. Solidification ensues, but the solid is much lighter than the subjacent liquid, and the ice forms a protecting roof over the living things below.

"Rumford obviously regarded this behavior of water as a solitary exception to the general laws of nature. 'Had not Providence,' he says, 'interfered on this occasion in a manner which may well be considered as *miraculous*, the solitary reign of eternal frost would have spread on every side from the poles.' . . .

"He begs the reader's candor and indulgence while he investigates the subject. 'I feel,' he says, 'the danger to which a mortal exposes himself who has the temerity to undertake to explain the designs of Infinite Wisdom.' But though he admits the enterprise to be adventurous, he contends that it cannot be improper." Professor Tyndall goes on to remark that "facts like those discussed by Rumford naturally and rightly excite the emotions. Indeed the relations of life to the conditions of life — the general adaptation of means to ends in nature — excite in the profoundest degree the interest of the philosopher. But in dealing with natural phenomena, the feelings must be carefully watched. They often lead us unconsciously to overstep the bounds of real knowledge, and to run into generalizations which are in perpetual danger of being overthrown. . . . Rumford was wrong in supposing that the case of water illustrated a miraculous interposition of Providence, for the case is not an isolated one." He then calls attention to an iron bottle, rent by the cooling of molten bismuth. "There is no life here to be saved, still the bismuth accurately imitated the behavior of water. Once for all it may be said that the natural philosopher, as such, has nothing to do with purposes and designs. His vocation is to inquire *what* Nature is, not *why* she is; though he, like others, and he, more than others, must stand at times rapt in wonder at the mystery in which he dwells, and towards the final solution of which his studies fail to furnish him with a clue."

75. Energy in a Body in Different States. — Since heat is energy, it is apparent that a body when expanded by being heated possesses more energy than when contracted; a given mass of any

substance contains more energy when in liquid than when it is in solid state, and more when it is in gaseous than when it is in liquid state, if the change from one state to another in each case is due to addition or abstraction of heat.

76. Absolute Zero.—While liquids, and especially mercury, are for many purposes convenient substances for a thermometer, gases are more accurate, especially when they are so rare as to conform closely to Boyle's law. A gas that exactly conforms to this law is called a perfect gas, and air and the so-called permanent gases are approximately such.

A given mass of such gas at 0° C. and under constant pressure changes in volume $\frac{1}{273}$ for a change of 1° C. in temperature. This fraction 0.003665 is practically constant for a wide range of temperature and is called the coefficient of expansion for gases. Also, if the volume were held constant, it is found that the pressure varies in the same proportion per degree centigrade. If this proportional change held good indefinitely, then at $-273°$, or at 273° below the temperature of freezing water, the gas, if kept at constant pressure, would shrink to nothing, or if kept at constant volume would lose all pressure. The first of these is inconceivable and the latter has not been realized; but this theoretical value ($-273°$ C.) is called the absolute zero, and serves as a starting point from which to reckon all temperatures on an absolute scale, although it is known that gases change their rates of contraction (or expansion), and many of them their states from gaseous to liquid, before they reach so low a temperature.

Now if temperatures are measured from this absolute zero, the pressure varies as the temperature. But we have seen (Art. 66) that when the number of molecules per unit of volume is constant, which is the case when the volume of a given quantity (mass) of the gas is constant, the pressure varies as the average energy per molecule.

Again, if we mix two gases, they come to the same temperature, and from mathematical principles the average energy for each set of molecules is the same. But in each case the energy is proportional to the common temperature; hence, whether we

take the same or different gases, the average energy of the molecules is proportional to the absolute temperature, or (8) "the temperature of a gas is proportional to the average energy of the molecules" (Wormell's *Thermodynamics*, p. 160).

This being so, the temperature may be regarded as an expression of the average energy per molecule, and it is so regarded for liquids and solids as well as for gases. Then, naturally, we see that the heat of a body is the total kinetic energy of its molecules, and these may be taken as the strict physical meanings of the two terms "heat" and "temperature."

77. **Avogadro's Law.** — Since for a gas of given temperature $p = \frac{1}{3} Nm\overline{V}^2$, where N is the number of molecules in a given volume, and m the mass of a molecule, if we have equal volumes of two gases at the same pressure and temperature, N_1 and m_1 being the number of molecules and the mass of a molecule of one gas, and N_2, m_2 the corresponding quantities for the other gas, then, since both have the same pressure p,

$$\frac{1}{3} N_1 m_1 \overline{V}_1^2 = \frac{1}{3} N_2 m_2 \overline{V}_2^2.$$

If these gases are mixed in a vessel of double the volume of each gas, the pressure will remain the same, the temperature does not alter, the product of pressure and volume for each gas is the same, and therefore the kinetic energy, or

$$\frac{1}{2} m_1 \overline{V}_1^2 = \frac{1}{2} m_2 \overline{V}_2^2.$$

These, substituted in the above equation, give $N_1 = N_2$; that is, *any two gases having the same temperature, pressure and volume contain the same number of molecules.*

This is the law of Avogadro.

78. **Density Not Determined by Closeness of Molecules.** — Here we see the reason for rejecting the idea that by density of a substance we are to understand the closeness of arrangement of its molecules. Equal volumes of two gases, say oxygen and hydrogen, at the same temperature and pressure would contain an equal number of molecules; on the average the molecules of the two gases would be equally separated, but we would

not say the two gases were of equal density. Since their masses would be in the ratio of 16 : 1, so would their densities.

79. Work of Expansion of a Gas; Combined Relation of Pressure, Temperature and Volume. — Suppose a mass of gas to be inclosed in a cylinder, stopped by a gas-tight, freely moving piston, whose cross-section has an area A. If p is the pressure per unit area against the piston, the total pressure is pA. With this pressure constant, if the gas is heated it will expand and move the piston a distance l. The work of doing this is $pA \times l$, and if we suppose the material of the cylinder to be unaltered in dimensions by the heat, this quantity $pA \times l$ is the work of expanding the gas. But $pA \times l = p \times Al$, and Al is the volume moved through by the piston, or the *increase in volume* of the gas. Thus the work of changing the volume of a gas is found by multiplying the change in volume by the pressure per unit area. If volume is cubic centimeters and pressure is dynes per square centimeter, work is ergs.

According to the law of Charles, if the volume of a given mass of gas is constant, its increase of pressure is proportional to the rise of temperature; and by the law of Mariotte, with constant pressure the increase of volume is proportional to the rise of temperature, — in each case $\frac{1}{273}$ of the pressure or volume at 0° C. Then, if we could reckon temperature from an absolute zero, each quantity, pressure p and volume v would vary as the temperature T, or $pv = RT$, where R is a constant depending on the units for p, v, and T. (This quantity R is sometimes called Regnault's constant.) Since this equation would be true for all temperatures, it would be good for the temperature 0° C., for which $T = 273°$; then, if p_0, v_0, are the pressure and volume of a gas at 0° C.,

$$p_0v_0 = R\cdot(273) \text{ or } R = \frac{p_0v_0}{273}.$$

Under the kinetic theory of gases we see that $pv = const. \times temperature$, and, by the experimental laws of Charles, Mariotte, and Boyle, we have here the means of determining the Const. R, and of expressing the second member of this equation numerically. If p = the pressure of one atmosphere in dynes, i.e., 1,013,250 (p. 82, foot-note), and v_0 = volume of one gram-molecule of gas, i.e., 22,400 c.c., $R = \frac{1{,}013{,}250 \times 22{,}400}{273} = 8.3 \times 10^7$. This value of R is sometimes called the universal gas constant. This is in terms of work units, ergs. For its value in heat units, see Ex. 6, p. 145.

EXAMPLES. —

1. The density of dry air at 0° and 76 cm. barometer pressure is 0.001293 g./c.c. Show that the density at 76.8 cm. pressure and 15° C. is 0.001239 g./c.c.

2. A mass of gas was cooled from 100° C. to 15° C. under constant pressure, and its volume was then 150 c.c. What was its original volume?

Ans. 194.27 c.c.

3. 1000 c.c. of air at standard atmospheric pressure and 0° C. is heated to 100° C., the pressure remaining constant. What is its increase in volume, and how much work was done in expanding?

Ans. 366.3 c.c.; 371×10^6 ergs.

80. Calorimetry; Unit of Heat.—With a given substance under given conditions, it is found that the same amount of heat (obtained by energy of mechanical action, or chemical action, or any other determinable means) produces the same change of temperature. Hence the unit by which to measure heat may be fixed by a temperature change in some standard substance. For measuring directly, a unit is generally of the same nature as the quantity to be measured; e.g., the unit to measure duration is itself a period of time; to measure distance, a length; etc. To measure heat, the unit is a quantity of heat. The c.g.s. unit of heat is the quantity of heat that will raise one gram of water from 0° to 1° C. in temperature. This unit is called a calorie (or sometimes a water-gram-degree). The British thermal unit (B.T.U.) is the heat required to raise one pound of water 1° F. in temperature.

81. Capacity for Heat. — By the capacity of a body for heat is meant not the quantity of heat it can contain, but the quantity of heat required to raise the temperature of the body one degree.

82. Specific Heat.—By this is meant the quantity of heat required to raise a given mass of any substance one degree in temperature compared with the amount of heat needed to raise an equal mass of water through the same temperature change. For water this is unity, and, as compared with water, the specific heat of any substance will be the ratio of the heat per gram to the change of temperature; i.e., it is numerically equal to the number of calories required to raise one gram of the substance one degree Centigrade. The specific heat of water is very nearly, but not quite, constant from 0° to 100°, decreasing slightly from

0° to 37°, and then slightly increasing. Measurement of heat is calorimetry. The commonest method of determining the specific heat of bodies is called the method of mixtures, though there are various other methods in use. The method of mixtures proceeds on the assumption that if two substances at different temperatures are mixed, then, if there is no chemical action involved, the heat that is lost by one is gained by the other as they come to a common temperature. If one of the substances is a standard, as water, the heat it gives out in a known rise or fall of temperature is simply the mass of water multiplied by the number of degrees it was changed in temperature; and this is the heat that produced the corresponding change of temperature in the other substance.

For a metal ball as an example:

Let sp. ht. $= c$;

Temperature of substance $= T$ (say, 100°);
Temperature of water $= t$ (say, 20°);
Temperature of mixture $= \theta$ (say, 23°);
Mass of substance $= M$ (say, 200 g.);
Mass of water $= m$ (say, 300 g.).
Then heat gained by water is $m(\theta - t)$;
Heat lost by substance is $cM(T - \theta)$.

Equating these, we get

$$c = \frac{m(\theta - t)}{M(T - \theta)}.$$

Using the above quantities, we should find $c = 0.0584$, which is approximately the value for tin.

Experiment No. 36, page 149. Specific Heat and Capacity for Heat.

EXAMPLES. —

1. 520 g. of water cools from 15° C. to 4° C. How much heat does it lose? *Ans.* 5720 calories.

2. A coil of copper wire at a temperature of 99.6° C. and weighing 180.4 g. is dropped into 240 g. of water at 10° C. The copper and water come to a common temperature of 16° C. Find the specific heat of the copper. *Ans.* 0.0955.

3. How much heat is required to raise the temperature of 416 g. of copper 50 centigrade degrees? *Ans.* 1988 calories.

Note. — In actual determination of sp. ht., the calorimeter (i.e., the vessel containing the water) undergoes the same change of temperature as the water in it, and in its capacity for heat it corresponds to a definite amount of water. This is called its water equivalent, and equals as many grams of water as the number of calories required to raise its temperature one degree. This must be added to the water placed in it, to get the equivalent mass of water actually heated or cooled.

4. A piece of silver weighing 25 g. was heated to 100° C. and dropped into a calorimeter containing 100 g. of water, the temperature of which was raised from 11° to 12.2°. The water equivalent of the calorimeter, stirrer and thermometer was 4.5 g. What was the sp. ht. of the silver?

Ans. 0.057.

5. What is meant by the statement that the sp. ht. of water is thirty times the sp. ht. of mercury?

83. Latent Heat of Fusion. — When a body begins to melt, the further application of heat does not cause any further rise of temperature in the mixture of solid and liquid until the former is all melted. The heat that is thus added to accomplish the liquefaction has been called the *latent* heat of fusion, and is again given out if the body returns from the liquid to the solid state.

The method of mixtures enables us to determine the latent heat of water or the latent heat of fusion of ice. By melting a known mass of ice at 0° in a known mass of water at $t°$, if the temperature of the mixture is $\theta°$, then the water in cooling has given out heat that has melted the ice and raised the temperature of the water, resulting from the melting of the ice from zero to $\theta°$.

For example, suppose 50 g. of ice is introduced into 200 g. of water at 60° C., and the resulting temperature, when the ice is melted, is 32° C. Call latent heat of water, L. 200 g. of water in cooling from 60° to 32°, i.e., through 28°, gives out 200 × 28, or 5600 calories; of this, it required to raise the 50 g. of melted ice from 0° to 32°, 50 × 32, or 1600 calories; the remaining 4000 calories were expended in melting the 50 g. of ice; therefore, the number of calories required to melt one gram of ice is $\frac{4000}{50}$, or 80 calories, which is the latent heat of fusion of ice. If heat is

energy, what has become of the energy when the heat is thus said to be latent?

84. Freezing (or Solidifying) is a Warming Process. — As heat is absorbed in the conversion of a solid into a liquid, so the change of a liquid into a solid is attended by the liberation of heat. For every gram of water that changes into ice at 0° C., 80 calories are set free and tend to warm the vessel or surrounding atmosphere. This delays the progress of freezing and may prevent the air from becoming so cold as to injure fruits or vegetables. Where such articles are stored in bins, advantage is sometimes taken of this fact when a severe fall of temperature is impending, by placing vessels of water offering a large surface in the vicinity of the articles that would be injured by the severe frost. The danger is mitigated by the heat liberated when the water freezes.

EXAMPLES. —

1. How much heat will be absorbed by 550 g. of ice in melting? How much heat will be given out by 880 g. of water in freezing?
Ans. 44,000 cals.; 70,400 cals.

2. How much water at 98° C. would be required to melt the 550 g. of ice in Example 1? *Ans.* 449 g.

3. Find the result of mixing 1 kg. of snow at 0° C. with 4 kg. of water at 30°. *Ans.* 5 kg. of water at 8° C.

4. What will result if 2 kg. of boiling water is poured on 2.5 kg. of snow at 0° C.?

85. The Two Specific Heats of a Gas.— With a given mass of gas so inclosed as to permit no expansion, the heat necessary to raise its temperature is all expended in increasing the kinetic energy of the molecules. The volume will be constant, but the pressure will increase with rise of temperature. The amount of heat needed thus to raise the temperature of one gram of gas one degree is called the specific heat at constant volume.

If the gas is free to expand as it is heated, it will push back its inclosing walls a certain distance with a force equal to the external (constant) pressure, thus doing work; and, in addition to this, the mean kinetic energy of the molecules is to be increased

as much as in the other case, for the same rise of temperature. In a gas thus heated, more heat is required per degree rise of temperature, and the heat needed thus to raise one gram of gas one degree in temperature is called its specific heat at constant pressure.

The latter is comparatively easy to determine directly; the former is difficult to determine directly, but, from known thermodynamic principles, it can be derived from the velocity of sound (!) in a gas at given pressure. This will be explained later. (See Watson, Arts. 204, 205 and 215.)

86. Change of Specific Heat with Change of State. — While the specific heat of most bodies is slightly different at different temperatures, the difference in specific heat of a substance in the various states of aggregation is considerably greater than it is for mere differences of temperature, and usually it is highest for the liquid state. For example, for water we find, solid, 0.50; liquid, 1.000; gas, 0.477. (See Watson, Art. 206.)

Dulong and Petit's law is that the product of the specific heat of an element in the solid state into the atomic weight is a constant. It is found, also, that the product of the atomic weight into the specific heat of a gas is approximately constant, and about half the value of the product in the case of solids. For solids the product is about 6.4, and for gases about 3.4. This product, called the atomic heat, varies considerably for a supposed constant, but it is pointed out that the specific heat of most solids seems to become constant near a certain temperature, and if the specific heat at this temperature were employed for getting the atomic heat the results would more nearly verify Dulong and Petit's law. (Watson, Art. 207.)

87. Change of State; Melting Point; Laws of Fusion. — The passage of a substance through the various states, solid, liquid and gaseous, together with the changes of energy involved, has been indicated in Arts. 73 and 75.

(*a*) For each substance there is a definite temperature at which the change of state occurs, that at which it changes from solid to liquid being known as its melting point.

(*b*) When the melting point has been reached, no further rise of temperature occurs by application of heat until the body is all melted.

Some substances, as wax and wrought iron, first become plastic, in changing from solid to liquid, and the exact melting point is hard to fix upon, since the real condition of liquidity is vague.

88. Influence of Pressure on Melting Point. — As will be pointed out later, when the boiling point of a substance is designated, it is necessary to specify the pressure under which boiling (or vaporization) takes place. For the other standard temperature, viz., that of exchanging liquid and solid states, it is usually not necessary thus to specify the pressure. Strictly speaking, however, the temperature at which this change of state occurs does depend somewhat on the pressure, and it varies in different measure with different substances.

It is readily seen that if a body contracts in melting, then the effect of applying pressure is to assist in diminishing the volume, and the substance more readily liquefies. That is, it will not remain in the solid state at the temperature at which it would have so remained if it had not been compressed; therefore, to keep it frozen under pressure it would have to be colder, or, the melting point is *lowered* by pressure. With bodies that expand on melting, the application of pressure makes it more difficult for them to melt, and they must be made hotter than would otherwise be necessary, or, the melting point is *raised* by pressure. In any case the change of temperature is very small even for a great pressure.

Water is a substance whose freezing point is lowered by pressure, the lowering amounting to $0.0076°$ per atmosphere, which does not continue in direct proportion to increase of pressure, but under 13,000 atmospheres a freezing point of water as low as $-18°$ has been reached.

Faraday discovered that when two pieces of melting ice are pressed together they freeze together. This phenomenon, which is illustrated in various ways, is called regelation. By suspend-

ing a heavy weight from a fine wire that passes over a block of ice — the pressure under the wire is great — the ice melts, heat (of fusion) is absorbed from the wire and the neighboring ice, and the wire sinks in the block. The cold water passes above the chilled wire and, with the pressure released, it again freezes solidly with the ice on each side.

Experiment No. 37, page 149. Regelation.

A block of ice five or six cm. square in cross-section, under a fine piano wire carrying fifteen pounds, will be cut through in from twenty to thirty minutes.

Professor Tyndall has shown that the movements of glaciers may be explained, perhaps, better on the basis of regelation than on that of viscosity.

Philosophy of making a snowball: Very cold snow is not good for snowballs, for pressure cannot be applied great enough to lower the melting point below the temperature of the snow, and regelation will not occur; but with snow at 0° it is precisely the regelation that makes the firm ball. So, too, with a heavy cart leaving a frozen track after the passage of the wheels.

The converse of this principle of regelation leads to some interesting deductions. Just as those solids which contract on melting take a lower temperature for melting under increased pressure, so those which expand upon melting, i.e., those whose density diminishes and which therefore sink in the liquid, require a higher temperature for melting when the pressure is increased. "Thus we can understand why it is that the materials forming the interior of the earth are practically rigid and perhaps solid, though at a temperature far above their ordinary melting points." There is little doubt that a few hundred miles below the surface of the earth the temperature is far above the melting point of lava; it is equally well shown that the earth as a whole is nearly as rigid as a globe of steel of the same size. These two apparently inconsistent conditions are at once reconciled if we suppose the average materials of the earth to be such as, like lava, expand on melting. They may remain solid even above a white heat, for "if the earth were liquid throughout and

of *uniform* density equal to its present mean density, the pressure within a few hundred miles of the surface would increase at the rate of somewhere about 800 atmospheres (or nearly $5\frac{1}{2}$ tons per sq. in.) per mile of depth" (Tait, *Heat*, pp. 124, 125).

It is interesting to note the effect of pressure in the opposite direction. It is a familiar fact that various substances vaporize directly from the solid state without passing through the liquid state (as when clothes "freeze dry" in an atmosphere below freezing). Camphor, arsenious acid, snow, are well-known instances. As a result of various investigations by himself and others, Dr. Thomas Carnelly in 1880–81 announced as a general proposition: "In order to change a solid body into a liquid, the pressure upon it must exceed a certain amount, which may be called the *critical pressure*, and below which the substance will not melt, no matter how great heat be applied to it" (Rosenberger, III, 2, p. 658). This pressure is not to be confounded with the pressure of a gas at the critical temperature. It is evidently a very low pressure for all ordinary solids. Carnelly found it to be 420 mm. for chloride of mercury, and 4.6 mm. for ice. Above this pressure ice will melt by applying heat; below, it passes into vapor by sublimation. (See also Watson, Art. 223.) He also asserts that if the pressure can be kept low enough and the heating done rapidly enough (i.e., to prevent the formation of vapor which would raise the pressure), ice can be heated far above the boiling point of water, even as high as 180° C. (Rosenberger, *loc. cit.*)

The passage of a solid directly into vapor without becoming liquid is called sublimation. On this subject see Preston, *Theory of Heat*, Art, 170; and Barker, p. 309.

The freezing point, then, under one pressure may be higher than the boiling point under another pressure. For example, benzene, which is liquid at ordinary temperatures under atmospheric pressure, and will freeze in a vacuum at a temperature of 5.3° C., will boil under a pressure of one atmosphere at 81°, but will freeze at 81.4° under a pressure of 3500 kilograms per sq. cm. or 3387 atmospheres.

89. Elastic Force of Vapors. — In passing from a solid to a gas a body generally goes through an intermediate liquid state; if not, the process is called sublimation. The change of a liquid to the gaseous state is vaporization.

A liquid introduced into a vacuum vaporizes instantly, so much of it passing into vapor in the inclosed space as to exert a definite pressure.

Experiments Nos. 38 and 39, page 150. Vapor Pressure.

By introducing small quantities of liquid into the space above the mercury column of several barometer tubes and noting the depression of the column, it is seen that at the same temperature the vapors of different liquids have different elastic forces.

A vapor denotes a substance in the gaseous form which, at ordinary temperatures, appears as a liquid or solid, while a gas denotes a substance which under ordinary circumstances appears in the gaseous form and which can only be reduced to the solid or liquid form by considerable pressure or lowering of temperature. Those vapors which are on the point of condensation are called saturated vapors, while those which can suffer a certain amount of compression or cooling without condensation are called unsaturated vapors. In this sense all gases are unsaturated vapors, for they can all be condensed by the simultaneous application of sufficient cold and sufficient pressure. (Daniell's *Physics*, p. 205.) In some cases a vapor condenses directly into a solid, e.g., arsenious acid. (*Ibid.*)

There are three processes of vaporization: (1) evaporation, where a liquid is converted into a gas quietly and without formation of bubbles; (2) ebullition, where bubbles of gas are formed in the mass of the liquid itself; (3) vaporization in the spheroidal state, where a liquid evaporates slowly, although apparently in contact with a very hot substance. While the first mode is being exemplified continually, we can do little to illustrate it just now in the lecture room; but we may state the relations of vapors and their pressures (or tensions) according to Dalton's laws.

I. "*In space destitute of air the vaporization of a liquid goes on only until the vapor has attained a determinate pressure depend-*

ent on the temperature, so that in every space void of air determinate vapor pressure corresponds to determinate temperature."

In the above experiment an attempt to reduce the space containing the saturated vapor will not increase the pressure but will result in condensing more vapor into liquid; to warm the vapor will increase the pressure. The pressure of a saturated vapor is called its maximum pressure (or tension) for that temperature. At 20° C. the maximum vapor pressure of

mercury	= 0.021	mm.	of	mercury	column;
water	= 17.39	"	"	"	"
alcohol	= 44.48	"	"	"	"
ether	= 433.26	"	"	"	"

II. "*In a space filled with air the same amount of liquid evaporates as in a space destitute of air, and the same relation subsists between the temperature and pressure of the vapor whether the space contain air or not.*"

This principle is involved in hygrometry, which we shall consider later; but, although unsaturated vapors obey Boyle's law, — and this law of Dalton's is equivalent to saying that the pressure of a mixture of vapors is the sum of the pressures due to each constituent as if the others were not present, — still this is found to be true only when the vapors are not very near saturation. "It would seem *a priori* that Dalton's law can only be an approximation, for otherwise it would mean that by introducing a sufficiently large *number* of different *kinds* of liquids into the same space we could produce as great a pressure as we please, a result that is unlikely to be true" (Watson, Art. 219).

90. Ebullition. — The layers of liquid first heated form vapor, which is condensed by the colder upper strata through which they rise. This rapid condensation of bubbles causes the singing of liquids before they boil. When the bubbles reach the surface ebullition has begun. The temperature of ebullition is dependent upon (1) external pressure, (2) nature of the vessel, (3) substances dissolved in liquid, (4) nature of liquid. The last two we need not discuss here at all; the second, only to call attention

to the fact that although the boiling water may differ in temperature in different vessels the steam will be of the same temperature if the pressure is the same. Therefore we give especial attention to the effect of pressure upon the boiling point of water.

91. Effect of Pressure upon the Boiling Point of Water. — The pressure of vapor during ebullition equals the external pressure, for in the apparatus shown in Figure 52 the mercury stands at the same level in the arms *a* and *b*. (This is true, no matter what liquid is in the vessel.) The temperature, then, will continue to rise until that point is reached for which the corresponding vapor pressure is equal to the external atmospheric pressure (or superincumbent pressure from whatever source; perhaps the pressure of its own vapor if inclosed as in a boiler). If this pressure is small, then the boiling point will be low; if great, the boiling point is high. Accordingly, as the barometer shows that the pressure of the atmosphere varies, the boiling of water under atmospheric pressure does not always occur at the same temperature. Furthermore, the temperature of ebullition being determined or the law of its change being determined, experimentally, as also the variation of elevation above sea level for a given change in barometer, a sensitive thermometer becomes available as a barometer. Such an instrument is called a hypsometer. This use of the thermometer is attributed to Wollaston, about 1800, but Fahrenheit had already suggested it as early as about 1700. We may use the term "boiling point" for any liquid, but when used without qualification it is usually with reference to water, and, in any case, it varies with the pressure.

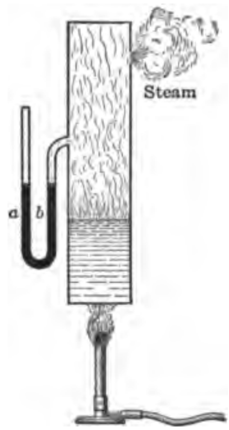


Fig. 52. Pressure of Steam equals that of Atmosphere.

"A liquid boils or passes into vapor at a temperature at which the pressure of its saturated vapor is equal to that which the liquid supports. If now the pressure of the vapor of any

substance at the freezing point is equal to or greater than one atmosphere, then this substance will not exist under atmospheric pressure in the liquid state, for as soon as the solid melts the liquid will pass off into vapor. Boiling will thus occur, as it were, at the surface of the solid. This will always occur at a given temperature if the pressure is less than that of the saturated vapor of the substance at that temperature; but if the pressure be greater than this value, the liquid form will be possible, and melting will occur if the given temperature is above the freezing point. Thus arsenic volatilizes without melting under the atmospheric pressure, but if the pressure is increased, fusion may be effected; and Carnelly showed that ice, mercuric chloride and camphor do not melt below a certain pressure peculiar to each substance, and which he proposed to call the critical pressure," as we have mentioned before. (Preston, *Theory of Heat*, Art. 170.) The volatilizing point of a solid under a given pressure is the maximum temperature at which it will remain in the solid state under that pressure.

A liquid, then, may boil at any temperature, depending on the pressure it supports, but when not otherwise stated the boiling point means the temperature of boiling under the pressure of one atmosphere, i.e., 76 cm. of Hg. In this restricted sense the temperature of boiling is not the same thing as the boiling point. The boiling point, in this sense, of water, air and other substances is given in Art. 94.

Experiments Nos. 40 and 41, page 151.—To set Water to Boiling by Cooling It.

Culinary paradox: Show renewal of boiling in a flask when superincumbent vapor is partly condensed, by application of cold water or ice. Also, partially exhaust the air of a receiver covering a beaker of warm water (say 60° or 80°), and have some water initially at the same temperature standing in a vessel outside the receiver; compare the temperature of the water in the two vessels after boiling that under the receiver. Also, ether under the exhausted receiver will boil at ordinary temperature.

These experiments show that water will boil at less than 100° if pressure is diminished below 76 cm. For higher pressures the temperature of the vapor and of the water rises, the increase being one degree C. for about 27 mm. added pressure at first. With an addition of eight atmospheres of pressure (about 120 pounds gauge), the temperature rises to 175° C. Papin's digester, with the safety valve which is used in the same form to-day, was described by their inventor, Denis Papin, in 1681. (When water is boiling it can be made hotter only in case greater pressure is put upon it.)

When vapor is produced a certain quantity of heat is required to convert the liquid into vapor of the *same temperature*, which is then said to be latent heat; and if heat is available from no extraneous source, it must be supplied by the substance from which evaporation is going on, and its temperature will then fall. Water may thus be frozen by its own evaporation. If the evaporation takes place rapidly under very low pressure, it may freeze while boiling!

Vaporization, then, absorbs energy and is a cooling process. That the cooling of a liquid due to its evaporation is in accord with the kinetic theory is shown thus: The temperature represents the average kinetic energy of the molecules; in evaporation only those molecules escape whose kinetic energy is above the average; consequently the average of those remaining is diminishing and the temperature falls. (Hastings and Beach, Art. 171.) Local anesthesia by evaporation of ether, etc.

92. Latent Heat of Vaporization. — When a liquid is brought to the point of boiling under a given pressure, a definite quantity of heat is required to convert a unit mass of the liquid into vapor at the same temperature. This is called the latent heat of vaporization. It varies with the temperature at which vaporization is effected. With water, starting at 0° C., the total quantity of heat required to raise a gram to a temperature $t°$ and convert it into vapor at that temperature is expressed by the formula:

$H = 605.5 + 0.305\,t.$ (See Hastings and Beach, Art. 168.)

This would give as the total heat of steam at 100°, 636 calories, and for the latent heat at 100°, 536 calories. Latest determinations give for the *heat of vaporization*, i.e., the *latent heat at any temperature*, $L_t = 596.73 - 0.601\ t$ (Watson, Art. 214), which at $t = 100°$, becomes $L = 536.63$. Conversely, condensation gives out heat; hence, philosophy of steam heating.

Experiments Nos. 42 and 43, page 152. Freezing Water by Evaporation.

Cryophorus: Freeze water in a warm atmosphere by its own evaporation. Call attention to the necessity for a very perfect vacuum in the cryophorus. When the temperature has fallen to zero, further evaporation can only occur (since the vapor would become saturated) if the pressure over the liquid is less than 4.6 mm.; also, if the temperature is to fall, heat must be taken from the water faster than it is supplied to it from the room. By evaporation of very volatile liquids intense cold may thus be produced. By the use of liquid sulphurous acid, or of liquid carbon dioxide, mercury may be frozen. When that process is applied at any particular place on the body, that spot may be so benumbed by the cold as to be insensible to pain, and minor surgical operations may be performed with the aid of such local anesthesia.

EXAMPLES.

1. What is meant by saying that the latent heat of steam is 536? How much heat is required to change 97 g. of water at 100° C. into steam at the same temperature? *Ans.* 51,992 cals.

2. How much heat is required to raise 100 g. of water from 0° C. to 99° C. and convert it into steam at that temperature? (Art. 90.) *Ans.* 63,569.5 cals.

3. How much heat is required to raise one gram of ice from −20° C. to 0° C., melt it, heat the water to 100° C., vaporize it, and heat the steam to 180° C.? *Ans.* 764.4 cals.

93. Internal Work and External Work in Heating a Body.— Except where a change of state is about to occur, we may regard the application of heat to a body as resulting in three kinds of changes:

(1) Rise of temperature, — an increase in the average molecular kinetic energy.

(2) Expansion, — an increase in molecular potential energy, since the positions of equilibrium for the oscillating molecules are forced farther from one another. (Counts for nothing in a gas.)

(3) Pushing back the medium exerting pressure on the body, in expanding.

(1) and (2) are called "internal work," (3) is "external work." In a change of state usually a considerable amount of energy is required or else liberated. Water could be represented in a rising scale of energy by ice, water, steam.

94. Critical Temperature of a Gas. — The conversion of a gas into a liquid, as we have seen, may be effected at various temperatures, depending on the pressure. Whenever the pressure exceeds the maximum vapor pressure for the given temperature, liquefaction ensues. This pressure is higher the higher the temperature, but for every gas there is a temperature above which no pressure will accomplish liquefaction. This temperature is called the "critical temperature." The pressure to produce liquefaction at the critical temperature is sometimes called the "critical pressure." Below this temperature, then, a body may be liquid or gas indifferently, or a given mass may exist at the same time partly in the liquid form and partly in the gaseous. The critical temperature for some gases is high, that of water being 365° C., at which temperature a pressure of 200 atmospheres is necessary for liquefaction, and above which no pressure will liquefy it. (If we lived normally in a temperature above 365°, we would know nothing of liquid water, and would consider steam a permanent gas, as we have been wont to consider air.) Such is our situation relative to oxygen and various other gases. For oxygen the critical temperature is −118° under a pressure of 50.8 atmospheres, although it will be liquid under one atmosphere pressure if its temperature is reduced to −181°, and that is said to be its boiling point. The following table exhibits various thermal conditions for several substances.

Critical temperature.	Critical pressure Atmos.	Temp. sat. vapor at 76 cm. (boil. point).	Freezing temperature.	Freezing pressure.	Density of liquid.
Water, 365° C., 689° F.	194.6	100° C. 212° F.	0° C., 32° F.	76 cm.	1 at 4° C.
Carbon dioxide, 31.1, 88	73	− 78.2, −108.8	−56, −69	76	0.83 at 0° C.
Air, −141, −220	39.6	−191.4, −312.6	....	....	0.933 at −191.4
Oxygen, −118.8, −182	50	−182.9, −296.4	....	....	1.124 at −181.4
Nitrogen, −146, −231	35	−195.7, −318.3	−214 C., −353.2	6	0.885 at −194.4
Hydrogen, −234.5, −401.7	20	−252.7, −422.8	....	....	....

(From Kaye and Laby, *Physical and Chemical Constants;* and Landolt and Bernstein, *Physikalisch-Chemische Tabellen*).

(See also Watson, Art. 232.)

Exhibit CO_2 tubes.

Adiabatic expansion of air from 60° F. (521 absolute) and 2500 pounds pressure to 14.7 pounds pressure, in doing work, would fall in temperature, with $K = 1.408$, to 117.5 Fahrenheit degrees on absolute scale or to −342.5° F. = (−208.6° C.).

To liquefy a gas, it must be cooled below its critical temperature. The gas is put under great pressure and surrounded by air also under such pressure and cooled to a low temperature by any convenient means; then the enveloping air is allowed to escape into the external atmosphere, and the work of its expansion against the external pressure causes its temperature to fall. The pipe containing the cold compressed gas is thereby cooled sufficiently further to liquefy its contents. This is called a "regenerative process." For further account of liquefaction processes see Watson, Art. 235.

95. Spheroidal State.—Rapid evaporation occurs from a liquid in the presence of and in apparent contact with a surface that is at a temperature higher than the boiling point of the liquid; but the contact is apparent only. A drop of liquid placed on a hot metal plate becomes a flat globule like mercury on a flat surface, and moves around or trembles with a rhythmic motion, evaporating at a moderate rate but not boiling. Liquids in this condition are said to be in a spheroidal condition. Close examination shows that there is a very thin layer of gas between the liquid and the surface of the hot plate, large enough to permit a beam of light to be passed between the two surfaces. Phenomena of this kind were formerly known as "Leidenfrost phenomena."

To produce them, it is necessary that the surface on which the liquid is placed shall be considerably hotter than the boiling temperature of the liquid. Water thus placed on a hot metal plate will remain in the spheroidal condition, itself several degrees below boiling temperature, evaporating not very rapidly while the plate cools until the plate is no longer hot enough to maintain the condition, when the liquid comes into closer contact with the plate and is immediately dissipated in vapor with an explosive force.

(Water may thus be evaporated on a thin metal plate which will finally become too cool to maintain the spheroidal condition and will be further cooled by the final vaporization of the water. But the plate may be found still hot enough to sustain a globule of ether in the spheroidal state. Ether may also be shown in the same condition on the surface of hot water.)

The globule is actually supported on a cushion of its own vapor formed at a rate sufficient to hold the liquid away from the hot surface. The explanation of these phenomena on the kinetic theory is as follows: "If a surface be heated, a molecule of gas striking against it is heated, and leaves the hot surface with increased velocity. If the surface is fixed, gas in front of it is driven off by bombardment of molecules which have touched the hot surface and, returning, strike fellow molecules; in front of the hot surface, therefore, the gas is under greater pressure than if the surface had been cold.

If the hot surface be the front aspect of a disk, the back of which is by some means kept colder than the front, and if the disk be suspended in a gas, the heat of the front surface increases the pressure towards the front, and the gas flows round to the back of the disk. Thereafter the disk is struck on the front by fewer molecules with greater velocities, and on the colder surface by a greater number of molecules with less velocity, and there is compensation: the disk is equally pressed front and back and does not move.

Now suppose particles recoiling from the heated surface do not meet other molecules, but impinge on walls of the vessel. A

layer of particles in such condition is called a "Crookes' layer." This will occur in two cases: (1) when the gas is so rarefied that the mean free path of the molecules exceeds the distance between the surface and the walls of the vessel; (2) when, whatever the density of the gas, the opposite wall is so near the hot surface that the distance between them is less than the actual mean free path of the molecules. These conditions, which are substantially identical, may concur; there may be both rarefaction of the gas and "approximation of the opposed surfaces." The latter is the case with the "spheroidal state." When the distance between the disk and the opposite wall is very small, the vacuum need not be very good; the effect of the repulsion may be made manifest even in the open air. (Daniell's *Physics*, 1st ed., p. 325, *q. v.* See also Preston, *Theory of Heat*, Art. 54; and Barker, pp. 329 and 397.)

Exhibit radiometer.

Explanation of boiler explosions by admission of cold water into a superheated boiler in which water has become too low.

May not gaseous molecules be wandering in interplanetary regions?

96. Van der Waals' Equation. — According to the law of Charles, if the volume of a given mass of gas is constant, its increase of pressure is proportional to the rise of temperature; and, by the law of Mariotte, with constant pressure the increase of volume is proportional to the rise of temperature; in each case $\frac{1}{273}$ of the volume at 0° C. per centigrade degree. Then if we reckon temperatures from an absolute zero, pressure p and volume v would each vary as the temperature T, or $pv = RT$, where R is a constant depending on the units for p, v and T. (See Art. 79.)

From this equation it is seen that, with volume constant, pressure decreases with fall of temperature until, at zero, the pressure becomes nothing; but with constant pressure, the volume diminishes with fall of temperature, until, at zero temperature, the volume would be nil, — a manifest absurdity. But the theory upon which this proceeds takes no account of the

size of the molecules, nor of a possible attraction or repulsion between them, which may vary with their distance from one another. That something of this latter may exist is indicated by the departure of gases from Boyle's law when greatly compressed; and that there is some size to the molecules is only to say that they occupy space. Various attempts have been made to formulate the relations of pressure, volume and temperature so as to take these conditions into account.

Van der Waals reached the conclusion that "the effect of the attractions between the molecules was to add a term to p, and he took it to be of the form $\frac{a}{v^2}$, where a is a constant. The effect of the finite size of the molecules was to virtually diminish the volume v, in which the molecules can move, by a constant amount b. His modified equation then took the form

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT."$$

(Watson, Art. 234, which see for fuller discussion.)

97. **Clausius' Equation of the Virial.** — To provide for the probability that the influence of one molecule upon another is not negligible, Clausius called the force that may exist between any two molecules, F, and the distance between them s; then Fs, being the product of a force by a distance, is a quantity of the order of work or energy. Since the entire number of such products would include the same product twice for each molecule, the effect of the interactions would be represented by $\frac{1}{2}\,\Sigma Fs$, which Clausius termed the "virial" of the molecules.

Under the kinetic theory we had

$$pv = \tfrac{1}{3}\,M\overline{V}^2, \text{ whence } \tfrac{1}{2}\,M\overline{V}^2 = \tfrac{3}{2}\,pv.$$

The first member of this equation is the kinetic energy of the molecules of the gas. Clausius puts this energy equal to the second member plus the virial of the interaction of the molecules, or

$$\tfrac{1}{2}\,M\overline{V}^2 = \tfrac{3}{2}\,pv + \tfrac{1}{2}\,\Sigma Fs.$$

Under ordinary conditions of such gases as hydrogen, nitrogen, etc., the virial is very small, but if the gas is greatly compressed it may naturally be expected to become relatively more significant. The equation may be written

$$pv = \frac{1}{3}\,\Sigma M\overline{V}^2 - \frac{1}{3}\,\Sigma Fs.$$

Art. 58 showed how the value of pv first decreased with an increase of p and then increased, which would mean, in the application of Clausius' equation, that up to a certain pressure ΣFs is a positive quantity, or the action between the molecules is an attraction, but beyond that pressure (or proximity of the molecules) the quantity ΣFs is negative, or the force between the molecules becomes a repulsion. (pv is at first a decreasing, afterwards an increasing, function of p.) (See Hastings and Beach, *General Physics*, Art. 252.)

For freezing point and boiling point of solutions, and for freezing mixtures, see Watson, Arts. 225–230.

98. Hygrometry; Humidity. — By hygrometry is meant the determination of the water vapor present at any time in the atmosphere.

The atmosphere is never free from moisture, and the presence of water vapor in a state of equilibrium means that this vapor exerts a definite pressure, which is a part of the entire pressure of the atmosphere as shown by the barometer. At a given temperature, the more moisture there is present the greater pressure will there be due to it, up to the point of maximum pressure of water vapor for that temperature. If the vapor is present in sufficient quantity to produce its maximum pressure, then the atmosphere is saturated with vapor. At that temperature no more vapor will exist as such; the effort to introduce more into a given space will result in liquefaction. Also, at the saturation point any lowering of temperature will result in liquefaction, since less vapor will produce the (smaller) maximum pressure that corresponds to the lower temperature. The term "humidity" may mean either of two things: (1) it may designate the actual mass of water vapor present in a given volume or given mass of

atmosphere, in which sense it is called the "absolute humidity;" or (2) it may mean the proportion of water vapor actually present in the atmosphere to the quantity that would be necessary for saturation, i.e., the quantity present, compared with that which could be present. This is the sense in which the term is ordinarily used, and is "relative humidity."

99. **Dew Point.** — If the temperature of the air at any place is lowered while the surrounding air exerts a constant pressure, the pressure of the air and vapor whose temperature is being lowered remains unaltered; but as the temperature is further lowered it finally reaches a value for which the pressure of the vapor present is the maximum or saturation pressure. If the temperature is further lowered, not so much moisture will remain in the form of vapor, but some will be precipitated in liquid form. The temperature at which this occurs is known as the dew point; the precipitation which occurs with further fall of temperature is dew. Dew, then, literally does "fall" upon objects from the atmosphere, and when the atmosphere has been cooled far below the dew point a good deal of moisture will be precipitated, so that the actual quantity of moisture remaining in the atmosphere will be less than before and the cold air will be "absolutely" drier than the warm was; but at the same time it will be saturated with moisture and consequently be "relatively" more humid, for the humidity of saturated air is 100 per cent, which will be the case with a small quantity of vapor when the temperature is low and with a large quantity when the temperature is high.

100. **Determination of Humidity.** — Various forms of instruments, or hygrometers, have been devised to determine the humidity or hygrometric state of the atmosphere. The principal ones are those known as dew-point instruments, and the psychrometer, or wet- and dry-bulb hygrometer. In the former (Fig. 53, Dines' hygrometer) the temperature of a polished surface in contact with the air is lowered until the dew appears upon the polished surface, and the temperature then is recorded. This is the dew point. The pressure of the vapor present is the same

as the maximum pressure of water vapor for the temperature of the dew point. This pressure may be taken from tables of

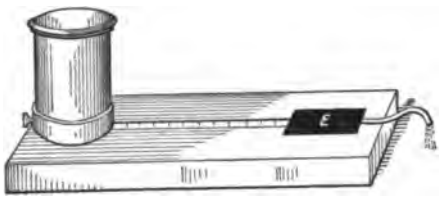


Fig. 53. Dines' Hygrometer.

maximum pressures of water vapor, which have been determined with great care. Also from those same tables is obtained the maximum pressure of vapor for the temperature of the air. That is the pressure the vapor would exert if the air were saturated. Now, unsaturated vapors obey Boyle's law, and the quantity of vapor in a given space is proportional to the pressure it exerts; therefore the quantity present, compared with that which could exist there, is just as the pressure of the vapor present is to the pressure that would be due to the vapor if the space were saturated. The ratio of these pressures, therefore, is the relative humidity.

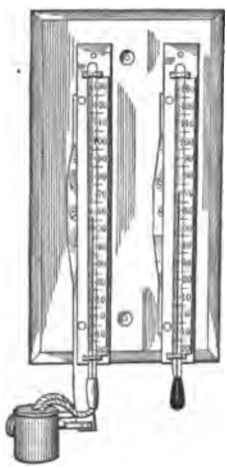


Fig. 54. Wet- and Dry-bulb Hygrometer.

(Make determination.)

The psychrometer (Fig. 54, wet- and dry-bulb hygrometer) consists of two thermometers, one of which has its bulb exposed directly to the air, while the bulb of the other is inclosed in a cloth which is kept moist, and from which, therefore, evaporation is constantly taking place. The effect of this evaporation is to lower the temperature of the wet bulb, and the extent to which the temperature shown by the wet bulb falls below that of the dry bulb will depend upon the rate at which evaporation takes place, and this in turn depends upon the dryness of the air. With little moisture in the air, the

evaporation is rapid and the difference of the readings is considerable; with much moisture present, evaporation is slow and the difference of readings is small. If the air is saturated with moisture, no evaporation occurs and the two thermometers read alike. But the rate of evaporation is influenced by local conditions, such as the size of the room, or taking the observations indoors or outdoors or in a draft; therefore an empirical table or formula is necessary to reduce the indications of the instrument to standard values. The effort is to find the pressure of the vapor present in the air, and a good general formula is

$$e = e' - k(t_1 - t_2)h,$$

in which e is the pressure of vapor present in millimeters of mercury, e' the maximum pressure for the temperature of the wet bulb, t_1 and t_2 the temperatures of the dry bulb and wet bulb respectively in centigrade degrees, and h the barometer reading in millimeters. k is a constant, suited to the conditions under which the observations are made. Its value ranges from 0.00077 to 0.0012, so that a mean value of 0.001 will give fairly accurate results. When the value of e is thus found, the maximum pressure f for the temperature of the air is to be taken from tables, and then the humidity H is the ratio of e to f, or $H = \frac{e}{f}$.

(Make determination.)

It is assumed that saturated air from around the wet bulb is continually giving place to unsaturated from the immediate vicinity. This would be the case if the two bulbs were in a passage-way through which unsaturated air moved from the dry to the wet bulb at such a rate as to maintain the saturation as it passed beyond the wet bulb. See Fig. 55.

Let t_1 and t_2 be the temperatures of the dry and the wet bulb, respectively, and e and e' the pressures of vapor in unsaturated and saturated air. Omitting small minor corrections, suppose one gram of air to pass the two bulbs. Entering at atmospheric (barometer) pressure h, the density of the vapor compared to that of air at the same temperature and pressure d (= .62); then, for one gram of air, the mass of water vapor entering the passage, say at A, is $\frac{e}{h}d$; the mass leaving at B is $\frac{e'}{h}d$; the mass

of water evaporated is the difference or $\frac{d}{h}(e' - e)$ and if L = latent heat of vaporization, the heat absorbed is

$$L\frac{d}{h}(e' - e). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If this is given up by one gram of air in falling from t_1 to t_2 in temperature, calling sp. ht. of air S (= .237), the heat so given up is

$$S(t_1 - t_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for one gram of air. Equating (1) and (2),

$$L\frac{d}{h}(e' - e) = S(t_1 - t_2);$$

whence

$$e = e' - \frac{S}{Ld}(t_1 - t_2)h.$$

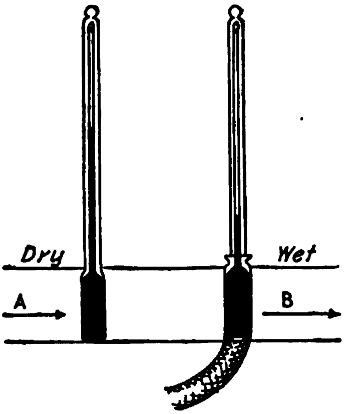


FIG. 55.—Principle of the Psychrometer.

For mean values, if t_2 = 15°, d = .62 and L = 587 (see p. 124), $e = e' - .00065(t_1 - t_2)h$ (Draper's *Heat*, pp. 157, 158). In the sling psychrometer, the swinging of the instrument makes certain that the wet bulb is continually placed in unsaturated air, so that moist air is not stagnant around the wet bulb. For such an instrument the value of h may be taken as 0.0007.

101. Buoyancy of the Air; Reduction of Weight in Air to Weight in Vacuo. — The density of dry air at 0° C. and 760 mm. pressure is 0.001293 gram per cubic centimeter, therefore a cubic meter weighs 1293 grams. Water vapor at the same temperature and pressure as air is found to have a density five-eighths that of air. It weighs, therefore, five-eighths as much as air that fills the same space at given temperature and pressure.

EXAMPLE.—Suppose the barometer reading is 750 mm., the temperature of the room 18° and the dew point 12°, find the weight of a cubic meter of the air in the room.

The vapor exerts the same pressure as that of saturated vapor at the temperature of the dew point, or 12°; this is 10.46 mm., and the actual amount of vapor present is the amount that would saturate the air at 12°.

The density is directly proportional to the pressure and inversely proportional to the absolute temperature. A cubic meter of air at a pressure of 10.46 mm. and a temperature of 18° C. would weigh

$$\frac{1293 \times 10.46 \times 273}{760 \times (273 + 18)}, \text{ or } 16.69 \text{ gm.},$$

and water vapor would weigh five-eighths as much as this, or

$$16.69 \times \tfrac{5}{8} = 10.43 \text{ gm.}$$

The pressure exerted by the air is

$$750 - 10.46 = 739.54 \text{ mm.}$$

The weight of one cubic meter is

$$\frac{1293 \times 739.54 \times 273}{760 \times (273 + 18)}.$$

This makes 1180.51, and the total weight of one cubic meter of atmospheric air under the conditions named is the sum of the weights of dry air and water vapor, or 1190.94 gm.

A cubic meter of dry air at the same temperature and total pressure would weigh $\frac{1293 \times 750 \times 273}{760 \times 291}$, or 1197.30 gm.; so that the presence of moisture in the air makes it lighter. In this instance the buoyancy of the air upon any body in it amounts to 0.00119 gm. per cubic centimeter of volume.

102. Atmospheric Conditions That Make for Comfort or Discomfort. — The principal atmospheric conditions that affect physical comfort are temperature, pressure and humidity, which are observed respectively by means of the thermometer, barometer and hygrometer. Cold air may be nearly saturated with moisture and therefore have high humidity, while containing not nearly so much water in it as would only partially saturate the air at the same pressure but at higher temperature. Place a thermometer outdoors on a cold day and it will show a low temperature, but will probably be itself dry, but on taking it into a warm room it is immediately covered with moisture precipitated from the air, although the air of the room is said to be drier than that outdoors. It is drier although it contains more moisture. It is the relative humidity that determines the feeling of dampness. When the air is high in humidity and low in temperature, it feels raw and chilly; on the other hand, if the air is very humid and at the same time very warm, as in summer, the moist air carries

away little moisture from the skin, and there is none of that relief from the excessive heat that is experienced when the air is dry enough to give the cooling effect that attends evaporation of the moisture from the surface of the body. It is the combination of heat and humidity that makes the suffocating, prostrating wretchedness of a hot day. In such cases, even getting into the shade will not bring much relief.

Instances are on record of men entering ovens or heated chambers, in one instance said to be as hot as 600° F., and in some others at temperatures of 330° F. (sculptor's oven) and 260° F. Explorers in the polar regions endure temperatures as low as −70° F.

The normal temperature of the human body being only a fraction over 98° F., and the boiling point of water 212° F., both the very high and the very low temperatures can be best endured if the air is dry. (2° below normal or 4° above is severe physiological derangement.) Men work in air chambers, and in submarine apparatus at depths below water of about 150 feet (as a limit), which corresponds to a pressure of over five atmospheres (about 5.4 atmospheres). The effect of rare air is more decided. Mountain sickness and dizziness are felt by some people at small elevation, even as little as 1000 feet being change enough in some cases to affect a weak heart; but balloonists have, in some instances, reached an elevation of five miles, where the barometer pressure is only about 10″; here the severity of the cold and the rarity of the air combine to produce severe distress to the system.

103. Transference of Heat. — Heat is transferred from one point to another by the three processes of conduction, convection and radiation.

104. Conduction; Conductivity. — In solids, energy is communicated by one molecule to its neighbor by direct impact without either one departing from its limited sphere of motion, but an increase of energy in a molecule is handed on from one to another until the energy is distributed throughout the body or communicated to another body. Such transference of heat through a body is conduction. The facility with which such

transference is performed by a substance is called its conductivity. This differs with different substances, and may be rated in two different ways. For instance, if two rods of different metal have one end thrust into a flame while the other end is kept at a constant lower temperature, and the temperature of the rods be observed at the same distances from the flame, one will be found to indicate a given rise of temperature at a given point sooner than the other will. Judged by that indication, the one may be said to be a better conductor of heat than the other; but the one that had the higher temperature might have a smaller specific heat (or capacity for heat), and, therefore, have reached a higher temperature with actually less heat than the other; so that the other may have actually transferred more heat without showing it by so great a rise of temperature. So a different mode of determining conductivity is preferable, namely, that by which the actual amount of heat transferred is considered. This gives the absolute conductivity. If a bar of metal has one end in boiling water so that its temperature is constantly, say, 100° C., and the other end is in contact with ice so that the temperature is constantly 0° C., the fall of temperature along the bar is 100°, and the fall per unit length, or the ratio of the total fall to the length, is called the temperature gradient. In such an arrangement as that just described the amount of heat transferred through the bar is determined by the amount of ice melted. Or the number of calories is the number of grams melted × 80.

If θ_1 and θ_2 are the temperatures of the ends of the bar, a the area of cross-section, d the length of the bar, and T the time conduction is going on, then it is found that for any given metal the total quantity of heat transferred is

$$Q = k\frac{\theta_1 - \theta_2}{d}a \cdot T,$$

k being a constant depending on the substance that composes the bar. If $(\theta_1 - \theta_2)$, a, d and T are each unity, then $Q = k$, and k is the number of calories transferred and becomes the measure of conductivity of that substance. Thus the absolute conductivity of a substance is expressed by the number of calories transferred

through a bar of the substance one centimeter long and one square centimeter in area of cross-section, in one second of time, when the two ends have a difference of one degree C. in temperature.

If the absolute conductivity is determined for various substances, their relative conductivities may be derived from those. Tables of conductivity are given in most textbooks, a few examples being silver, 1.006; iron, 0.16; sawdust, 0.00012; horn, 0.00009. It is the poor conductivity of wood and organic substances that makes wood or paper suitable for matches or tapers, for they can burn down nearly to the end without burning the fingers; but paper, if very *thin*, will conduct heat well enough to make it possible to boil water in a paper cup by direct application of a flame, without burning the paper.

Experiments Nos. 44, 45, 46, pages 152 and 153. Illustrating Conductivity.

Liquids are poorer conductors of heat than are solids, and gases are still worse, being, in fact, almost perfect nonconductors.

Experiment No. 47, page 153. Conductivity of Water.

(For conductivity of gases, see Watson, Art. 241; and Barker, p. 332 to foot p. 338.)

EXAMPLES. —

1. If 1,150,000 calories are transmitted in an hour through an iron plate one centimeter in thickness and 100 sq. cm. in area when the sides are kept at 0° C. and 20° C., what is the thermal conductivity of iron?

Ans. 0.16 cal. per cm. per deg. per sec.

2. When the pressure in a steam boiler is 120 lbs. gauge pressure, the temperature is 175° C. If the boiler is of iron 5 mm. thick, and the outside is at a temperature of 150° C., at what rate is heat lost through the boiler, for each square centimeter of surface? *Ans.* 8 cals. per sec.

3. An ice house has walls 20 cm. thick, with an area of 200 square meters. If the conductivity of the walls is 0.00051, and the air in contact with the outside has a temperature of 77° F., how much ice may be expected to melt in a day? *Ans.* 1377 kg.

105. Convection. — When a heated molecule travels from one place to another it carries heat, and this process is called convection. This is the commonest mode of transference of heat by liquids and gases, and occurs most effectively when there

is best circulation of the fluid. If a vessel containing water is heated at the top, the bottom may stay ice-cold for a long time while the water at the surface is boiling, but if the water at the bottom is heated the warm water rises through the colder and the liquid is gradually heated throughout by convection. The same applies to gases, the unequal heating of the air at different points of the earth giving rise to the innumerable and varied winds.

It is by convection that heating by hot water is effected, and the ventilation of buildings accomplished.

As examples of meteorological processes, if air is not saturated with vapor evaporation takes place at the surface of water; the vapor rises into the colder upper regions, where it condenses into clouds; these are carried by winds to distant regions, and if they are further cooled they are precipitated in the form of rain or snow, on mountains sometimes, whence they return to the sea or lakes to undergo a repetition of the process endlessly.

Heavy clouds of vapor and smoke may form a covering which will confine the air beneath it. In the San Francisco fire after the earthquake there was no wind, no water was to be had, and a heavy pall of smoke spread above the city. The air rising to the height of this cover became there hotter and hotter, and always hotter than the regions below it, until this layer of atmosphere became hottest at the top and was coolest at the bottom. Tall buildings remote from the flames ignited before low ones that were nearer, and always at the top first, e.g., the top of a steeple or the cornice of a tall building. Fairmount House on a hill took fire before Palace Hotel on lower ground although far from other burning buildings.

Flame under a vessel is never in actual contact with the metal of the vessel. (See Barker, pp. 339 to 341.)

106. Radiation. — The third mode of transference of heat is by radiation, and is displayed throughout the universe on a scale far transcending the other methods.

This process implies that the energy of motion of a hot body is imparted directly to the universal medium, the ether, through which it is distributed in the form of wave motion until it is again communicated to gross matter through the close inter-dependence or connection between the ether in and around an ordinary body and the molecules of the body. The inter-rela-

tion of ether and gross matter is one of the most recondite subjects of physics, but is necessarily involved in all forms of radiation through the medium of the ether. While in transit from one body to another the energy is that of wave motion, but when this is absorbed by a body it then becomes energy of molecular motion of the body, or *heat.* Just to the extent that the body does absorb this radiant energy, to that extent is the body heated. No heat is evidenced by the radiation falling upon a body if the body does not take it up. It is then either reflected by the body or else it passes through the body. Heat that is thus reflected by a body produces no effect of heating the body, nor does that which passes through it. Bodies through which radiant heat does not pass are called athermanous, those through which radiant heat will pass are called diathermanous. Good radiators are good absorbers and bad reflectors. Good reflectors are bad radiators and bad absorbers. The absorbing and radiating power of a surface are equal. General law of radiation that of inverse squares. (Barker, p. 376.)

Diathermanous bodies do not transmit heat equally from all sources, the readiness with which it passes through increasing with the temperature of the source. Thus glass transmits readily the heat radiated from the sun, but intercepts that from bodies of only a moderately high temperature, or what are called "black bodies." The heat from the sun will enter a greenhouse and heat the air and bodies within it, but the heat radiated from these is not transmitted but is intercepted by the glass, the latter being athermanous to heat from sources below 100° C in temperature. The aqueous vapor in the atmosphere readily transmits or permits passage of the heat of the sun, which thus warms the earth, but the heat radiated from the latter is intercepted by the vapor and the clouds, and the earth thus cools more slowly than it otherwise would on removal of the sun's rays. This is noticeable in the rapid cooling of the earth and the air at high elevations. No dew on cloudy nights. For mutual relations of radiation, reflection and absorption, see Barker, pp. 373 to 375.

In the Dewar bulb (Fig. 56 *a*), nitrogen, more volatile, evaporates from liquid air, and the liquid takes on a bluish hue, the color of liquid oxygen. Heat admitted to the interior by conduction and radiation is very little; only loss is by evaporation, from free surface of liquid, which is slow if vessel does not receive heat. Rate of evaporation is from 5 to 15 per cent of what it

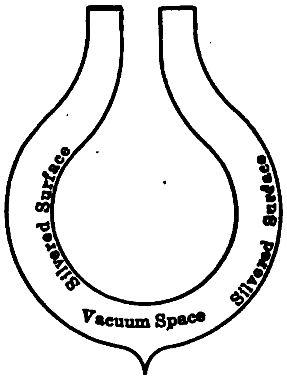


Fig. 56 (a). The Dewar Vacuum Bulb.

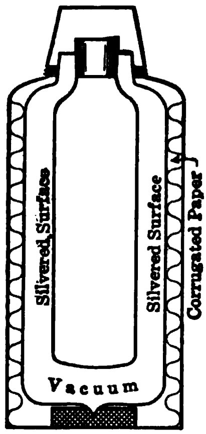


Fig. 56 (b). The Thermos Bottle.

would be from a single-walled vessel, unsilvered. Inner surface of outer vessel and outer surface of inner vessel are silvered. Evaporation of liquid oxygen surrounded by vacuum bulb, unsilvered, 170 c.c. per minute. Evaporation of liquid oxygen surrounded by air chamber, unsilvered, 840 c.c. per minute. With a silvered vacuum bulb evaporation is less than one-half above rate.

Similar conditions hold in the Thermos, Hotakold, and other vacuum bottles, in which none of the three processes is available for the passage of heat to or from the material within.

107. Prevost's Theory of Exchanges. — This theory states "that bodies radiate heat at all temperatures, and that the amount radiated depends on the body itself and not on surrounding objects; that a red-hot ball radiates the same amount

of heat whether placed in the middle of a furnace or hung up in an ice house; that ice radiates the same amount whether in an ice house or hung up in front of a furnace. Bodies also receive heat from surrounding objects. When a body radiates more than it absorbs, its temperature falls; when it absorbs more than it radiates, its temperature rises. If the radiation equals the absorption, there is thermal equilibrium, and the temperature is constant" (Wright, p. 90). See also, on this subject, Preston, *Theory of Heat*, Art. 229, and Barker, p. 372.

108. Adiabatic Curves and Ratio of Specific Heats. — The thermodynamic condition of a body is determined by the three things, pressure, temperature and volume. In liquids and solids, a very considerable change in pressure or temperature might be attended by a change in volume so small as to be often negligible; but it is not so with a gas.

If, however, one of these three quantities is kept constant while the other two change, their relative values may be examined and plotted as a curve.

In case the temperature remains constant, the curve showing the relation of pressure to volume is called an *isothermal* curve, and this has already been discussed to some extent in connection with gases. It is to be noted, however, that with a decrease of volume in a given mass of gas the temperature will rise unless some means is provided to carry off heat, and, *vice versa*, if the gas expands (against pressure) its temperature will fall unless at the same time heat is communicated to it from some other body. If, then, the gas could be so confined in a nonconducting chamber that no heat could be admitted to it or could escape from it, such a vessel would be *adiabatic*, the changes in the condition of the gas with varying pressures and volumes would be adiabatic changes, and the curve showing the relative values of p and v would be called an adiabatic curve.

In the isothermal, this relation is expressed by the simple equation $pv =$ const. and the curve is an equilateral hyperbola.

In the adiabatic, it may be shown by a somewhat difficult demonstration that the relation is given by the equation

pv^k = const., in which k is the ratio of the specific heat of the gas at constant pressure to that at constant volume, or $k = \frac{C_p}{C_v}$. (See Watson, Art. 259.)

A gas that is compressed or rarefied without change in the quantity of heat, i.e., adiabatically, would undergo change of temperature, and of course would not conform to Boyle's law; neither would the relation of the change of pressure to the change in volume be the same as if the temperature were constant, but it can be shown that this ratio would be k times as great in an adiabatic change as in an isothermal change. (See Watson, Art. 259, and Hastings and Beach, Art. 241, "The Two Elasticities of a Gas.) This ratio, is, however, $\frac{\text{stress}}{\text{strain}}$, which is the measure of the elasticity. The elasticity, then, of a gas that is compressed or rarefied adiabatically is k times as great as if the gas could lose or gain heat at such a rate, while being compressed or expanded, as to keep its temperature constant. In the latter case we have seen (Art. 59) that the elasticity of a gas is numerically equal to the pressure; in an adiabatic change the elasticity equals k times the pressure. (See *infra*, Art. 125, "Velocity of Sound in a Gas.")

109. Laws of Thermodynamics; Dynamical Equivalent of Heat.—The fundamental relations of heat and mechanical energy are embodied in two principles, known respectively as the First and the Second Law of Thermodynamics.

The first law is that *any definite amount of mechanical work is convertible into a definite quantity of heat, and, conversely, any given quantity of heat is capable of performing a definite amount of work.* The exact ratio of the amount of work to the quantity of heat it can produce is called the mechanical (or dynamical) equivalent of heat. It is the number of work units that correspond to one heat unit. This number will obviously depend upon the units in which both work and heat are measured. It might be the number of foot-pounds necessary to produce one British thermal unit (which would be 778), or it might be the number of gram-meters that would produce one calorie (which would be 427), or

in absolute c.g.s. units it is the number of ergs that produce one calorie (4.184×10^7 ergs).

The establishing of this law was the beginning of modern physics as a science of energy. It was the culmination of work that may be said to have begun when Count Rumford showed that heat could not be material, and so displaced the earlier "caloric" theory of heat. The theoretical principles of heat as a form of energy were developed by a number of philosophers, prominent among them being Rankine, Clausius, Julius Robert Mayer and others, but the experimental investigations establishing the exactness of relationship between heat and work are due chiefly to James Prescott Joule of Manchester, England. An account of the labors of Mayer is given in the last lecture in Professor Tyndall's *Heat a Mode of Motion*, and the work of Dr. Joule is published in full in two large volumes. The account of this work forms one of the most interesting and most important chapters in the history of physical science.

Dr. Joule determined the mechanical equivalent of heat by a series of experiments that are now classical, but, besides the direct determinations, he made indirect ones by means of heat produced electrically and also chemically, and thus showed not only the connection between heat and mechanical energy, but also the interrelation of both of these forms with chemical action and electric currents. This was the first unfolding of the physics of to-day, and dates from about 1850. (*Vide infra*, Art. 198.)

Many redeterminations by various methods have been made since Dr. Joule's reports, the best being summarized in Watson, Art. 251, and in Hastings and Beach, Arts. 226 and 227.

The second law of thermodynamics embodies certain principles that are by no means so simple as the one constituting the first law, although the second law has been cast in two or three forms that sound simple enough. Clausius puts it: "*It is impossible for a self-acting machine, unaided by any external agency, to convey heat from one body to another at a higher temperature.*" This is so paraphrased as to say that heat cannot of itself pass from a cold body to a hot body, and if by any means we cause heat to be

transferred from a body to another at a higher temperature, we must in the process supply the system with energy from some outside source. So Lord Kelvin states the law: "It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of surrounding bodies." (See, on this subject, both Watson, and Hastings and Beach.)

This law has been the subject of much discussion, but its bearings are chiefly in abstruse relations of thermodynamics or else in the possibilities of heat engines, and therefore will not be further considered in this course.

EXAMPLES. — Take 1 calorie = 427 gram-meters, or 4.19×10^7 ergs.

1. What is the heat value of 10,000,000 ergs? *Ans.* 0.2387 cals.

2. If the water at Niagara Falls drops 50 meters, how much is its temperature raised by its stopping? *Ans.* 0.117° C.

3. A weight of 100 kg. in descending 20 meters rotated a stirrer in a calorimeter containing 984 g. of water, and the temperature was raised 4.7° C. If the water equivalent of the calorimeter was 16 g., what does this give for the mechanical equivalent of heat?

Ans. 425.5 gm.-meters/cal.

4. If one-half of the heat from the impact of a leaden bullet against a wall at 16° C. is taken up by the lead, what must be the velocity of the bullet that it may be melted? (Take sp. ht. of lead, 0.032, melting point 326° C., and latent heat of fusion 5.36 calories per gram.)

Ans. 506.5 meters per second.

5. The combustion of 1 gram of coal liberates 8000 heat units (calories). If 2,000,000 liters of water are to be pumped from a well in which the surface of the water is 30 meters below the level to which it is to be raised, how much coal must be burned, supposing the engine can apply 15 per cent of the heat to useful work in raising the water? *Ans.* 117 kg.

6. From the value in ergs of the gas-constant R, given in Art. 79, show that it is almost exactly 2 calories.

110. Sources of Heat. — We recognize heat as resulting from various processes, every one of which is a transformation of energy from some other form into this form, as:

Chemical action, of which the most common instance is combustion; but there are many other chemical combinations in which energy is liberated in the form of heat; this energy is then

said to be energy of chemical separation possessed by the bodies before their combination.

Mechanical action, whenever it increases the molecular motion within a body. It may be stirring, hammering, twisting, bending; or the increased molecular motion may be brought about by:

(*a*) Friction;

(*b*) The compression of gases;

(*c*) The passage of an electric current;

(*d*) Direct radiation from the sun.

(Illustrate with fire syringe; incandescent electric lamp immersed in water; sun glass. For chemical action see Barker, pp. 352–357.)

Most of the effects of chemical action (certainly all of combustion), and most of mechanical action (other than efforts of animals), can be traced ultimately to the sun; for the vegetation which supplies fuel, whether the coal formed in past ages or the wood of to-day, is due to sunlight; and hence also is all our steam power. The power of winds and of waterfalls is also, as we have seen, directly attributable to the ceaseless action of solar radiation, and our sources and our supplies of energy other than these are inconsiderable.

111. **Solar Constant.** — But, apart from the chemical effect of sunlight, the heat emitted by the sun as such, and the quantity received from the sun by the earth, is measurable, and when expressed quantitatively, the latter is called the solar constant of radiation. The latest determinations of this quantity make it very nearly 2 calories per square centimeter per minute on a normally exposed surface above the atmosphere, which is reduced about one-third at sea level by atmospheric absorption; or, on the surface of the sun, enough heat is radiated to burn 1400 pounds of coal per hour on every square foot of the sun's surface.

To account for the maintenance of such a supply of energy thus dissipated into space is a problem that has long taxed physicists and has called forth numerous hypotheses. (See, on this subject, Barker, pp. 357–361, and *The New Knowledge* by Robert Kennedy Duncan.)

EXPERIMENTS TO ILLUSTRATE CHAPTER II.

Experiment No. 32, Art. 72. Nos. 32 to 35, incl., illustrate Expansion and Contraction.

Gravesande's ring (Fig. 57) is a flat ring of brass that passes easily but closely over a hollow sphere of copper. On heating the ball with the flame of a lamp or Bunsen burner, if the ring is brought up from below the ball, the latter is too large to pass through the ring and is raised up. As the ball cools and the ring is warmed, the contraction of the former and the expansion of the latter presently permit the ball to drop through. The experiment holds good with the ball in any position, showing expansion in all directions.

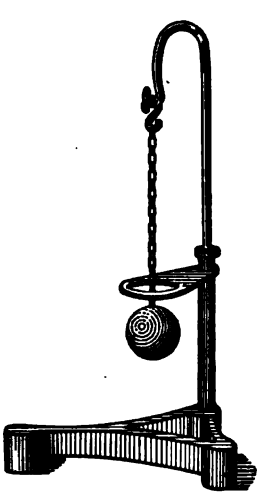


Fig. 57.

Experiment No. 33, Art. 72. Compound Expansion Bar.

A strip of iron about 60 cm. long, 15 or 18 mm. wide and 2 mm. thick is riveted at intervals of about 5 cm. to a brass strip of the same size.

At a certain temperature this compound bar is straight and will lie flat on the table. If it is heated, however, preferably by several burners at different points, the two metals in contact undergo the same rise of temperature, but the bar acquires a considerable curvature, and will turn like a rocker on the table. The brass being more expansible is on the outer or long side of the bow, but if the bar had been much lowered in temperature, say, by immersing it in a freezing mixture, the curvature would have been in the opposite sense; the brass would be shorter than the iron and consequently would be on the inner side of the arc.

Experiment No. 34, Art. 72.

Trevelyan's rocker (Fig. 58) is a prism and bar of brass or copper that rocks easily on the edges of a groove along one side of the prism. The prism is heated nearly to the melting temperature of lead, and then laid across the edge of a triangular prism of that metal, the latter, as also the end of the bar, resting on the table. On giving a slight rocking movement to the

brass prism, the rocking is maintained by the sudden jutting up of micro scopically minute nodules of lead against the edge of the groove that is fo

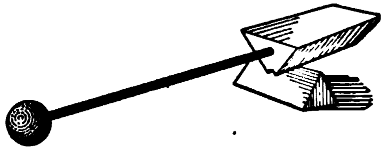


Fig. 58.

the instant in contact with the lead, and the instrument sings a clear but varying note.

(The secret of success is to have the edges of the groove and also the contact edge of the lead clean and bright. The lead prism should lie firmly on the table.)

Experiment No. 35, Art. 72.

The contraction of rubber on being heated may be shown as follows:

A piece of soft (black) rubber tubing T (Fig. 59), about 2 ft. in length and a quarter-inch bore, is fitted at each end over a short pipe (preferably brass), to which it is secured by wrapping it with cord. The upper end is clamped, or suspended from a hook at the height of 4 or 5 ft. above the table or floor. From the lower end suspend a weight of 3 or 4 lbs. to stretch the tube T until the weight lightly rests upon the table or floor. By a string s connect the lower end of T with the short arm of a pointer P, and adjust the length of the string so as to set the index end of the pointer at the zero of a scale. Arrange a boiler as in the figure, near the upper end of the rubber tube. A connecting tube b will permit steam to pass from the bent tube a of the boiler to the rubber tube T.

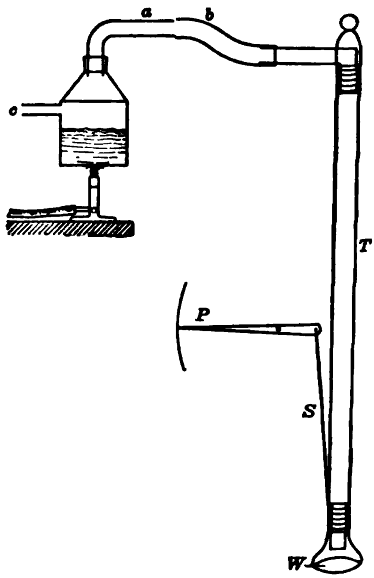


Fig. 59. Rubber Contracts when Heated.

When steam is issuing freely from a of the boiler, connect b with a and close c. The steam passing through the rubber tube T causes it to contract, raising W probably 3 or 4 cm. The contraction is more plainly seen in the movement of the pointer over the scale.

When the tube has been stretched in the first place by the weight W, the latter should rest on the table *lightly* just when the steam is admitted at b; otherwise the tube continues to be drawn out for some time by the weight.

Experiment No. 36, Art. 82. Differences in Thermal Capacity.

Balls of copper, iron, zinc, tin and lead of equal mass will not differ greatly in diameter, except the lead, which will be much smaller than the others. These balls in a bath of boiling water will acquire a common temperature of about 100° C.

Fig. 60. Capacity for Heat.

At this temperature place them upon a cake of beeswax about 4 mm. in thickness. They will all begin to melt the wax and they will cool off by loss of the heat of fusion of the wax, but they will continue the melting so long as their temperature is above 62° C., i.e., while their temperature falls through 38 degrees.

If they all had the same capacity for heat, they would all give out the same amount of heat in changing through this common range of temperature, and consequently all would melt the same amount of wax. It is seen, however, that the iron ball will melt its way through (and possibly a second time), the copper will melt through and partly again, the zinc will probably about melt through, the tin scarcely halfway, while the lead, which would require a smaller orifice than any of the others, will make very little impression.

Observe, it is not at all a question of which one melts the wax the *fastest*, but the *most*, i.e., which has the most heat to supply in a given change of temperature. The one which has the least might be a better conductor than another and so part with its heat more rapidly.

Instead of beeswax, a somewhat thicker cake of paraffin may be used. Its melting temperature is 53° C.

Experiment No. 37, Art. 88. Regelation.

Fig. 61. Freezing Point of Ice is Lowered by Pressure.

The wire (Fig. 61) passes through the ice by the process described in Art. 88, but it is always solidly incased in ice; the latter is not cut apart. Air which is in separate large bubbles in the block is

now seen along the plane of section in minute bubbles, giving a gray appearance like that of ground glass.

Experiment No. 38, Art. 89. Nos. 38 and 39 show Vapor Pressures.

If several barometer tubes (Fig. 62) are filled with mercury and inverted over a reservoir of that liquid, the column in each tube will stand at a height, under normal atmospheric pressure, of 76 cm. If, now, a drop or two of water be introduced into a tube as in the figure, the water rises to the top and in the vacuum it instantly changes into vapor. The pressure of this vapor forces the column of mercury down, against the external atmospheric pressure. Introducing other liquids into the other tubes, each vapor will be found to exert a pressure peculiar to itself. Thus, if the temperature is 20° C., water vapor will force the top of the column from 76 cm. down to 74.26 cm., exerting a pressure of 1.74 cm.; alcohol to 71.55, showing a pressure of 4.45 cm.; and ether will force it down to 32.67, with a pressure of 43.33 cm. Use tubes of at least 4 mm. bore and use different pipettes for inserting the different liquids.

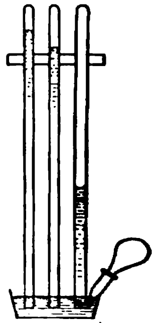


Fig. 62. Vapor Pressure Tubes.

Experiment No. 39, Art. 89.

On a tube T (Fig. 63) blow a thin bulb B, about 2 cm. in diameter. Fill the bulb half full of water (most simply done by allowing it to draw water in as the bulb cools), and draw the end of the tube to a capillary C. A bottle F of about 200 c.c. capacity and rather thick glass has a rubber stopper with two holes. Through one orifice passes T, through the other a manometer tube containing oil. When the stopper is in place, the manometer tube is raised or lowered through A until the pressure in the bottle is equal to that of the atmosphere, which is shown by the oil being at the same level in both branches. The air in the bottle should be dry. The capillary end C is now sealed by the flame of a Bunsen burner or a blowpipe. By manipulating the tube T the bulb B can be pressed or rubbed against the side of the bottle and be broken and the water liberated. The vapor saturates the space in the bottle and the vapor pressure is at once shown by the movement of the oil in the bent tube.

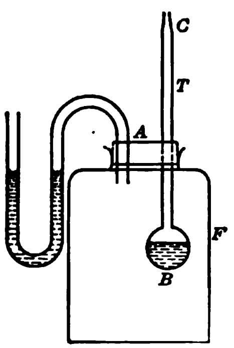


Fig. 63. Vapor Pressure Bottle.

Experiment No. 40, Art. 91.

(*a*) Culinary Paradox; to make water boil by cooling it.

Boil water in a flask of thin glass (Fig. 64), stop neck of flask securely, invert flask, and soon ebullition ceases, as the temperature falls. The application of cold water to the part of the flask inclosing the vapor condenses the latter, lowers its pressure, and the water again boils violently.

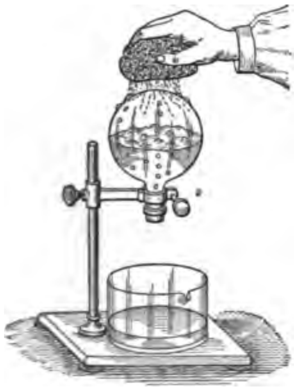


Fig. 64. The Culinary Paradox.

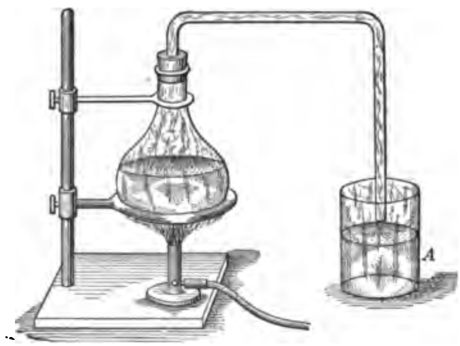


Fig. 65. Boiling is Renewed under Reduced Pressure.

Experiment No. 41, Art. 91.

(*b*) *Experiment* No. 40 is well modified as follows. (Carhart's *University Physics*, Part II, Art. 45.)

Arrange apparatus as in Fig. 65; have flask stopped tightly, and boil the water until steam issues freely from the tube. Remove the burner, and when the water is no longer boiling, raise the vessel *A* containing cold water, until the tube dips well down in it. As the vapor in the flask cools, its pressure decreases; water rises through the tube, and, entering the flask, greatly reduces the vapor pressure, and violent ebullition ensues.

Experiment No. 42, Arts. 91 and 92.

Freezing water by its own evaporation, or by the evaporation of a liquid surrounding it (*a*), by the cryophorus (Fig. 66).

Have the upper bulb of the cryophorus less than half full of water to prevent breaking of the glass when the water freezes. Pack the lower bulb in a freezing mixture of coarse salt and crushed ice, about one volume of salt to two of ice. Condensation of vapor in the lower bulb reduces the pressure upon the liquid above and evaporation goes on rapidly though quietly in the upper bulb, absorbing heat from the liquid there, and if this

absorption of heat is more rapid than the communicating of heat from the air of the room, the temperature falls and the liquid freezes, — usually in twenty minutes to a half hour.

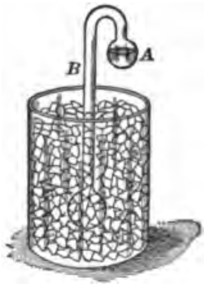


Fig. 66. The Cryophorus.

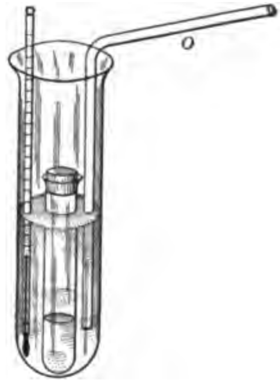


Fig. 67. Freezing Water by Evaporation of Ether.

Experiment No. 43, Arts. 91 and 92.

In a large test tube (Fig 67), containing sulphuric ether, place a small stopped tube containing one or two c.c. of water. Agitate the ether by blowing through a tube *O*; it rapidly evaporates and cools the water to freezing. A thermometer in the larger tube shows the progress of the fall in temperature.

Experiment No. 44, Art. 104. Heat Conduction.

Place a wire gauze containing 50 or more meshes to the inch about an inch above the top of a Bunsen burner, turn on the gas and ignite it above the wire gauze. The gas burns above the metal but does not extend below it, the metal conducting away the heat fast enough to prevent the under surface from getting hot enough to ignite the gas.

Experiment No. 45, Art. 104. Nos. 45 and 46 illustrate Differences in Conductivity.

From a massive ring of brass (Fig. 68), radiate rods of various metals about 5 cm. long and 4 mm. thick. On the ends of these are placed small pieces of phosphorus, and the central ring is heated by a gas flame. The phosphorus ignites when the end of the rod rises to the necessary temperature, and the difference in conductivity of the rods is seen in the longer time required by one than by another to produce ignition.

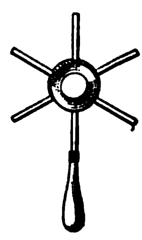


Fig. 68.

Experiment No. 46, Art. 104.

If a wooden cylinder (Fig. 69) be fitted into the end of a brass tube of the same external diameter, and a single thickness of paper be made to en-

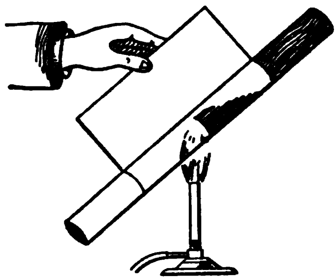


Fig. 69.

circle snugly both the brass and the wood, the flame of the burner at the junction will char the paper around the wood, but not that inclosing the brass which rapidly conducts the heat away.

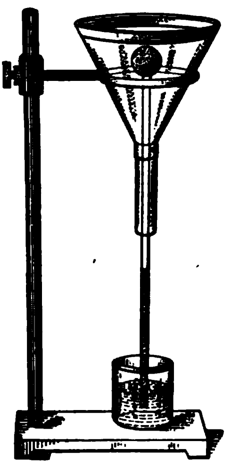


Fig. 70.

Experiment No. 47, Art. 104. Poor Conductivity of Water

An air bulb of about 5 cm. diameter (Fig. 70), has its stem dipping into a liquid (colored water), and is covered with water to a depth of about 3 mm. above the bulb. The index liquid stands about halfway up the tube. The air in the bulb is sensitive to very small changes of temperature; the warmth of the fingers placed upon the bulb at once moves the liquid down the tube. Pour a very thin layer of ether on the water over the bulb and ignite it. The flame spreads over the surface of the water, but no heat is conducted to the bulb, though the heat is intense and the depth of liquid very small. When, finally, the index does show a warming of the bulb, it is an effect of radiation rather than of conduction.

CHAPTER III.

WAVES AND WAVE MOTION.

(PRELIMINARY TO SOUND, LIGHT AND ELECTRICITY.)

112. Introduction and Use of Wave Theories. — Probably the earliest appreciation of waves was derived from the disturbance in the surface of a body of water, and this would be immediately extended to liquids generally. Waves in gases could hardly have been recognized until much later, and waves in solids must have been unthinkable without a considerable development of the science of elasticity. However, elasticity, as a property of bodies, must have been known early, perhaps before inertia itself, since early ideas were to the effect that a body in motion would "naturally" come to rest of itself — an erroneous conception in regard to inertia.

The idea of waves in physics has become so extended and generalized as to lead to this statement: "A wave is a progressive form due to the periodic vibration of the particles of the medium through which it moves. . . . . . To prove that any phenomenon is due to wave motion it is sufficient to show, first, that it is periodic; second, that it is propagated with finite velocity." (Hastings and Beach, pp. 514, 606.)

An examination of this broad statement will at once show that wave motion attends a wide range of physical phenomena, connected possibly with every medium in the universe, or wherever an effect is produced in a body on one side of a medium by an agent on the other side of it, for so far as known, all such disturbances require time for their transmission, and many of them are periodic in character.

113. Vibratory Motion and Wave Forms. — If motion of a body or particle is vibratory, that at once means periodicity; but periodicity does not necessarily mean motion to and fro in a

straight line. It is a recurrence to a given position at regular intervals and a successive repetition of its movement through various positions in the same order. It will always give the effect of a to-and-fro movement in a straight line if the component of its motion in a given plane could be viewed from a point in that plane. For example, if a body moved in a circle, Fig. 71,

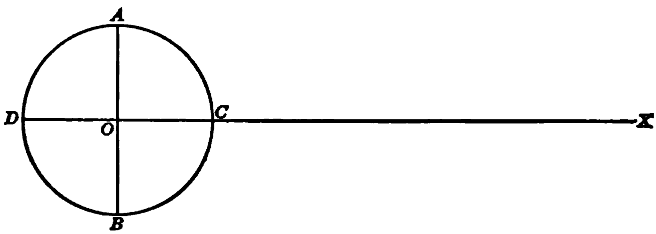


Fig. 71. Vibration in Simple Harmonic Motion.

describing successive revolutions in equal periods of time, from A through C, B, and on again through A, this motion would be periodic, and, to an eye in the plane of the circle and far distant, the body would vibrate across a diameter. If the circle itself were at the same time turning about an axis, as AB, then, to an eye on the fixed line OX, the body might describe a complex figure, but its movement would be periodic, and at any instant could be resolved into two straight-line vibratory motions along two diameters of the circle.

If we start with the uniform motion of a particle around a circle, as above, its motion is a combination of two S.H.M.'s along two diameters. The amplitude of vibration is the extent of the departure from its mid-position, in this case the radius of the circle; and the period is the time that elapses between the passage of the particle through a given position and its next passage through the same position *in the same direction.*

If, now, we consider the motion of this particle to be communicated to neighboring particles in the same medium, and we follow the effect in one direction, say, to the right of the figure, let us suppose a certain time to be required by each particle in succession to acquire the motion from its predecessor and transmit it

to its successor; then a chain of particles will have been set in similar motion by the time one particle, as A, will have described one revolution; i.e., in one period T. The length of this chain is the distance the disturbance travels in one period, and is a wave length, λ. The particle next following the last in this

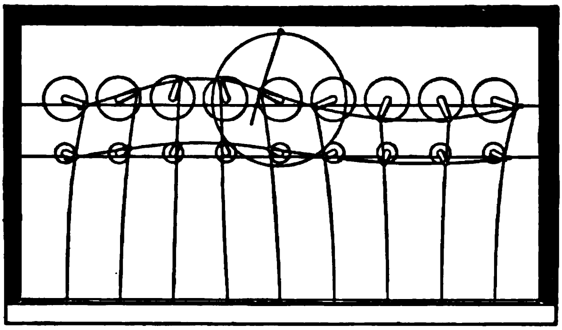


Fig. 72. Lyman's Apparatus illustrating Waves in Liquids.

chain is starting upon its motion as A is beginning its second period. The motion of any one particle may be regarded as vibratory, but the *wave* comprises all the particles set in vibration in one period. If these particles are in a liquid with a free surface, the effect is to put the surface into the form usually recognized as a wave. (Shown by the circular motion of particles in a line, as in Fig. 225, Watson, p. 343, and also on the Lyman's wave apparatus, and in the water waves on the Columbia wave machine.)

If, in Fig. 71, the amplitude of vibration along CD diminishes while that along AB remains the same, the circle becomes an ellipse with its longer axis vertical, and if we consider this change carried to an extreme we get simply a motion up and down on the line AB. This motion, communicated to the particles in one direction, say, as before, to the right, will be taken up and transmitted by the particles successively, and the wave form will be the sine curve of equal crests and troughs, every particle describing S.H.M. across the line of propagation of the disturbance. The wave in this case is said to be due to *transverse vibration*, and

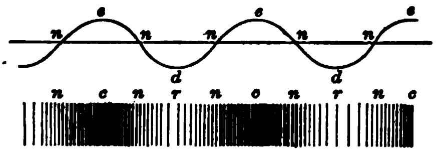


Fig. 73. Wave Forms due to Transverse and to Longitudinal Vibration.

is sometimes called a transverse wave. It is shown in the upper part of Fig. 73.

But if, in the same Fig. 71, we consider the amplitude in the vertical direction to be diminished, while that in the horizontal direction is unchanged, it would result ultimately in a to-and-fro motion along the horizontal diameter, and the effect upon the particles along a horizontal line in the direction of motion, say, to the right, would be a crowding together, followed by a withdrawing or separating of the particles; a crowding so long as the initial particle was moving from left to right (or during half of a period), and a separating so long as it was moving from right to left (or during half of a period). One half of the wave, then, would be in a state of condensation, and one half in a state of rarefaction, every particle describing S.H.M. along the line of propagation of the disturbance. The wave in this case is said to be due to *longitudinal vibration* and is called, sometimes, a compressural wave, — more commonly a wave of condensation and rarefaction. This is shown in the lower portion of Fig. 73.

Experiment No. 48, page 168.— The waves resulting from these three modes of movement are all exhibited on the Columbia wave apparatus.

While we may thus examine the vibration of any single particle, and, if complex, resolve it into its simple elements, the wave form is the outline of the positions occupied at any instant by all the successive particles in one line that have been set in motion successively during the time that the initial vibrating particle has required to complete its motion, and return to its first position to start upon a repetition of its motion. A simple vibratory motion back and forward continually in the same line can be traced along any line as a succession of simple waves, but a complex vibration will give a complicated form that can be resolved into simple ones of various periods and amplitudes.

For example, if the particle at *O*, Fig. 74 *a*, should describe a figure 8 in the rectangle by following the direction of the arrows, it will complete its vibration by crossing the rectangle four times horizontally and twice vertically. The wave movement along a line of particles would be twisted and difficult to show in a diagram, but it could be resolved into motion in the vertical and the horizontal planes separately, as shown in Fig. 74 *b*, in the length of one wave of that component of the motion which has the longest period; i.e., the one with the vertical movement.

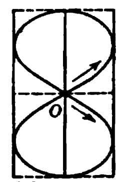


Fig. 74 *a*.

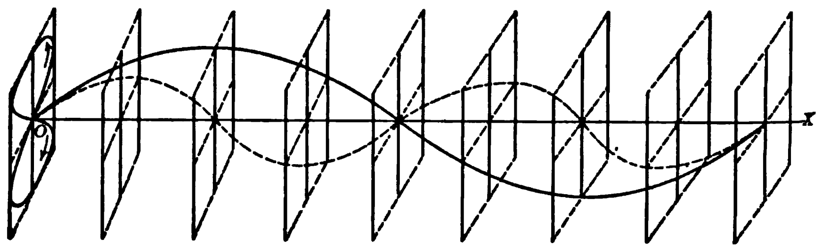


Fig. 74 *b*. Wave Motion in Two Planes.

It has been shown by Fourier (*Théorie Analytique de la Chaleur*) that any complex periodic curve which consists of an exact simple number of vibrations will present along a line of particles a complex wave form that can be analyzed as a series of simultaneous component wave forms of simple character.

114. Wave Front. — When a vibrating particle communicates its motion to the next one, and that to another, and so on along a given line, the resulting wave may be traced along that line, but this "next one" receives the motion only because it is "next," and there is such an one in all directions around the one first considered. A wave motion then is started and proceeds throughout a medium in all directions, and if the disturbance travels at the same rate in all directions, the boundary of disturbance at any instant will be the surface of a sphere about the initial point of disturbance as a center. In the case of waves at the surface of a liquid the boundary becomes a circle. This boundary is called the wave front.

As the disturbances advance there is at any instant a line or surface of disturbed particles, and each of these particles is regarded as itself the source of a disturbance that is to be propagated further. The complete investigation of such propagation through a medium is exceedingly abstruse, but it can be shown that the effect is to produce little or no disturbance at a given instant except in the wave front. Thus a disturbance proceeding from a single point, as *O*, Fig. 75, will presently have advanced to *ABC*. If each point of this line *ABC* be taken as a center and circles be described with equal radii, then in the time needed for the disturbance to travel the length of such radius, it will lie chiefly in the tangential boundary *DEF*, which again is a sphere or circle with *O* as its center.

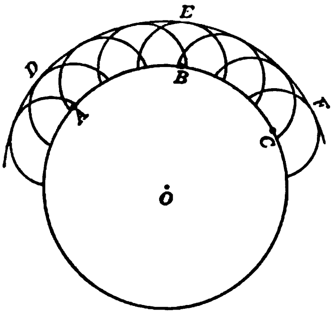


Fig. 75. Huyghens' Construction of Wave Front.

If *ABC* were a straight line in the surface, as of water, *DEF* would be a parallel straight line; and if *ABC* represented a plane surface, *DEF* would also be a plane wave front, parallel to *ABC*, and might be regarded as proceeding from a point *O* at an infinite distance.

115. Velocity of Propagation. — The conventional form chosen to represent a wave is the same whether the wave results from transverse or longitudinal vibration, or from circular motion, and consists of alternate crests and troughs.

In the case of transverse vibrations this form properly corresponds to the actual position of successive particles in the line of propagation. In the case of longitudinal vibration, where the particles are alternately crowded together and separated, the ordinates of this sine curve must be understood as corresponding to the displacement of the successive particles from their normal position, but laid off in a transverse direction. The curve does not directly represent the shape of the wave.

In all cases the wave length λ is the distance the disturbance advances through the medium in the time of one vibration. It is the distance from any particle to the next particle that has the same displacement and is moving in the same direction. It is the distance from crest to crest or from trough to trough; from condensation to condensation or from rarefaction to rarefaction; from crest to trough or from condensation to rarefaction is half a wave length, $\frac{1}{2}\lambda$.

If T is the period, i.e., the number of seconds for one vibration, then in one second there will be $\frac{1}{T}$ vibrations. Calling the number per second n, this is the frequency or the vibration rate. If n waves per second follow upon one another, each of a length λ, the disturbance in one second along a line of particles will be a train of n waves, and the distance the disturbance will have traveled is $n\lambda$; or the velocity of propagation is $v = n\lambda$; from which relation if either two quantities are known the third may be determined. Also, if for n we put its value $\frac{1}{T}$ we have $v = \frac{\lambda}{T}$.

116. Reflection of Waves. — Suppose a wave front, for simplicity, say, a plane wave front, as it advances through a given medium, encounters the surface of a medium of different density. If, for example, this be a wall, the advancing waves will be reflected. If the advancing wave front is at an oblique angle, the reflected waves will proceed in such a direction that the reflected wave front makes an angle with the reflecting surface equal to that made by the incident wave front (measured the other way). In Fig. 76, if the full lines represent advancing wave fronts, the broken lines will represent the reflected ones after the encounter with the surface AB, and it is seen that for even such a limited wave

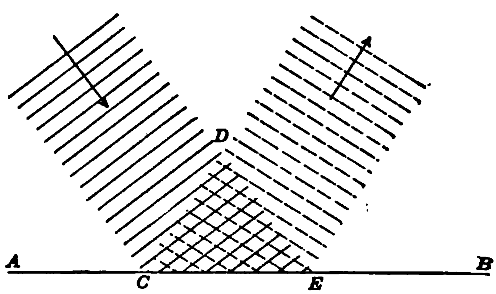


Fig. 76. Reflection of Plane Wave Front.

front, there is a portion of the medium *CDE* which will be disturbed by both systems of waves, the incident and the reflected. If *AB* were the surface of an elastic medium, part of the energy of the advancing waves would go to setting up vibrations in this medium, and waves would continue to advance in it, the reflected waves possessing less energy than if *AB* were unyielding.

If we consider a single line of waves instead of a wave front, and the reflecting surface is normal to the line of waves, the reflected waves will retrace the same line of particles along which other waves are advancing. The result of this will be that if a particle in the advancing waves is met by the returning wave so that the particle would be impelled to move in the opposite direction (say, the advancing one moving upward and the reflected

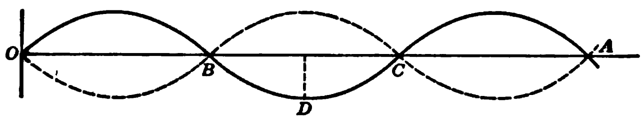


Fig. 77. Stationary Waves.

one downward), then if the amplitude of vibration were alike for each system, that particle would be undisturbed, as at *B*, Fig. 77, but a quarter of a wave length further along from the reflecting surface, the advancing one would not be so far up (it would be in its lowest position), and the reflected one would be farther down (it would be in its lowest position), and the depression would be doubled in extent as at *D*; a quarter of a wave length nearer the origin of motion the particle in the advancing wave would be on the axis line but going down and that in the reflected wave in the same position but going up, and again the particle would be stationary as at *C*. At that particular instant the line of particles would have the position of the full line *ACBO*; a half period later they would have the position of the dotted line *ACBO*, the particles at *A*, *C*, *B*, and *O* remaining stationary. These points are called nodes, the broad places or those of greatest disturbance being loops or ventral segments. Such waves are called stationary waves.

A similar effect is produced if the waves are due to longitudinal vibration, the nodes being due to motion imposed upon a particle in opposite directions at the same time, and corresponding, therefore, to condensation or rarefaction but minimum movement, while the loops correspond to freest movement in either direction along the line of propagation. The nodes, as in the former case, are half a wave length apart, are places of rest, and are alternately condensed and rarefied conditions, while the loops are places of largest movement alternately in the direction of the advancing and the reflected waves. From node to node or from loop to loop is half a wave length. From node to loop is one quarter-wave-length.

Experiment No. 49, page 168. — Stationary waves with spiral rope.

117. Interference. — A train of reflected waves is like an independent set of waves proceeding from a different source from that of the incident waves. When such waves retrace the same line as the incident ones traverse, the effect is the same as if the line had been traversed by waves from two different sources. When waves proceed from two sources it may happen that at places in the medium where there would be the crest of a wave from one source there would be a trough from the other. At this place the waves are said to "interfere" and neutralize each other. In the case of stationary waves there is interference at the nodes. Midway between the nodes the effect of either wave is heightened by the other, constituting what is termed "reënforcement."

If waves proceeding from two independent sources and spreading throughout a medium, are alike in period, they will produce interference continually at the same places, so that these quiescent places may be traced out as "interference bands." If the periods are not alike or commensurate in some simple ratio, the positions of interference are shifting and may be difficult to recognize.

Waves proceeding from one point may be brought to a focus elsewhere by reflection from a curved surface. Thus if they

proceed from one focus of an ellipse they will converge upon the other focus. A train will thus be sent out from each focus and they will produce interference bands. (See Watson, Fig. 229.)

Experiment No. 50, page 168. — Interference of waves in surface of mercury, projected by reflection.

118. Phase of Reflected Waves. — We confine our attention to waves that meet a reflecting surface perpendicularly, and are therefore reflected directly upon themselves. In the case of water waves, an examination of the movement of particles so as to form the wave will show that if the reflecting surface is met by the crest of the advancing wave this will be reflected also as a crest, and the reflected wave is said to be *in the same phase* as the incident. But if the waves are due to the elasticity of the medium and are advancing along any line *OD*, which is fixed at *D*, this point is of necessity a node, and whatever the phase in which the advancing wave arrives at *D*, the returning wave must have at *D* such a phase of vibration as to neutralize vibration there. For this, the reflected wave must be reversed in phase, and must retrace the line *DO* as if it had undergone a retardation of a half wave length.

When a transverse vibration travels from a rarer to or into a denser medium, that part of the vibration that is reflected, and therefore returning through the rarer medium, is reversed in phase, or moves as if retarded a half period; but in proceeding from a denser into a rarer medium, no such reversal occurs.

When a longitudinal vibration travels from one medium into another, the circumstances of reversal of phase are just contrary to the above. (See Watson, Arts. 277, 300; and Hastings and Beach, Art. 471; see also *infra*, Art. 243, Newton's Rings.)

119. Waves in the Surface of a Liquid. — When a liquid is heaped up it seeks to come to a level by gravity. It is not then a question of elasticity as the restoring force, hence waves produced under such conditions are called "gravitational waves."

120. Rate of Travel of a Wave.— (*a*) *Water waves.* — If the water is of a depth greater than the wave length, and if the wave length is considerable, say, 10 cm. or more, it is found that the velocity of propagation is given by the equation $v^2 = \frac{g\lambda}{2\pi}$, which is applicable to liquids of any density. In general, the velocity of waves in a liquid is given by $v^2 = \frac{f\lambda}{2\pi}$, where f is the acceleration due to the downward force and this consists of gravity plus surface tension. For gravity waves (where surface tension is neglected) if r is the radius of the circle described by one particle, the wave length, $\lambda = 2\pi r$. It is found that the velocity of travel of the wave equals the velocity acquired by a body in falling $\frac{1}{2}r$; as $r = \frac{\lambda}{2\pi}$, this gives $v = \sqrt{\frac{g\lambda}{2\pi}}$. Also, the time t, to travel the length of one wave, is $\frac{\lambda}{v}$,

or
$$t = \frac{\lambda}{\sqrt{\frac{g\lambda}{2\pi}}} = \frac{2\pi r}{\sqrt{\frac{g\,2\pi r}{2\pi}}} = 2\pi\sqrt{\frac{r}{g}}.$$

If T = the time of oscillation of a pendulum of length r, $T = 2\pi\sqrt{\frac{r}{g}}$. Hence the time required for a gravitational wave to travel one wave length is the same as the time required for the complete oscillation of a pendulum whose length is the radius of a circle of which the circumference equals the wave length. (This is also the time required by a body to fall freely a height $= \pi(2\pi r)$, or $\pi\lambda$, which is the circumference of a circle of which λ is the diameter.)

Illustrate by Lyman's wave apparatus, Fig. 72.

But while gravity is in such cases the chief cause tending to restore the disturbed surface of the liquid to a level, there is also a pressure aiding this, due to surface tension. This pressure is greater the smaller the radius of curvature, and, as in long waves the radius of curvature is large, the effect of surface tension is so

small relatively to that of gravity as to be negligible. The expression for the velocity due to surface tension is $v^2 = \frac{2\pi T}{\lambda\rho}$, where T is the surface tension and ρ is the density of the liquid; and where both causes are taken into account $v^2 = \frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}$; if the wave is very short, say, 0.4 cm. or less, the first term of the second member is negligible compared with the last. Such waves are called capillary waves or ripples. For more complete discussion of liquid waves and ripples, see *Encyclopædia Britannica*, Art. Capillarity; Watson's *Physics*, Arts. 270–273, 279, 280; Hastings and Beach, Arts. 476, 477.

(*b*) *Transverse Wave along a Stretched String.* — Suppose *the wave* to be traveling along the cord from right to left with a speed v while the cord does not travel. So far as the forces in the cord are concerned it is the same as if the cord were made to travel from left to right at the rate v, and the crest of the wave remained at the same place. That would be the same as if the cord rested on the rim of a pulley whose radius R was the radius of curvature at A, and which was rotating with a velocity of circumference equal to v, as in Fig. 78.

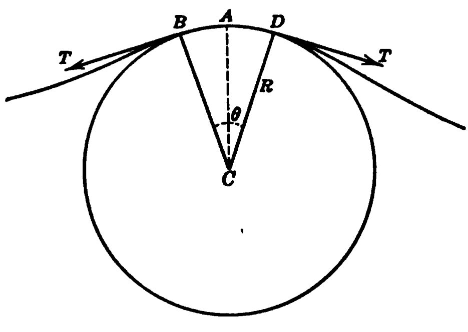


Fig. 78. Travel of a Transverse Wave along a Cord.

Let m be the mass per unit length of the cord, and let BD represent a very short element of the cord, subtending an angle at C equal to θ. Then $BD = R\theta$. That A should just keep its contact with the rim of the wheel, BD must be under a pull towards the center just equal to the centrifugal force upon it. If T is the tangential pull at each end of BD, i.e., the tension in the cord, each of these T's may be resolved into a component along

AC and another perpendicular to AC. The latter two neutralize each other, the former, added together, make the pressure toward the center. This centrally directed component of each is $T \sin \frac{1}{2} \theta$, or for both,

$$2 T \sin \frac{1}{2} \theta.$$

The centrifugal force is $\frac{mBDv^2}{R}$, and, equating, we have

$$\frac{mBDv^2}{R} = 2 T \sin \frac{1}{2} \theta.$$

BD should represent an infinitesimal arc, for which $\sin \frac{1}{2} \theta$ is $\frac{1}{2} \frac{BD}{R}$, or $\frac{BD}{2R}$, therefore $\frac{mBDv^2}{R} = \frac{T \cdot BD}{R}$, whence $v^2 = \frac{T}{m}$, or $v = \sqrt{\frac{T}{m}}$. That is, the velocity equals the square root of the quotient obtained by dividing the tension by the mass per unit length. In absolute units, taking T as dynes and m as grams per centimeter of length, v will be the velocity in centimeters per second.

(*c*) *A Longitudinal Wave in an Elastic Fluid.* — The demonstration for this is omitted as tedious though not too difficult. It is important, however, and students would do well to examine the demonstrations given in Hastings and Beach, *General Physics*, Art. 473, or in Watson's *Physics*, Art. 281.

It is shown that the velocity is given by the equation $v = \sqrt{\frac{E}{\delta}}$, in which E is the elasticity $\left(\frac{\text{stress}}{\text{strain}}\right)$ of the medium and δ its density. Inasmuch as this expression for v does not involve the wave length, it follows that, in the same medium, waves of different length travel at the same rate.

EXAMPLES. —

1. A tidal wave 100 meters in length is started in mid-ocean, in what time will it arrive at a port 300 km. distant? (Art. 120(*a*).)

Ans. $6\frac{2}{3}$ hrs.

2. Ripples 3 mm. long are formed in the surface of mercury. If the surface tension of mercury is 540 dynes per centimeter, how fast do the

ripples travel along the surface? How fast would ripples of the same length travel in water, surface tension being 81 dynes per centimeter? (Art. 120(*a*).) *Ans.* In mercury, 28.8 cm./sec.
In water, 41.2 cm./sec.

3. A steel wire 100 meters long, 1 mm. in diameter, and of density 8 g./c.c. has a weight of 2 kg. suspended from it; with what velocity will a transverse wave travel along the wire? If the wire is struck transversely at the lower end, in what time will the wave be felt at the top? (Art. 120 (*b*).) *Ans.* 55.86 m./sec.; 1.8 sec.

4. If the wire in Ex. 3 were suddenly jerked downwards at the lower end, at what rate would the impulse be transmitted, and in what time would it be felt at the top? (Art. 120 (*c*).) Take $E = 20 \times 10^{11}$ dynes per square centimeter. *Ans.* 5000 meters/sec.; 0.02 sec.

EXPERIMENTS TO ILLUSTRATE CHAPTER III.

Experiment No. 48. Art. 113. Three Typical Wave Forms.

In the Columbia wave apparatus (Fig. 79) three horizontal rows of particles are subject to the same periodic motion, any one particle being one eighth of a period in advance of the following one.

In the upper row the particles describe circles and the line assumed by

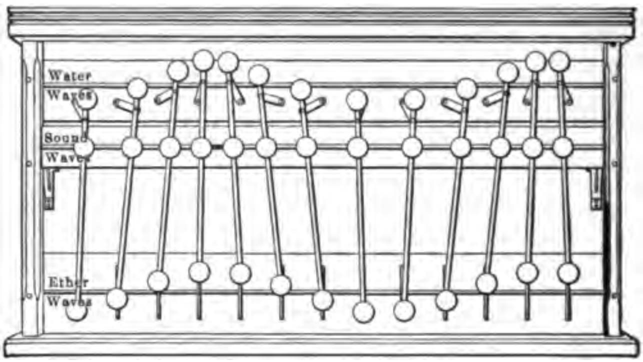


Fig. 79. Columbia Wave Apparatus.

them at any instant has the form of a water wave, which progresses as the particles describe their circles.

In the second row the particles are constrained to move only to and fro in the same line with the particles, giving the form at any instant of a sound wave which is a wave of condensation and rarefaction, that moves along as the particles vibrate.

In the third row the particles are constrained to move only in a direction transverse to the line of the particles, giving the form of ether, or light waves, the wave traveling along the horizontal line while the vibration is vertical.

Experiment No. 49, Art. 116. Nodes and Loops of Stationary Waves.

Attach an elastic cord, as, e.g., a coiled wire, 3 or 4 meters in length, to a hook in the wall, and, drawing the cord moderately tight, send waves along it by shaking the end. With care, the cord may be thrown into vibration with stationary waves, showing one, two, or even half a dozen segments.

Experiment No. 50, Art. 117. Interference of Waves.

A shallow dish (Fig. 80) of elliptical or other suitable geometric form, contains mercury. A funnel tube is drawn out to a fine opening, placed over one focus of the ellipse, and mercury is poured into the funnel. The

drops form waves which proceed as circles expanding from one focus and converging upon the other, which point in turn acts as an origin of waves, and these, crossing the set from the first focus, produce distinct bands of reënforcement and interference, showing as secondary ellipses in the surface of the mercury.

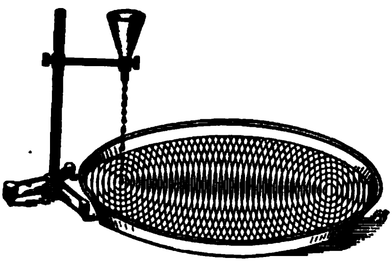


Fig. 80. Interference of Liquid Waves.

The waves may be excited by means of a slender iron stylus attached to the end of one prong of an electrically driven tuning fork. Even without the funnel dropper or the stylus, the mercury surface will show the waves, foci, and interference by tapping with the knuckles on the vessel containing the mercury, or simply on the table on which the dish stands.

With an opaque projection apparatus they can be shown upon a screen, or upon the ceiling by an ordinary large convex lens, in the reflected beam of nearly parallel light from a lantern.

CHAPTER IV.

SOUND.

121. Sound as a Phenomenon and as a Sensation. — The word "sound" is used to express a physical fact as well as a mental perception. It may be considered, therefore, both objectively and subjectively. It is with the former view that physics has to do chiefly.

122. Sound a Wave Phenomenon. — Sound should be classed among wave phenomena because it obviously requires time for its transmission, and there is abundant evidence that it is periodic in character. A sounding body is always a vibrating body: it may be a solid, as a bell; a liquid, as a waterfall; or a gas, as the wind; or the siren, or wind instruments of music, though the last are assisted by casing of thin solid. A vibrating body is not necessarily a sounding one, at least it may not be audible, but vibration is the only difference necessary to convert a silent body into a sounding one, and neither in the body that is causing the sound, nor in the medium transmitting it, is any change perceptible except vibration.

123. Complexity of Sound Phenomena. —

Mechanical	For the production of sound, then, vibration is essential. For the transmission of sound is needed a medium capable of receiving the vibrations and impressing them on the ear.
Physiological	For the reception of sound is required an organ of hearing, as the ear, or teeth and nerves.
Psychological	For the perception of sound is needed the brain to act in response to the nerves.

Thus the complete consideration of sound (as also that of light) involves three processes.

Experiment No. 51, page 199. — Necessity of medium for conveyance of sound is shown by bell under receiver of air pump.

Observe especially that the medium which brings the vibrations to the ear and actuates the organ of hearing is that in which the ear is embedded, as air or water, and *not* the ether. Sound failed by removal of air from the receiver though ether was still there.

A distinction is sometimes made between sound and noise. Where such a distinction is employed, sound results from a series of regular vibrations or a combination of such regular vibrations as have frequencies related to one another in simple ratios; noise results from a jumbling of abrupt single impulses having no definite numerical relation to one another.

It may be called sound if the action is sustained only long enough to give to the ear a sense of rhythm.

Experiment No. 52, page 199. — Illustrate sound and noise by the graduated blocks of wood.

Vibration of an elastic body (or medium) takes place in obedience to a force of restitution that is proportional to displacement; the motion therefore is S.H.M.

124. Sound Due to Longitudinal Vibration. — Whatever the character of the vibration that is to result as sound, whether the vibrating body is itself vibrating transversely or longitudinally, the wave that finally affects the ear is one of compression and rarefaction. A stretched string may itself vibrate transversely, and in so doing may be scarcely audible; if it is heard at all it is by means of compression and rarefaction which it produces in the air, and which will be slight because there is but little surface to the vibrating string to disturb the air; but if the string is so mounted as to set vibrating a larger surface like a sounding board the sound is more pronounced. A rod may be set vibrating transversely, but its vibrations will constitute sound only when the transverse vibrations have been converted into longitudinal ones either in the rod itself or in the subsequent media. If the rod is stroked longitudinally it is at once set vibrating longitudinally.

Experiment No. 53, page 199. — A sensitive flame shows the variations in pressure at the orifice of the gas jet.

Experiment No. 54, page 200. — Projecting a Crova's disk upon the screen shows the actual formation and progress of sound waves.

125. Velocity of Sound. — The velocity with which sound will travel, then, in any medium is the velocity of propagation of a compressural wave in that medium. This velocity, for any homogeneous elastic medium, was shown by Newton to be $v = \sqrt{\frac{E}{\delta}}$ (see Art. 120(*c*)), where E is the measure of the elasticity of the medium and δ is its density. Note that E and δ are the elasticity and density of the medium in which the sound is traveling, which usually is not the vibrating body from which the sound is proceeding.

For transmission in a long narrow body as a wire, the elasticity E is simply Young's modulus, but for a fluid it is necessary to use the volume elasticity. That is the ratio which the stress (or increment of pressure per unit of area pressed) bears to the strain produced by that increment of pressure, and the strain is the ratio which the diminution of volume bears to the original volume. If V is the initial volume of the fluid, p the additional pressure per unit of area applied and v the diminution in volume, then p is the stress, $\frac{v}{V}$ is the strain and $E = p \div \frac{v}{V} = \frac{pV}{v}$. For water at 4° C. the density is 1; an increase of pressure equal to one atmosphere or 76 cm. of mercury (1,013,300 dynes per square centimeter) diminishes the volume 0.000049 part. Then stress = 1,013,300, strain = 0.000049, $E = \frac{1,013,300}{0.000049}$, $\delta = 1$ and

velocity of sound in water $= \sqrt{\frac{E}{\delta}} = 143,800$ cm./sec.

Various experimental determinations for water at slightly higher temperatures give a velocity of about 143,500 cm./sec.

In a similar manner the velocity of sound in any other liquid may be determined. Owing to the minute compression and the high specific heat of liquids their temperature in transmitting sound waves undergoes practically no change.

In a gas at constant temperature, as we have seen (Art. 59), the elasticity is equal to the pressure. For air at 0° C. and standard barometer pressure, $\delta = 0.001293$, and $E = 1,013,300$, and these in the formula $v = \sqrt{\frac{E}{\delta}}$ give, for the velocity of sound, 27,995 cm./sec. As compared with experimental determination of the velocity of sound in air under those conditions, this is in error by 20%, the true value being 331.3 m./sec., an error quite too large to be ascribed to fault in experimentation, and showing that there is some fundamental error in the formula itself or in its derivation.

In a gas the temperature changes considerably with moderate changes of pressure, and as the elasticity equals the pressure only for constant temperature, it is in the effect of temperature upon the elasticity that we should first look for the error. Now if the changes from condensation to rarefaction and the reverse through the initial condition take place too rapidly for the heat to be carried from or to the portion of gas affected, the changes of density are occurring adiabatically, and the elasticity of the gas is not equal to the pressure. In Art. 108 it was seen that in an adiabatic change of a gas, PV^k = constant. Now if P is increased by the small pressure p, the volume of gas will be diminished by the small quantity v, and we shall have

$$PV^k = (P + p)(V - v)^k$$

and, by expanding $(V - v)^k$ and neglecting the higher powers of v,

$$\begin{aligned} PV^k &= (P + p)(V^k - kV^{k-1}v) \\ &= PV^k - kPV^{k-1}v + pV^k - kV^{k-1}pv, \end{aligned}$$

and, omitting the term containing the product of the two very small quantities p, v, we get

$$kPV^{k-1}v = pV^k \quad \text{or} \quad kP = \frac{pV}{v}.$$

But it has just been shown above that $\frac{pV}{v} = E$; therefore in adiabatic compression and rarefaction of a gas $E = kP$ (see

also Watson, Art. 287), and in this view we may write for the velocity of sound in a gas, $v = \sqrt{\frac{kP}{\delta}}$. k is the ratio of the specific heats of the gas, and for air is found to be 1.41. This gives for sound in air at 0° C., $v = \sqrt{1.41\frac{P}{\delta}} = 332$ m./sec., agreeing very closely with experiment.

This correction, due to Laplace, seems so far warranted that it is now taken as the best means of determining the ratio of the two specific heats of any gas. The velocity of sound in the gas at a given pressure and density can be determined, and from the above formula for v the value of k is computed.

(See Watson, Art. 287, for the effect of pressure upon the velocity of sound in a gas for which Boyle's law does not hold; that is, for gases when near the point of liquefaction.)

126. Effect of Temperature on Velocity of Sound in a Gas.— If a gas is so rare as practically to conform to Boyle's law, then any increase of pressure decreases the volume or increases the density in exactly the same proportion if the temperature is not altered, so that in the formula $v = \sqrt{\frac{kP}{\delta}}$, the ratio of $\frac{P}{\delta}$ is not affected by change of pressure for *any* given temperature; but under any pressure whatever, a change in temperature changes the density inversely per degree, as much as the coefficient of expansion, say α, so that if v_0 is the velocity in air at 0° C., and δ is its density, at any temperature $t°$ the density will be

$$\frac{\delta}{1 + \alpha t},$$

and

$$v_t = \sqrt{\frac{kP}{\delta}(1 + \alpha t)},$$

$$= v_0\sqrt{1 + \alpha t},$$

in which, for air, $\alpha = 0.003665$. With a rise of one degree in temperature, the velocity of sound in air is $\sqrt{1.003665}$ times that at 0° C., an increase of 60 cm./sec.

As in wave motion generally, if λ is the distance the disturbance travels in the time of one vibration, it is the length of one wave; and if n is the frequency of vibration, i.e., the number of vibrations per second, then the velocity v is equal to $n\lambda$; or if T is the time of one vibration, $n = \frac{1}{T}$, and $v = \frac{\lambda}{T}$. In these equations either quantity may be expressed in terms of the other two.

(For consideration of the subject of the last two articles in connection with the kinetic theory, see Daniell's *Physics*, Art. "Propagation of Sound in Gases according to Kinetic Theory.")

EXAMPLES. —

1. Taking the velocity of sound in air at 0° C. as 331.3 meters per second, what will be the velocity at 20° C.? *Ans.* 343.2 meters per sec.

2. When the temperature of the air is 0° C. the steam escaping on blowing the whistle of a locomotive is seen three seconds before the sound is heard; how far is the observer from the engine? *Ans.* 994 meters.

3. How much time would be required for the sound to reach the observer if the temperature were 30° C.? *Ans.* 2.8+ secs.

4. An obstruction in a pneumatic mail tube was located by discharging a pistol in front of a membrane stretched across the open end of the tube, and observing accurately when the pulse that traveled along the pipe was returned to the open end after reflection from the obstruction. If this time was 2.25 seconds and the temperature of the air in the pipe was 20° C., how far from the open end was the obstruction?

Ans. 386.1 meters.

127. Three Characteristics of a Sound. — Sounds other than those so confused as to be termed noise are distinguished by the three characteristics pitch, intensity (or loudness) and quality (or timbre). Variation of either of these attributes without regard to the others will cause a perceptible difference in the sound, easily recognized by the ear and determinable by physical measurement.

128. Pitch. — The difference between one note and another in pitch is that difference which is usually expressed by high or low (not loud or soft). One note may be loud and another scarcely audible, yet they may both be of the same pitch, and if

a note sounded by a musical instrument is repeated by the voice, the sound of the voice will probably differ greatly from that of the instrument except that it will have the same pitch, and will then be said to be in tune with it. The pitch, that is to say the highness or lowness of a tone, has reference solely to the rate or frequency of vibration, the number of vibrations per second producing it. This may be shown to be a fact, and at the same time the actual number of vibrations corresponding to any pitch may be determined by a suitable counting apparatus.

Experiment No. 55, page 200. — The siren determines it by counting the puffs of air; Savart's wheel, by the taps on the teeth of a wheel; different tuning forks, by tracing their vibrations on a smoked surface.

129. **Musical Scale; Intervals.** — A musical ear recognizes distinctly notes of different pitch as differing in the same degree and manner, when they have the same *relative* rates of vibration, no matter what their actual rates may be. The difference in pitch, then, of two sounds is expressed not by the arithmetical difference in their rates of vibration, but by the ratio of their frequencies, and this difference in pitch is called the interval between the tones, the ratio of the vibration rates being the measure of the interval. Not only is the interval recognizable when the notes are sounded in succession, but the effect of sounding two together depends for its smoothness or pleasantness upon the interval between them. If one note has a frequency twice as great as another, the interval between them is 2, and this interval is called an octave. It makes no difference what the actual frequency of one may be; if it is one half or double that of the other, they are separated by an interval of an octave. Between a given note and its octave above, the ear accepts naturally a number of intermediate intervals rising in successive simple ratios with the first frequency, and forming a so-called natural or diatonic scale. If we call the first note the unit, then the rates of vibration or the intervals within the octave are as follows, the first line being the common names of the notes, the second the intervals between each note and the first, and the third line the intervals between the successive notes:

do		re		mi		fa		sol		la		si		do
1		$\frac{9}{8}$		$\frac{5}{4}$		$\frac{4}{3}$		$\frac{3}{2}$		$\frac{5}{3}$		$\frac{15}{8}$		2
	$\frac{9}{8}$		$\frac{10}{9}$		$\frac{16}{15}$		$\frac{9}{8}$		$\frac{10}{9}$		$\frac{9}{8}$		$\frac{16}{15}$	

These seven steps are seen to be not quite equal although the first, second, fourth, fifth and sixth, are nearly so and are called whole tones, while the third and seventh are smaller and are called half tones or semitones.

It may seem queer that a gamut of seven steps in intervals so irregular should be a "natural scale." This irregularity is easily shown to be simply the mathematical result of shifting the beginning note, to form sets of notes in simple ratios. Nothing is more common in sound than the production of an octave up or down, and this means doubling or halving the initial frequency. Three notes whose vibration rates are as $4 : 5 : 6$ make an agreeable combination, forming a (major) natural triad. Why this combination should be natural or agreeable we cannot tell, though the fact has been discussed, notably by Pythagoras among ancient philosophers, and by Euler and others among the moderns, but their views are chiefly speculations.

If we start with a note of definite pitch which we may call C and its frequency the unit or 1, the triad will be numerically $1 : \frac{5}{4} : \frac{3}{2}$, 2 being the octave. If we produce a like triad beginning with the third one of this series, the numbers will be as $\frac{3}{2} : \frac{3}{2} \times \frac{5}{4} : \frac{3}{2} \times \frac{3}{2}$, or $\frac{3}{2} : \frac{15}{8} : \frac{9}{4}$; and if we insert a similar set of which the octave 2 shall be the uppermost, this triad will be numerically $\frac{4}{3} : \frac{5}{3} : 2$ as these are in the ratio $4 : 5 : 6$. We now have from our beginning note C = 1, intervals built up from the ratios $4 : 5 : 6$, as follows:

C	D	E	F	G	A	B	c	d
1 :	( ) :	$\frac{5}{4}$:	$\frac{4}{3}$:	$\frac{3}{2}$:	$\frac{5}{3}$:	$\frac{15}{8}$:	2 :	$\frac{9}{4}$

The octave below the last of these is $\frac{9}{8}$, occurring between C and E, in the parenthesis, and completing the scale of seven intervals in the octave from C to c.

130. Temperament. — The scale might begin with a note of any frequency and progress by the intervals just given. As the interval between the third and fourth notes is called a half tone, and equals $\frac{16}{15}$, the whole tone intervals might each be subdivided into half tones, making twelve intervals in the octave, but two successive intervals of $\frac{16}{15}$ would not make a rise of exactly $\frac{9}{8}$, for $\frac{16}{15} \times \frac{16}{15}$ equals 1.138 instead of 1.125. If we letter the notes of the scale C, D, E, F, G, A, B, c, the half tone above C would be called C sharp (C♯), and a half tone below D would be properly D flat (D♭), and, allowing $\frac{16}{15}$ for a half tone, C♯ would be higher than D♭. Again if absolute numerical values were given for the pitch of the notes according to the natural intervals, beginning the scale with C, then those numbers would not make the correct intervals if we wanted to begin with C♯ or D. An instrument with fixed pitch for each key that is struck, like a piano, would require an enormous number of sets of keys and strings or pipes to make music with true intervals in different "keys" (i.e., starting the octave with different frequencies as the keynote). With an instrument like a violin on which a note of any pitch may be produced, this is not the case.

To escape this difficulty, scales have been adopted in which some of the intervals have been made slightly false. A scale thus altered is called a tempered scale. The mode of tempering almost universal now is to make an even-tempered scale or scale of equal (or "just") temperament. This is produced by choosing some value for one of the notes and then dividing the octave, for which the whole interval is 2, into twelve equal intervals of $\sqrt[12]{2}$, or 1.059 each. That makes some of the notes a little too high and others a little too low, the actual comparison being as follows:

Natural scale,	1;	1.125;	1.25;	1.333;	1.5 ;	1.667;	1.875;	2.
Equal temperament	1;	1.12 ;	1.26;	1.325;	1.498;	1.682;	1.882;	2.
	C	D	E	F	G	A	B	c

In ordinary conversation the voice is modulated in musical intervals, the most frequent being the octave with which the final word of an assertion is uttered, the voice dropping or rising

through an octave involuntarily. It is interesting thus to examine the intervals in the modulation of the voice by different speakers. (Illustrate.)

Stage recitation is sometimes done to music, making it difficult sometimes for the listener to decide whether the performer is speaking or singing.

A musical voice is one in which the sound is neither harsh nor husky, and which is so modulated that the intervals in its tones are exact.

The International Standard Pitch gives the middle A of the pianoforte 435 vibrations per second, from which, by natural intervals, the c above it would have 522 vibrations per second, but by equal temperament it has 517.3, and this latter number would be the proper frequency for a c fork to be used in tuning for an even tempered scale with the International Standard Pitch. The New York Philharmonic Society uses the Stuttgart Standard for which A has a frequency of 440 vibrations per second.

EXAMPLE. — Starting with the middle C of the pianoforte as 260 vibrations per second, the pitch of an octave above E was fixed by equal temperament, and from this the note c (i.e., the octave above C) was deduced on the natural scale; how much did it differ from the piano?

Ans. 4.12 vibrations per sec. higher.

131. Doppler's Principle. — The pitch of the note issued by a sounding body is determined by the rate of vibration of the body; the pitch of the note perceived by the ear is that due to the number of vibrations per second received by the ear, and these two pitches are not necessarily alike. If a body sends out n waves per second, and the source of sound and the ear are at a constant distance apart, the medium transmitting the waves being at rest, then the ear will continually receive n waves per second, but if the ear and the sounding body are approaching each other the ear will receive more than n waves per second and the pitch will be higher, or, if they are separating, the ear will receive fewer than n waves per second and the pitch will be lower. Suppose the velocity of sound is V and the

sounding body (supposed to be stationary) is emitting n waves per second. The wave length λ is $\frac{V}{n}$. If the observer is stationary he will receive n waves per second, but if he moves toward the sounding body a distance v per second, he will in this distance increase the number of waves he receives by $\frac{v}{\lambda}$, so that, in all, he receives in one second $n + \frac{v}{\lambda}$, or $n + \frac{nv}{V}$, that is, $n\left(1 + \frac{v}{V}\right)$. In the same way, if he is receding from the source of sound at a rate v, he will receive in one second $n - \frac{v}{\lambda}$, and the pitch of the note will be $n\left(1 - \frac{v}{V}\right)$.

This change of pitch and its explanation are known as Doppler's Principle. It has important applications in connection with light. Other changes of pitch ensue when the transmitting medium is in motion. See Barton's *Text Book of Sound*, Arts. 60, 61.

EXAMPLE (from Watson's *Physics*.) — "A locomotive whistle emitting 2000 waves per second is moving towards you at the rate of 60 miles per hour on a day when the thermometer stands at 15° C. Calculate the apparent pitch of the whistle."

$$\text{Velocity of sound at } 0° = 1093 \text{ ft./sec.}$$

$$V = 1093\sqrt{1 + 0.003665 \times 15} = 1122.5$$

$$60 \text{ miles per hour is } 88 \text{ ft./sec.} = v \quad \text{and} \quad \frac{v}{V} = 0.0784.$$

$$\text{Apparent pitch} = 2000 \times 1.0784 = 2156.8.$$

Curious results are deducible for cases in which the body producing the waves is moving at the same time faster than the waves themselves travel. (See Watson, Art. 299.)

132. Velocity of Sound Independent of Pitch. — The formula for the velocity of a compressural wave (Art. 118) is independent of the rate of vibration, thus indicating that waves of all frequencies progress with the same speed. This is verified in the case of sound waves. Notes of different pitch travel with the same velocity, as is evidenced by the fact that notes in a musical performance played in definite time, following upon one

another in well defined periods, follow each other in just the same periods when the music is heard at a long distance from where it is produced.

If, then, v is the velocity of sound for all frequencies, and n is the frequency and λ the wave length for any given note, n such lengths make the distance v, or $v = n\lambda$, or $\lambda = \frac{v}{n}$.

133. Intensity of Sound. — A note of any pitch may be strong or feeble, the difference in this respect being called a difference in *intensity*, and the difference of sensation produced is a difference in *loudness*. The intensity does not depend upon the frequency, but upon the energy of vibration. In general the energy of vibration for any body is proportional to the square of the amplitude of vibration, so that the intensity of sound produced by a given body will vary as the square of the amplitude of its vibration; but if the same body be so mounted as to expend its energy upon a limited mass of the conveying medium, as the air in a tube, instead of dissipating it in a large sphere of air, the sound at the end of the tube will be more intense than at that distance in air unconfined. Again, if the vibrating (or sounding) body is mounted upon a sounding board or box, its energy is applied to disturb a greater mass of the air near it, and the sound at a distance is correspondingly more intense. It is plain that in such case the vibrations of the body will die out sooner, or its energy be sooner exhausted, than if it were agitating less mass. (Illustrate by tuning forks: small one in air, scarcely audible; on table, plainly heard.)

That the pitch remains the same while the intensity diminishes, and that the intensity falls off with a decrease in the amplitude of vibration, may be shown by successive tracings of the vibrations of a tuning fork as the sound dies out.

The energy of vibration of air in the bounding surface of a sphere can be shown to decrease as the square of the distance from the source of vibration increases, and so the *physical* intensity of sound varies inversely as the square of the distance (Watson, Arts. 312, 313), but the loudness, which is a *physiological*

sensation, does not vary in any such definite proportion, though it does increase with an increase in intensity.

134. Quality of Sound; Overtones. — There is a rate of vibration for every body that may be called its natural or normal rate, and which is the slowest rate at which it will vibrate without constraint. This evidently depends upon the nature of the material and upon the form and dimensions of the body. The tone which it emits when vibrating in its lowest frequency is called its fundamental tone. Most bodies, however, are capable of being put into other and higher rates of vibration, by dividing up into vibrating segments, and the tones due to these more rapid vibrations are called overtones. If a body is emitting at the same time its fundamental and overtones along with it, the effect, to the ear, is different from that due to the fundamental alone, although the pitch would be said to be the same in both cases. This difference is called a difference in *quality* (*timbre, Klang-farbe*). The lowest rate of vibration present in the note determines the pitch. If the frequencies of the overtones are those obtained by multiplying that of the fundamental by the natural numbers 2, 3, 4, . . ., they are called harmonics. The actual quality of the tone is determined by the particular overtones that are present with the fundamental, and their respective intensities. An analysis of sound consists, for the most part, in resolving a compound tone into its constituents and determining them.

Experiments Nos. 56 and 57, page 201. — Composite vibrations of sound shown by manometric flames; fundamentals and overtones produced with tuning forks.

135. Reflection and Interference of Sound. — Sound waves undergo the changes due to reflection, interference, etc., that have already been presented in considering wave motion; the most familiar illustration of reflection of sound being the echo. If sound travels, say, 340 meters per second, then a sound from a point opposite a wall that is at a distance of 170 meters will return to the source in just one second. If various sounds were being produced at intervals of one second, the echo would

cause confusion. At one tenth the distance (or about 55 feet), sounds emitted at the rate of ten a second would be in confusion with the echo, and articulation would be indistinct. This would also be the case at any point in a room where the echo of one sound arrived at the same instant that a succeeding sound arrived in its direct path. Failure to recognize these principles is responsible for a good deal of the bad acoustics of halls intended for public speaking. In the construction of such halls such arches or domes in the ceiling as would produce focal regions or axial focal lines in the audience by reflection of waves proceeding from the rostrum should be especially avoided.

A sound wave traveling in one medium and arriving at the surface of a second medium of different density is reflected in part, if not wholly, and if the reflecting surface is perpendicular to the line of progress of the waves, the reflected waves directly retrace the path of the advancing waves and interference will result. That is, at regular intervals, where the motion of the advancing waves would crowd the particles, say, to the left, that in the reflected waves would crowd them toward the right and there would be stationary particles in condensation; half a wave length further along there would again be stationary particles due to a tendency to separate in both directions; half a period later the former of these places would still be a place of rest but of rarefaction, while the latter would be in rest but in condensation. In fact these places would be permanently places of rest due to interference, while between them would be places of maximum motion alternately in one direction and in the opposite. This is a case of stationary waves of sound, the points of no motion, or rather of minimum motion, being nodes, and between them loops, the distance from node to node or from loop to loop being half a wave length.

EXAMPLE. — With the temperature of the air 20° C., a man standing some distance from the face of a cliff fires a pistol and hears the echo two seconds later. How far is he from the cliff? *Ans.* 343 meters.

136. Longitudinal Vibration of Rods. — The elasticity and density of a rod will determine the speed with which a sound

wave will travel along it. If the rod is fixed at one end and free at the other, then when the rod is stroked the fixed end is necessarily at a node and the free end at a loop. The longest wave the rod can then represent will be four times the length of the rod itself, and, v being the velocity of sound in the material, the frequency of vibration will be $\frac{v}{4l}$. If the rod is clamped in the middle and free at the ends, the mid-point will be a node and each end a loop. The length l of the rod is then half a wave length or $\frac{\lambda}{2}$, the wave length is $2l$, and the pitch is that of n vibrations where $n = \frac{v}{2l}$. By means of these formulæ, if the pitch of the note is determined, the velocity is at once calculable, and from the measured density the elasticity may be found; or, with rods of different material tuned to the same pitch, if the densities are known the values of E may be compared. Let n, E, δ and l represent respectively the frequency, elasticity, density and length of rod, then

$$n_1 = \frac{\sqrt{\frac{E_1}{\delta_1}}}{2l_1} \text{ and } n_2 = \frac{\sqrt{\frac{E_2}{\delta_2}}}{2l_2}.$$

If these values of n are equal, since δ_1, δ_2, l_1, l_2 are known, the ratio of E_1 to E_2 becomes known.

EXAMPLE. — An iron rod having a mass of 92.7 g., length 80.5 cm., diameter 0.436 cm., when held in the middle by the thumb and forefinger and gently stroked with a rosined piece of chamois, emitted a note that was the third octave above the G of an adjustable tuning fork. The frequency of this G was 389 vibrations per second. The pitch of the note emitted by the rod, therefore, was $n = 389 \times 2 \times 2 \times 2 = 3112$.

$$\lambda = 80.5 \times 2 = 161 \text{ and } v = 161 \times 3112$$
$$= 501{,}000 \text{ cm./sec.}$$

Also $$\delta = \frac{\text{mass}}{\text{volume}} = \frac{92.7}{11.994} = 7.73 \text{ g./c.c.}$$

and since $$v = \sqrt{\frac{E}{\delta}}, \quad E = v^2\delta = 193 \times 10^{10} \frac{\text{dynes}}{\text{sq. cm.}}$$
$$= 200 \times 10^4 \text{ kg./sq. cm., nearly.}$$

137. **Kundt's Tube.** — By inclosing air or any gas as a column in a tube so that the length of the column can be adjusted, and inserting a rod of metal carrying a loosely fitting piston at the end in the tube while half the rod or more extends outside the tube, stroking the rod while it is held firmly at its middle point puts it into longitudinal vibration, and the light piston on the end of the rod imposes waves of condensation and rarefaction of the same period upon the column of inclosed gas. v being the velocity of sound in the gas and n the frequency of vibration, the wave length in the gas of such waves is $\frac{v}{n}$; a half wave length is $\frac{v}{2n}$. If the length of the air column is divisible by this number it will break up into segments of this length.

Experiment No. 58, page 202. — Kundt's tube experiment.

138. **Interference. Can Two Sounds Produce Silence?** — Stationary waves are due to interference of reflected with advancing waves. These two trains may be regarded as proceeding from two different sources, though in fact all originating at the same source. It is possible, however, so to employ two different sources of sound that the waves will interfere at definite places. If a wave proceeding from one source arrive at a certain point in a phase, say, of condensation, and a wave of equal intensity from another source arrives at the same point in a phase of rarefaction, the effect of the two at that point will be *nil*. If, furthermore, the sounds are of the same pitch (i.e., frequency), this place will be one of continuous silence. One of the simplest illustrations of interference of sound waves is with an ordinary tuning fork. Suppose O (Fig. 81) to be between the ends of a

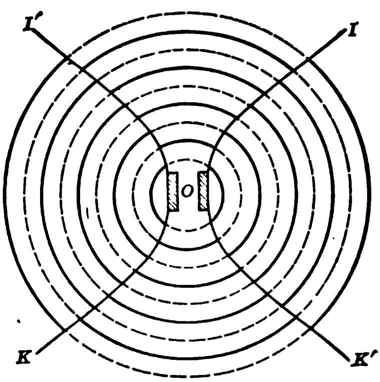


Fig. 81. Interference of Sound Waves.

vibrating tuning fork. The prongs alternately approach each other and recede, causing the air between them to be alternately condensed and rarefied, while that at the back of them is at the same time alternately rarefied and condensed. A series of waves will proceed outward, represented in the plane of the paper by heavy (condensed) and dotted (rarefied) circles. At four points around each circle there will be a tendency at the same instant to both condensation and rarefaction, and interference will be produced along the lines *IOK* and *I'OK'*. Such a vibrating fork held vertically before the ear and slowly rotated will give four distinct positions of silence in one rotation of the fork.

139. **Resonance.** — Strictly speaking, resonance could only apply to sound, for it would be impossible for any body to be resonant unless it or some other body were first sonant, but the similarity of occurrences in other branches of physics to those in acoustics has warranted an extension of the term to cases in which sound plays no part.

If any rhythmic action in one body excites in another, whether directly connected with the first or apparently disconnected from it, rhythmic action of like periodicity, the second body is said to be in resonance with the first.

In the case of sound the body that is primarily vibrating may be emitting but a feeble sound, but may awaken such vibrations in an inclosed air space as to make a loud sound. Such a space is called a resonance chamber or resonator, and the air in it a resonance column. If a large surface of solid or liquid has been put into vibration it is called a sounding board.

To resound to a note of given pitch the resonance column must be of proper dimensions to vibrate at the rate corresponding to that pitch. A column of air in a tall vessel *AD* (Fig. 82) will resound to a fork vibrating above it if the wave after reflection from a surface, as *B*, arrives at the mouth *A* in such phase as is then being produced there by the vibrating fork. When the tine *T* begins to move downward it starts a pulse of *condensation* down the tube; this, arriving at *B*, is there reflected and returns to *A*. If it reaches *A* when the prong of the fork is just ready

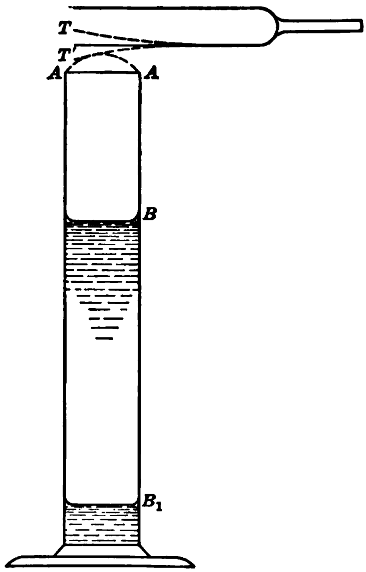


Fig. 82. Resonance of Air Column.

to return the pulse must travel from A to B and back while the prong of the fork swings from T to T', or the time is one half the period of vibration. From A to B and back would then be one half the length of the sound wave in air.

The prong of the fork, however, will be starting in the opposite direction from that in which it was going when it sent the pulse down the tube, in one half a period, or in $\frac{3}{2}$, or $\frac{5}{2}$, or any odd number of half periods. If the returning pulse is in time with any of these returns of the fork, resonance will occur. That is, the distance from A to B and back must be an odd number of half wave lengths, or AB must be an odd number of quarter wave lengths. If the distance from A to the reflecting surface B, B_1, B_2, etc., is l, the column will resound if $l = (2x + 1)\frac{\lambda}{4}$, where x is any number, and λ is the wave length for the pitch of fork used. The simplest and strongest resonance occurs for $x = 0$, or AB = one quarter-wave-length. When the value of x is known, and also the rate of vibration of the fork, the velocity of sound in the gas in the tube can be determined, for $\lambda = \frac{v}{n}$, and therefore $l = (2x + 1)\frac{v}{4n}$. For the first or fundamental resonance, $x = 0$, and $v = 4nl$; for the next resonance, i.e., for the next greater length of tube, AB_1, $x = 1$, and $l = \frac{3v}{4n}$, whence $v = \frac{4nl}{3}$, and so on.

Owing to the spherical form which a wave assumes as it

proceeds from a point of disturbance, and also probably on account of some retardation by friction along the sides of the tube, the wave front of air at the mouth A is curved somewhat upwards, so that the actual length to be counted for $\frac{1}{4}\lambda$ is AB increased by a quantity that varies somewhat with the diameter of the tube, from 0.6 to 0.8 of the radius.

Experiment No. 59, page 202.—Resonance of tube with adjustable water column.

The whistling of a key, or of the cap of a fountain pen, and the popping of a bottle when uncorked are instances of resonance.

Pitch of Nature. — The general contour of the country, the presence of trees, brooks and houses, the varied occupations of men, the movement of machinery, all combine to produce in the ear of a listener a dominant tone of which the pitch may be identified.

At Dorollis, N. Y., in August, 1910, the wind in the trees and the humming sound made up of this, and the murmuring of the brook, and other noises, which, with their reverberation from the hills, combined to make up the tone of nature, produced a note of which the pitch was a half tone under the octave below the note emitted by the cap of a fountain pen. The cap was 1 cm. in diameter and 5.9 cm. deep. Adding 0.35 to 5.9 gives 6.25 cm. for one quarter wave length, or 25 cm. as the wave length of the note it emits. At 20° C. the velocity of sound is 342 m. per second, and the pitch of the note from the cap is $\frac{34{,}200}{25}$, or 1368 vibrations per second; about the third F above middle of pianoforte scale. The octave below this is 684 vibrations per second, and the half tone under that is $684 \div 1.059$, or 645, which was the "pitch of nature" as then observed, approximately the second E above the middle C of the piano.

On Jan. 18, 1908, in City Hall Park, New York City, between the City Hall and the Court House, with Park Row and the clamor of the other streets and the Brooklyn Bridge terminal on the east and Broadway on the west, the sounds from the east seemed to reduce to a note of 940 vibrations per second, while the sound from the west or Broadway side was one full tone lower, corresponding to 840 vibrations per second.

Forks for experimentation are mounted on resonance boxes closed at one end and open at the other. These boxes are usually of a length that is approximately $\frac{\lambda}{4}$ for the period of the fork they carry (somewhat less than $\frac{\lambda}{4}$ on account of their width). The

fork vibrating on the cover of the box has its vibrations transmitted to the air within by the elasticity of the cover, but if the fork be dismounted and held, while vibrating, before the mouth of the box, the resonance is pronounced. (The hole in the top of the cover must be stopped.) Here the interference effect described in Art. 138 is plainly shown by rotating the fork before the mouth of the resonance box.

Experiment No. 60, page 203. — Interference of sound.

It is to be noticed that a given resonance chamber will not resound to all tones, but only to those of a certain period, and first and most forcibly to its fundamental tone. Perhaps the most remarkable resonance cavity is that which sustains the voice and is formed by the mouth, throat and nasal passages. The variations in its size and form by the muscular action of the person speaking or singing accounts for nearly the whole range of modification which vocal sounds receive. The vibrations producing the sound are due chiefly to the vocal cords and the lips, but the variations in tone are due to the resonance qualities of this chamber.

An interesting illustration of complex vibration in a single elastic rod and the facile action of the mouth in resounding is shown by the jew's-harp. (Illustrate.)

There is, then, a pitch of voice to which any room or hall readily responds. A speaker using this as the dominant pitch in speaking feels his voice strengthened or sustained, and a skillful elocutionist readily discovers the pitch appropriate to the hall in which he speaks. The same room, when filled with people, may call for a different pitch of voice from that to which it resounds when empty.

EXAMPLES. —

1. The fork above the tall vessel in Fig. 82 makes 320 vibrations per second, and gives strongest resonance when the air column AB is 25.4 cm. long. If the diameter of the vessel is 4 cm., what is the velocity of sound? What length of air column AB would give the next resonance? (Add $0.7r$ to AB for $\frac{1}{4}\lambda$). *Ans.* 343 m./sec.; 79 cm.

2. A shrill note is produced by blowing across the end of a key. If the

bore is 3 mm. in diameter and 1.2 cm. deep, the velocity of sound being 345 meters per second, what is the frequency of the note emitted? (Add $0.8r$ to depth of bore for $\frac{1}{4}\lambda$.) *Ans.* 6534 v./sec.

140. Sympathetic Vibration. — Usually resonance is regarded as strengthening a sound or bringing it out, but resonance may occur without doing this, and a body may be in resonance with another in consequence of having a like fundamental period, and continue sounding, though perhaps feebly, after the first one ceases. Such vibration is called sympathetic vibration. It may be shown very readily by depressing a key of a piano, thus removing the damper from that string, and then striking the key one or two octaves higher. Although the sound from the string that is struck is immediately stopped by releasing its key, if the other key is held down its string gives out the note of that which was struck.

Experiments Nos. 61 and 62, pages 203 and 204. — Sympathetic vibration.

141. Selective Absorption. — If a large number of wires were strung across a room and tuned to rates of vibration that are multiples or submultiples of the pitch of the notes produced by an orchestra, a piece of music performed by the orchestra at one end of the room would be much weakened, if not wholly destroyed, for listeners at the other end of the room, by the absorption of the energy of vibration by the wires. Each wire, however, would take up only that order of vibration to which it was itself tuned. Such a process of taking up energy is called "selective absorption."

142. Corti's Fibers. — In the liquid which fills the internal ear are a large number of fibers or threads that are termini of the auditory nerve. These fibers are of various lengths and mass and respond to various periodicities of disturbance in the liquid. It has been supposed that any vibration of the drum of the ear was imparted by the latter to the liquid within, and these fibers, selecting the vibrations of their own respective periods, furnished at once the stimulus by which the brain perceives and distinguishes the different sounds, either simple or complex. It is now thought that these fibers accomplish only

a part, and a minor part at that, of the act of hearing, the sympathetic vibration really occurring in a so-called basilar membrane which extends along one of the chambers of the cochlea, and upon which rest the Corti fibers and also other fibers connected with the auditory nerve.

143. Vibration of Air in Pipes.—(*a*) *Open Pipes.*—In a pipe open at both ends, the air at each end is free and therefore will vibrate as at a loop or ventral segment. The node nearest each end will be at a quarter wave length from the end, and if the air column breaks up into stationary waves it must so divide as to have a quarter wave length at each end and an integral number of half wave lengths between them in the pipe. The simplest division is with a node in the middle, as at C (Fig. 83 (*a*); here the length of the pipe is two quarter wave lengths, or $2\frac{\lambda}{4}$. The next simplest division is as in (*b*), in which $AC = \frac{\lambda}{4}$, $CD = \frac{\lambda}{2}$, and $DE = \frac{\lambda}{4}$; in (*c*), $AC = \frac{\lambda}{4}$, $CD = DE = \frac{\lambda}{2}$, and $EF = \frac{\lambda}{4}$, and there are six quarter wave lengths in the total length l. It is readily seen that the subdivision may be extended, but always so as to give an even number of quarter wave lengths. That is, the length of the pipe l equals

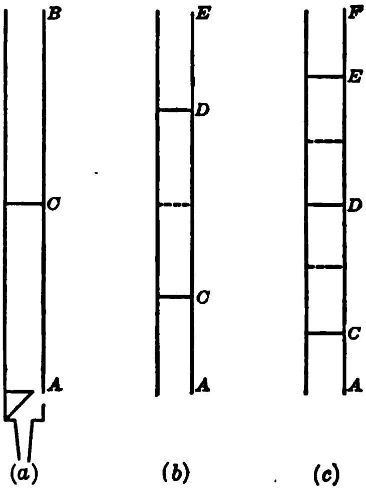


Fig. 83. Division of Vibrating Air Column in an Open Pipe.

$$2\frac{\lambda}{4},\ 4\frac{\lambda}{4},\ 6\frac{\lambda}{4},\ \ldots\ 2n\frac{\lambda}{4},\ \text{or}$$

$$\lambda = 2l,\ l,\ \frac{2}{3}l,\ \frac{1}{2}l,\ \ldots\ \frac{2}{n}l.$$

The rate of vibration of the air column is inversely as the length of the wave, or the vibration rates for such a pipe are as the numbers $\frac{1}{2l}, \frac{2}{2l}, \frac{3}{2l}, \ldots \frac{n}{2l}$, or as the natural numbers, 1, 2, 3, . . . n. An open pipe, then, will give for its fundamental a tone for which $\lambda = 2l$, and it is capable of emitting all the harmonics.

(*b*) *Stopped Pipes.* — A resonance column or box closed at one end, like those described in Art. 139, Resonance, is like an organ pipe that is "stopped." The stopped end is of necessity a nodal point, while the air at the open end is free, and that is therefore the position of a loop. In such a pipe the simplest mode of vibration, producing the fundamental tone, is when the entire length of the tube equals $\frac{\lambda}{4}$, or $\lambda = 4l$ (Fig. 84*a*). In any further subdivision of the column, since the uppermost segment must always be a half wave length, and the lowermost a quarter wave length, the pipe will contain one, three, five, or the odd numbers of quarter wave lengths. That is,

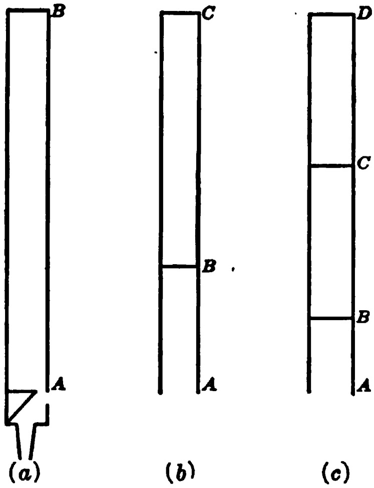


Fig. 84. Division of Vibrating Air Column in a Stopped Pipe.

$$l = 1\frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}, \ldots (2n+1)\frac{\lambda}{4},$$

and
$$\lambda = 4l, \frac{4}{3}l, \frac{4}{5}l, \ldots \frac{4l}{2n+1},$$

and the vibration rates are as

$$\frac{1}{4l}, \frac{3}{4l}, \frac{5}{4l}, \ldots \frac{2n+1}{4l},$$

that is, as the odd numbers 1, 3, 5, . . . $(2n+1)$. Such a pipe, therefore, can give out only the odd harmonics besides its fundamental.

If an open pipe were of the same length as a stopped pipe, its fundamental would have a wave length half as great as the fundamental of the stopped pipe, or it would be an octave higher in tone. The fundamental of an open pipe whose length is l is of the same pitch as that of the stopped pipe whose length is $\frac{l}{2}$.

This may be simply but strikingly illustrated as follows: Select a plain open pipe of glass or metal 3 or 4 cm. in diameter and from 30 to 60 cm. long. Close one end suddenly with the palm of the hand and as suddenly jerk it away. Repeat this rapidly, and a rapid alternation of a note and its octave is produced. The lower note is that of the pipe when the hand closes it, and the higher when it is opened.

Since an open pipe is capable of producing all overtones and a stopped one only those corresponding to the odd numbers, although an open pipe of length l and a stopped one of $\frac{l}{2}$ have the same pitch, the sound of the former is richer than that of the latter, because it possesses certain overtones which cannot be present in the sound of the latter.

Also illustrate with organ pipes, and exhibit (project) Crova's disks: (*a*) showing fundamental of open pipe; (b) showing second overtone of stopped pipe.

Experiments Nos. 63 and 64, page 204. — Singing flames; resonant pipes.

EXAMPLES. —

1. What length of closed organ pipe will give a wave 2 meters long? What length of open pipe will give the same wave length?

Ans. 50 cm.; 100 cm.

2. In the pipes of Ex. 1 what would be the wave length of the first overtone produced by (*a*) the closed pipe; (*b*) the open pipe?

Ans. (*a*) 66.67 cm.; (*b*) 100 cm.

3. If the velocity of sound in air at 0° C. is 332 meters per second, what change is produced in the note of an open organ pipe 50 cm. long, when the temperature rises from 10° C. to 35° C.?

Ans. The frequency increases from 338 per second to 353 per second; the pitch rises a little less than a semitone.

144. Vibration of Strings. — Gases can receive or transmit only compressural waves, resulting from longitudinal vibration; solids may undergo transverse or longitudinal vibration. If the

latter, the waves traversing the solid are sound waves, as they were exemplified in Kundt's tube and in Art. 136. With strings or wires the vibration is usually transverse, and the sonant effect is only obtained by some mounting of the string which will set vibrating a sounding board or the cover of a resonance box which thus imposes sound vibrations upon the air. A string itself will take on rates of vibration of various periods, having a fundamental rate corresponding to its vibration as a whole, and at the same time possibly breaking into segments of rates to produce overtones. If the string is stretched between two fixed points, A and B (Fig. 90; see Experiment No. 65), the distance AB is considered the length l of the string. The slowest vibration is when the whole string is swinging to and fro, the largest movement being midway between A and B. The fixed points are necessarily nodes, and in this vibration the length of the string is half a wave length, or $l = \frac{\lambda}{2}$. Then $\lambda = 2\,l$, and if the string is making n vibrations per second, the velocity with which such waves travel along the string is $n\lambda$. An obstacle holding the string at its middle point D will cause it to vibrate in two segments, or in waves whose length is l instead of $2\,l$, and by imposing a position of rest at proper points, the string can be made to produce vibrations of any period for which it can be divided into an integral number of segments, i.e., as the successive numbers 1, 2, 3, . . . n. From Art. 120, the rate at which transverse waves travel along a string is given by the equation $v = \sqrt{\frac{T}{\Delta}}$, where T is the tension of the string and Δ the mass per unit length. If T is dynes, and Δ is grams per centimeter, v is centimeters per second. Then, since $v = n\lambda$,

$$n\lambda = \sqrt{\frac{T}{\Delta}}. \qquad \text{(A)}$$

Also if s is the number of segments into which the string divides itself, $\frac{l}{s}$ is the length of one segment, and $\lambda = \frac{2\,l}{s}$;

therefore,
$$\frac{2\,nl}{s} = \sqrt{\frac{T}{\Delta}}. \qquad \text{(B)}$$

If δ is the density of the material and the string is circular in cross-section with radius r, then (1) πr^2 is the volume of unit length, and $\pi r^2 \delta$ is the mass per unit length, or the value of Δ in Eq. (B), from which we obtain

$$n = \frac{s}{2\,lr}\sqrt{\frac{T}{\pi\delta}}. \tag{C}$$

For a given value of s, l and δ, n varies as the square root of T, and for the fundamental rate s is unity; so the pitch of the string varies as the square root of the stretching force. If the string is stretched by a weight w a stretching weight of $4\,w$ will produce a tone an octave higher, etc. (Compare this with the fact that a change of weight makes slight difference in the note if the vibrations are longitudinal.)

With the same material, i.e., δ being constant, and keeping the same value of T while s is unity, doubling the length will halve the frequency or lower the tone an octave, and conversely; also, with l constant, doubling the radius will halve the frequency. All these relations for transverse vibration may be summed up as follows:

With a string vibrating as a whole, the rate of vibration (pitch) is proportional,

(*a*) *Directly to the square root of the stretching force.*
(*b*) *Inversely to the square root of the density.*
(*c*) *Inversely to the radius.*
(*d*) *Inversely to the length.*

These principles all have direct application in stringed instruments of music.

Experiment No. 65, page 205. Melde's experiments.

EXAMPLES. —

1. A string stretched by a weight of 2 kg. vibrates 200 times per second. With what weight must it be stretched to give a note an octave higher?

Ans. 8 kg.

2. How should the length of the string in Ex. 1 be altered to produce the octave when stretched with the 2 kg. weight?

Ans. It must be one half as long.

3. The strings of a violin are tuned to G, D, A and E. Suppose the D

string to be of like material and length with the A, but with twice the area of cross-section, how does its tension compare with that of A? (Refer to Eq. (A), Art. 144, and to intervals of natural scale, Art. 129.)

Ans. 0.911 as great.

145. Beats; Combination Tones. — If two sounds of equal intensity and of like period issue from independent sources so situated that at a given point the sounds are in exactly opposite phase of vibration, a condensation arriving at the point from one source at the instant that a rarefaction arrives there from the other, there will be complete interference and no sound will be perceived. On the other hand, if the phase of vibration from both were the same, both condensation and rarefaction would be greater and the sound would be rendered more intense, or be reënforced. But if the two sounds differed slightly in period, they would agree at one instant and there would then be reënforcement, but gradually the one would gain in phase upon the other until it would be a half period in advance; it would then be in opposition, or producing a condensation where the other would be causing rarefaction, the two would interfere, and the sound would be destroyed; after a while, however, the faster one would gain another half period when they would again reënforce each other and a loud sound would result. While the vibrations are maintained there will be a succession of loud sounds intermitted by silences, or rather a series of alternate crescendos and diminuendos. These are called "beats." When the two sounds have almost exactly the same period (frequency), both the interference and the reënforcement hold for a considerable length of time. Two organ pipes thus sounded together, if very slightly out of unison, may seem for a considerable time to be not sounding at all, but with the stopping of either the other will immediately be heard. If one sound has one more vibration per second than the other, it will gain one vibration per second, and there will be one beat (one swelling and one shrinking of sound) per second; if the rates differ by two vibrations per second there will be two beats per second, and so on. If the difference in rate is, say, as great as ten or twelve a second, the beats occur

as rapid throbs, and their presence gives a roughness or huskiness to the combined sounds. These throbs, however, may correspond in period to the rate of vibration of a hall or building and set the entire structure to vibrating. When the beats are as frequent as twenty or more a second they produce the sensation of a separate low tone called a beat tone, or a difference tone. Similar effects may be produced by sounding simultaneously two notes of very nearly an exact simple ratio, as, for example, a note and its octave, and other combination tones called summation tones may be produced.

(The classic on this subject is Helmholtz's *Tonempfindungen;* in the English translation, *Sensations of Tone.*)

Experiment No. 66, page 206. — Beats produced by tuning forks, also by pipes.

EXAMPLE.—The rate of vibration of a tuning fork decreases with a rise in temperature of the fork. Two forks are in unison, each making 256 vibrations per second at 20° C. When one is heated to 100° C., and the two are then sounded together, 20 beats are heard in 9 seconds. In what proportion is the frequency lowered per degree? *Ans.* 0.000108.

146. Limits of Audition. — There are from 16,000 to 20,000 Corti's fibers which unite in a common auditory nerve, but of which each is capable of conveying a separate distinct sound to the brain. The lowest rate of vibration to which the human ear is responsive is given by Preyer as 16, by Helmholtz as 34 per second; while the upper limit is given by Despretz as 32,000, and by Preyer as 40,000. If we take the inferior limits, i.e., 34 and 32,000, as more nearly normal, the range of audition is about ten octaves.

(Illustrate with Galton's whistle.)

147. Musical Instruments.—These are of the utmost variety, but they can be arranged in classes, as:

(*a*) Wind instruments, in which air columns are set vibrating by the edge of a mouthpiece, as in organ pipes, the flute, etc.; or by the lips, as in the cornet; or by a reed tongue, as in reed organs, the clarionet, etc. In all the pitch is varied by means of keys except in the trombone and the trumpet.

(*b*) Stringed instruments, in which the transverse vibrations of strings are converted into sound vibrations by a sounding board, as in the piano (seventh or ninth node struck out); or by a sounding head made of elastic membrane, as in the banjo; or by a surface of the instrument constituting one side of a resonance chamber in the body of the instrument, as in the violin or guitar.

(*c*) Those in which the vibrating surface is directly struck, as the membrane of a drum (if that is a musical instrument) or the rods of a xylophone. Bells and plates come under this category, while various forms of acoustic apparatus, as the phonograph and the telephone, act by means of a vibrating disk and are often used to produce or reproduce music.

(See, on the subject of this article, Hastings and Beach, *General Physics*, Chapter XXXV.)

EXPERIMENTS TO ILLUSTRATE CHAPTER IV.

Experiment No. 51, Art. 123. Sound not transmitted through a Vacuum.

Under the receiver of an air pump support an electric bell on a felt cushion or by an elastic suspension, so as not to communicate its vibrations to the plate of the apparatus, and connect through an external circuit to the battery.

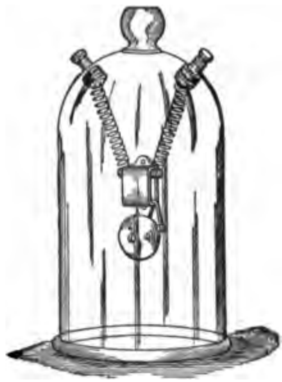


Fig. 85. Bell in a Vacuum.

As the air is exhausted from the receiver the sound of the bell becomes fainter, and when a good vacuum is reached it is nearly if not quite inaudible. As air is readmitted the sound grows louder and regains its full intensity when the air as a transmitting medium is completely restored.

Experiment No. 52, Art. 123. Sound vs. Noise.

A stick of dry pine wood 15 cm. long, 15 mm. thick, and of width equal to or greater than the thickness, if dropped upon the lecture table will give out momentarily a note of about 780 vibrations per second (g of the pianoforte scale). A number of such pieces of the same length and width may be tuned by varying the thickness, so as to produce the successive notes of the musical scale, the thickest being the highest.

With these laid out in order on the table, a simple air, the notes of which lie within the compass of the pieces, may be picked out by simply dropping the pieces upon the table. Any one of these notes separately may be characterized as sound, but if the pieces are all gathered in the hand and let fall together on the table the result is noise.

Experiment No. 53, Art. 124. Sensitive Flame.

Heat a piece of glass tubing of ½ inch bore, about 3 inches from one end, and draw it down to about half its diameter. When cold cut it in two at the narrowest place, and soften the narrow end in a flame until the orifice is from one and a half to two millimeters in diameter. Connect this nozzle to the gas pipe by a rubber tube and clamp it upright above the table. One or two inches above it support a piece of wire gauze with about thirty meshes to the inch. On turning on the gas at full pressure and lighting it above the gauze the flame flares and burns above the gauze noisily with a blue color. By turning the gas cock, cut down the pressure gradually. Presently the flame will burn steadily, 3 or 4 inches in height, yellow except at the base. Now increase the pressure nearly but not quite to the point of making the

flame flare. It is now sensitive and drops down on the production of any noise in its vicinity, responding most readily to the vowel sound *ah* or the pronoun *I*. The slight variation in pressure at the tip of the nozzle, due to the vibrations of the air, is sufficient to break the unstable equilibrium of the flame, causing it to burn with the quick rustling that comes with its sudden shortening. The flame will dance to a succession of staccato sounds.

Experiment No. 54, Art. 124. Movement of Sound Waves.

On a cardboard about 30 cm. in diameter a small central circle has a diameter of, say, 2 cm. If a number of points, say 16, are spaced equidistant on the circumference of this circle, and these points are taken in succession as centers from which to describe circles, each with a larger

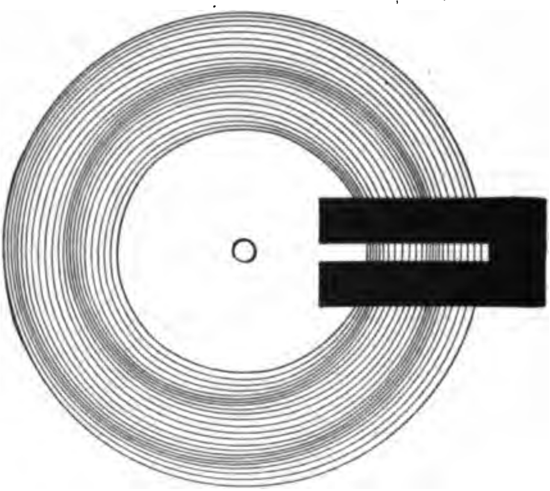


Fig. 86. Crova's Disk.

radius than the preceding one, a figure is formed having a spiral shaped crowded appearance that spreads into an expanded portion. If this disk is rotated before a slit, the portions of the curved lines seen through the slit will show a travel of condensations and rarefactions from one end of the slit to the other, like the progress of sound waves.

Similar disks may be constructed by proper shifting of centers, so as to exhibit various conditions of reflected sound waves.

When prepared on glass disks of about 12 cm. diameter, they may be projected with an ordinary projection lantern.

Experiment No. 55, Art. 128. Determination of Pitch.

If a siren disk or a toothed wheel (Fig. 87) is mounted on the axle of an electric motor or other rotator, the pitch is easily seen to rise or fall with an increase or decrease in the rate at which puffs of air pass


Fig. 87. Siren, and Savart's Wheel.

through the orifices of the disk, or taps occur on the teeth of the wheel against which a card is held lightly.

If the apparatus have a counting device to determine its rate of rotation, the actual number of vibrations per second for any recognized pitch is readily determined.

Experiment No. 56, Art. 134. Manometric Flames.

A manometric capsule is a small chamber, one part of which is separated from the other by a flexible membrane as rubber or paper. A tube leads gas into one side of this chamber, and the gas passes out through a tube that is provided with a fine pinhole tip. When the gas is admitted, and lighted at the tip, the jet should be reduced to about 3 cm. in height. To a tube leading into the other side of the chamber is attached a funnel like a speaking trumpet. On singing or speaking into this funnel, the membrane forming the partition is set vibrating and the resulting variations in the pressure upon the gas flowing through the other compartment cause a dancing of the jet of flame.

A mirror rotated before the flame shows a band of light that is broad and even when the jet is undisturbed, but which becomes serrated when a note is sounded in the funnel. The serrations are regular or irregular, deep or slight, according to the complexity of the sound.

Experiment No. 57, Art. 134. Production of Overtones.

If a heavy tuning fork is sounded by drawing a violin bow across one prong near the top it emits its fundamental almost exclusively. If overtones are present they are feeble and die out quickly. If the fork is bowed near the lower end it can be made to produce an overtone, loud and clear; by gently bowing near the top to produce the fundamental, and then bringing out the overtone, both may be heard together, with the overtone much stronger than the fundamental. When the overtone is sounding the vibrating prong has a node about one-third or one-fourth of its length from the end; this note therefore may be produced by holding the finger against the prong at this place and bowing close to the top.

If the fork is mounted in a horizontal position, with a stylus attached to one prong to make a record of the vibrations on smoked glass that is drawn along under it when the overtone and fundamental are sounded together, the tracing will show plainly the small waves of the former superposed upon the longer ones of the latter.

Experiment No. 58, Art. 137. The Kundt's Tube Experiment.

For comparing the velocity of sound in different media, the experiment of Professor Kundt with dust figures is suitable.

A glass tube (Fig. 88) a meter or more in length and about 2 cm. in diameter, is closed at one end by a stopper through which passes a rod of glass or metal about a meter long and 4 or 5 mm. in diameter, held at its middle by the stopper or a clamp. The end of the rod within the tube carries a disk of cork which fits the tube loosely enough to permit vibration. At the other end of the

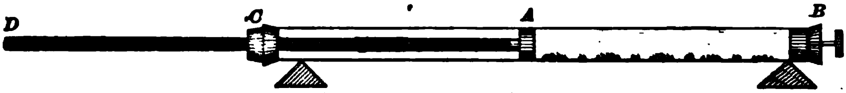


Fig. 88. Kundt's Tube.

tube is a plunger that can be moved into the tube or withdrawn, over a range of 15 or 20 cm. Scatter within the part *AB* of the tube a small quantity of cork dust or lycopodium powder; stroke the portion *CD* of the rod with a piece of rosined chamois skin so as to produce a clear sound. The disk *A* vibrates with the longitudinal vibration of the rod and sets up vibration of the air in *AB*. If *AB* is of a suitable length to divide into segments of stationary waves, it will so divide, and immediately the powder will gather into spindle-shaped segments, indicating nodes and loops, each such segment being a half wave length in the air, for the note emitted by the rod.

If on stroking the rod the dust is not thus disturbed, move the plunger *B* in or out as the rod is sounded until this result is attained. The rod itself is a half wave length in the metal for its rate of vibration; therefore the velocity of sound in the rod and in the air is as the wave lengths, or as the half wave lengths, i.e., as the length of the rod compared to the length of one segment of the air column. By measuring the distance in the tube occupied by several segments a tolerably correct average is obtained.

With rods of different materials, the velocity in each can be compared to that in air, and hence to that in any of the other rods. Also, if one rod is sounded in air, and then the tube is filled with some other gas, as illuminating gas, and the same rod is sounded in it, the velocity of sound in the two gases may be at once derived by the ratio of the segments or half wave lengths in them.

Further extension of the experiment is discussed in advanced acoustics. See Barton's *Text-Book of Sound* (The Macmillan Company).

Experiment No. 59, Arts. 138 and 139. Resonance.

To the lower end of a tube 3 or 4 cm. in diameter and about a meter in length, attach an outlet pipe with a stopcock. Starting with the tube nearly full of water, hold above it a vibrating tuning fork as in Fig. 82, Art. 139, and, opening the stopcock, let the water sink slowly. Mark the level at which the first strong resonance occurs. The *air column* in the tube above this level is the resonance column; the water has nothing to do with

the experiment except to afford an easy means of varying the length of the air column. To the length of the latter, at the first resonance, add about 0.8 times the radius of the tube; the sum makes one quarter-wave-length in air for the frequency of the fork used. This quarter-wave-length multiplied by 4, and that by the number of waves per second made by the fork, gives the velocity of sound in air at the temperature in the tube.

Continuing to let the water escape, resonance occurs again at a lower level, where the air column plus 0.8 times the radius of the tube equals three quarter-wave-lengths, from which again the velocity of sound may be computed.

A tube a meter in length will give both the first and second resonances for a fork whose frequency is 256 or higher; for one of 430 vibrations per second it will give a third resonance length.

Instead of using an efflux tube, a tall narrow jar of water may be used, into which the resonance tube dips to just such depth as makes the air column in it above the water to resound. With this arrangement a metal tube can be used instead of glass, for the height of the resonance column can be measured as well on the outside of the tube as on the inside.

Note. — The correction on account of width of the tube is slight, and in a demonstration may be neglected. It varies with the size of the tube, from 0.8484 R for a narrow tube to 0.7854 R for a wide tube. It is due to the fact that the wave front at the open end of the tube is convex outward.

Experiment No. 60, Arts. 138 and 139.

Rotate a vibrating tuning fork before the open end of its resonance box, or over the top of the air column that resounds to it, as in Fig. 89. For four positions in one rotation silence results from interference. While the fork is held in one of these positions, carefully slip a cylinder of paper or cardboard over one prong without touching the other. The pipe or box resounds to the vibration of the exposed prong; on removing the cylinder interference again causes silence.

Fig. 89. Interference of Sound.

Experiment No. 61, Art. 140. (Nos. 61 and 62 illustrate Sympathetic Vibration.)

Stretch a heavy cord, 8 or 10 meters in length, across the room, and about 2 meters from each end suspend a pendulum of about 2 meters in length, both having the same period of oscillation. Set one pendulum swinging; the other takes up the motion and is soon swinging in unison with the first.

A similar effect is produced if the two swinging bodies are bobs suspended from helical springs and oscillating vertically in the same period. The latter arrangement will work if the weights are suspended from two points of a flexible lath supported at its ends on the backs of two chairs.

Experiment No. 62, Art. 140.

If two tuning forks are exactly in unison, either will set the other vibrating. Place their resonance boxes facing each other at a distance of one or two meters apart; sound one fork and after a few seconds stop its vibrations with the finger. The other fork will be sounding.

Experiment No. 63, Art. 143. Singing Flame.

When the flame is adjusted to sensitiveness in Experiment No. 53, hold vertically above the flame, and resting on the wire gauze, a tube 25 to 30 mm. in diameter and 40 to 50 cm. in length. The flame will probably flutter and the pipe will emit a loud clear sustained note. If it does not at once do this, raise the gauze and tube together carefully a little higher over the gas nozzle; a position will presently be reached at which the flame will flutter and the pipe will sing. A metal tube is preferable, as a glass one is apt to break.

Support in a vertical position a tin pipe about 3 inches in diameter and 4 or 5 feet long (a piece of ordinary rain pipe will answer); attach a rose gas burner to a half-inch gas pipe, and that, by a rubber tube, to the gas cock at the lecture table. When the burner is lighted, insert it in the lower end of the pipe and gradually raise it. At the distance of 10 or 15 cm. from the lower end, it produces a deep roaring sound in the pipe.

Experiment No. 64, Art. 143. Resonance Pipe.

Use the siren disk of Experiment No. 55, Art. 128, and a tube of glass or metal 3 or 4 cm. in diameter and, say, 80 cm. long, open at both ends.

Support the tube perpendicularly to the plane of the disk and close to it on the opposite side from the nozzle giving the jet of air. With air blowing through the nozzle, when the disk reaches a sufficient speed the tube will resound. At the temperature of 20° C., with a tube of the above dimensions, the first resonance will occur when the siren is producing about 210 puffs per second. There is then a node in the middle of the tube, and the length of the tube is half a wave length. A second resonance will occur with double the speed of the disk and again when the first speed is trebled, and so on.

If the same pipe have the end farthest from the disk stopped, it will resound first at about 105 vibrations per second, for which the length of the pipe is one quarter-wave-length; next at three times this speed; then at five times, and so on. Other sizes of pipe may be used to vary the experiment. (See Ames and Bliss'; *Manual of Experiments in Physics*, Exp. 43.)

Experiment No. 65, Art. 144. Melde's Experiments.

Attach one end of a light braided string AB, about a meter in length, to a prong of a tuning fork (a fork that is driven electrically is preferable if available), carry the string over a light pulley B and hang a weight W at the end. If B moves freely W is the tension in the cord AB. Set the fork vibrating and adjust the weight until the string gives the appearance of two spindles, as ADB. It is then vibrating as a whole in unison with the fork.

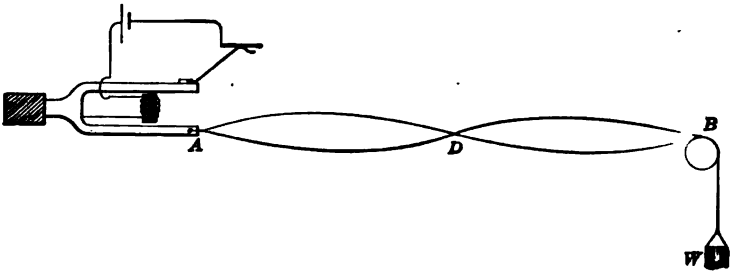


Fig. 90. Transverse Vibration of Strings, by Melde's Experiments.

In one complete vibration of the fork a disturbance travels along the string a distance of one wave length, creating the form of the heavy line ADB; if this is reflected from B it gives the light line BDA. If W is made one fourth as great the string will vibrate in four segments or two wave lengths. The wave now travels from A to B in two vibrations of the fork, or it requires twice as long to set the entire string vibrating as before; i.e., the number of vibrations of the string in a given time is only one half as great if the weight is one fourth as much as at first; if W is reduced to one ninth of its first value, there will be six spindles, or the string will make one third of a vibration in the period of the fork, and so on, the rate of vibration of the string varying as the square root of the tension.

With a given cord, if a certain length and tension give a certain number of segments, twice the length of string with same tension will give twice as many segments (i.e., half as frequent vibration of the whole string), three times the length of string will give three times as many segments, or one third as many vibrations of the string, and so on, the rate of vibration of the string being inversely proportional to the length.

Again, if a vibration with a definite number of segments be obtained for a length l of thin cord or fine wire with a definite tension T and definite mass per unit length, then with another cord of the same length and tension, but with a mass per unit length four times as great, it will vibrate just half as fast, i.e., give double the number of segments, and a cord of nine times the mass per unit length will vibrate one third as fast, or show three times as many segments, and so on, showing that the rate of vibration of the cord is

inversely as the square root of the mass per unit length. This is illustrated by beginning with a light cord of single strand, then using four strands of same length and material, then nine.

For the case where the motion of the fork is in the direction of the length of the string instead of transverse to it, see more advanced works on Physics or Acoustics.

Experiment No. 66, Art. 145. Beats.

Sound two forks that have the same frequency. Place a little wax on the prong of one fork, thus reducing its rate of vibration. When both forks are sounding, now, there will be a slow alternation of swelling and diminishing sound; on adding more wax, these alternations become more rapid.

Two pipes in unison may be made to produce beats by partially covering the open end of either one.

A conductor's whistle is often made of two small pipes of slightly different pitch. When sounded together they give beats so rapid as to produce a tremolo, as if a ball or some obstruction in the pipe had caused the interruptions in the sound.

CHAPTER V.

POTENTIAL; MAGNETISM; ELECTRICITY.

148. Theory of Potential. — Under the general terms Potential and Field of Force are to be developed several special ideas underlying the study of energy as it is manifested in the various branches of Physics. The presentation of elementary principles in the following eight articles is adapted in large part from Cumming's *Theory of Electricity*.

149. Field of Force. — Near any material system, if we try to move a mass of matter from one position to another, the movement is either resisted and work has to be done in moving the mass, or if we move it in the opposite direction a force assists the movement and would, if the mass were free to obey the force without friction, generate in it kinetic energy or do work on it during the fall. To express this condition in any space the term "Field of Force" is used. Defined thus: Field of force is any bounded or unbounded region in which any two points A, B being taken, work has to be done to move a mass from A to B, while kinetic energy is generated if the mass be allowed to fall without friction from B to A.

In either case the numerical measure of the result is the same. The field of force is specially characterized by the form of energy in consequence of whose manifestation the field of force exists; for example, it may be gravitational, thermal, magnetic or electrical, in which the amounts of work or kinetic energy indicating the fields of force will depend upon different relations; but there are several fundamental conceptions which are general and which apply alike to fields of force whatever may be the form of energy concerned.

150. Line of Force. — At any point in a field of force there is a definite direction of the resultant force at that point, along which a mass of matter left to itself will tend to fall. By

choosing points near enough together, such that the line joining each two consecutive points shows the direction of the resultant force at a point on that line, we shall have a broken line through the field so that its direction at every point shows the direction of the resultant force near that point. If the points be taken close enough together this broken line becomes a continuous curve, such that the tangent at every point shows the direction of the resultant force at that point. This curved line is then a line of force; hence the definition: Line of force is a line in a field of force such that the tangent to the line at any point shows the direction of the resultant force in the field at that point.

One line of force passes through every point in the field, and lines of force cannot intersect at any point where there is one resultant force, for if they could there would be at the point of intersection more than one direction of the resultant force.

151. Strength of Field. — The magnitude of the force by which a mass of matter is urged along a line of force depends jointly on the field or system of force and on the quantity of matter. To compare force at different points in the field, we should place at those points a unit of mass and find the force it experiences. In the c.g.s. system the unit of mass is the matter in one gram.

Strength of field at a point is the magnitude of the force (number of units of force) experienced by a unit of mass when placed at that point in the field of force. If the systems of bodies and forces are of the kind to which Newton's laws of motion apply, the strength of field is clearly the force per unit mass, and this is numerically equal to the acceleration which a body would acquire in the field, since Force = Mass $\times$ Acceleration. Thus the strength of the earth's gravitational field at the surface of the earth is 980 dynes, since a gram experiences 980 units of force as it undergoes an acceleration of 980 cm./sec.^{2} (see Art. 28). When we know at every point in a field of force the strength of field and the direction of the line of force, our knowledge of that field of force is complete.

152. Potential. — Our study is simplified if we can express both the direction of the line of force and the strength of field in terms of one quantity. The name given to the quantity expressing the condition of a field of force in these two respects is "potential," and potential is studied by comparison, referring to some arbitrary level for zero, or by directly examining difference of potential at any two points. This is somewhat analogous to observing the difference of temperature between two bodies without regard to the absolute temperature or the actual amount of heat of either body.

By the potential at a point is to be understood the amount of work that would have to be done to bring a unit body (unit mass if the force is gravitational) from an infinite distance, or from without the field of force, to this point, against the forces of the field.

153. Difference of Potential. — If any two points, *A*, *B* (Fig. 91), be taken in a field of force, and a unit of matter be carried from *A* to *B* against the force in the field, a certain amount of work will be done on the unit, and if the unit of mass be allowed to pass back from *B* to *A* by frictionless constraint, the particle will acquire an equal amount of energy in its fall. The principle of the Conservation of Energy shows that the amounts of work or energy will be the same, whatever path be pursued from *A* to *B* or from *B* to *A*, respectively; for if more energy were acquired in falling along a path *BCA* than along another path *BDA*, then by allowing the particle constantly to fall along *BCA*, and to return along *ADB*, we should have unlimited source of energy.

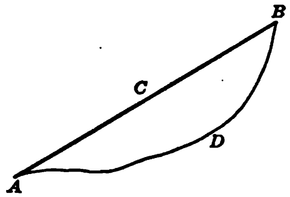


Fig. 91.

The distribution of energy in a field of force is for the most part studied by examining not the absolute potential at any point, but the difference of potential between two points.

Difference of potential at any two points is the work done in carrying a unit of mass from one point to the other, and is

independent of the path traversed, depending only on the position of the two points in the field.

Zero potential is the potential at a certain point chosen as a standard of reference; any point which requires work to be done to bring the unit of mass from the zero point to it will have positive potential, and any place which requires work to be done to bring the unit of mass from it to the zero point will have negative potential.

Let A and B be two points in a field of force, and let F be the average force along AB. Let V be the difference of potential between A and B, then $V = F \times AB$, or $F = \frac{V}{AB}$. But if V is the total difference of potential between A and B, then $\frac{V}{AB}$ expresses the space rate of change (i.e., change per unit of distance) in potential from A to B. Hence the average force along any line is given by the average rate at which the potential changes along the line, and if the line be made *very short* we may say that the force at a given point in a given direction is the rate of change of potential at that point and in that direction. Since the resultant force at a point is in the direction of the line of force, the force along any direction at an angle with the line of force less than a right angle will be less than the resultant force: it follows that the potential changes most rapidly along the line of force, and less and less rapidly in directions more and more inclined to the line of force, while in a direction at right angles to the line of force the rate of change of potential must vanish.

154. **Equipotential Surface.**—If, then, a surface be drawn through the field of force which everywhere cuts at right angles lines of force, the rate of change of potential along such surface will be zero, or the surface will be an equipotential surface, and the force resolved along it will always vanish, so that no work is done in moving matter along such a surface. Due to the earth's gravitation, the surface of the sea is an equipotential surface, and so far as the effect of gravity is concerned no work is required to move a body along the surface of water.

An equipotential surface, then, is a surface drawn through all points in the field at which the potential is the same; it will everywhere cut lines of force at right angles.

155. **Measure of the Potential at a Point.**— The actual work involved in transferring a body from one point of a field of force to another, or to bring a body to a given point in the field, will depend upon the law of force that exists between the bodies or things concerned, and in consequence of which the field of force exists. In a gravitational field the force between bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, and a similar law holds in connection with magnetism and with electricity. This law, therefore, is the one we are to take into consideration.

The mass upon which work is to be done in transferring it is unity, and if a mass m is at a point O (Fig. 92), the potential due to m, at any point whose distance from O is r, equals $\frac{m}{r}$. Without the calculus this may be demonstrated as follows (see Cumming's *Theory of Electricity*, Art. 39, Chapter II, Theory of Potential).

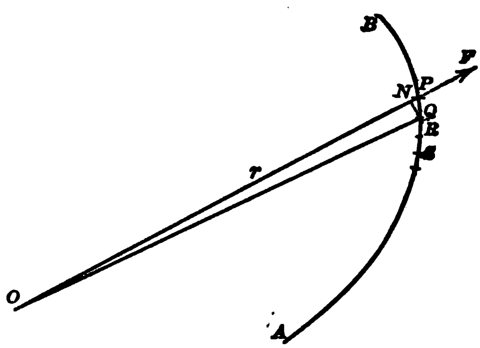


Fig. 92. Difference of Potential between Two Points.

"To find the work done in carrying a gram against the attraction" (or repulsion) "of any system of particles from one point to any other point, or to find the difference of potential between two given points." The mass m is at O, B and A are two given points, and PQ is a very small element of the path from B to A. Draw QN perpendicular to OP. The mean attraction (or repulsion) on a gram between P and Q is $\frac{m}{(OP) \cdot (OQ)}$, a mean pro-

portional (not arithmetical mean) between $\frac{m}{OP^2}$ and $\frac{m}{OQ^2}$. This force, resolved along PQ, is $\left(\frac{m}{OP \cdot OQ}\right) \cos OPQ$, or $\frac{m}{OP \cdot OQ} \times \frac{PN}{PQ}$, and the work of carrying the gram from P to Q is

$$\frac{m}{OP \cdot OQ} \cdot \frac{PN}{PQ} \cdot PQ = \frac{m \cdot PN}{OP \cdot OQ} = \frac{m(OP - OQ)}{OP \cdot OQ}$$

$$= m\left(\frac{1}{OQ} - \frac{1}{OP}\right).$$

Similarly, the work to carry the gram from Q to R is

$$m\left(\frac{1}{OR} - \frac{1}{OQ}\right),$$

and from R to S it is

$$m\left(\frac{1}{OS} - \frac{1}{OR}\right),$$

so that from P to S it is

$$m\left[\left(\frac{1}{OQ} - \frac{1}{OP}\right) + \left(\frac{1}{OR} - \frac{1}{OQ}\right) + \left(\frac{1}{OS} - \frac{1}{OR}\right)\right],$$

or

$$m\left(\frac{1}{OS} - \frac{1}{OP}\right),$$

and so from B to A it is

$$m\left(\frac{1}{OA} - \frac{1}{OB}\right).$$

If there are other masses m_1, m_2, etc., at points O_1, O_2, etc., a similar expression represents the work of transferring the unit mass for each of these, and the total work to take a gram from P to Q is $\sum\left[m\left(\frac{1}{OQ} - \frac{1}{OP}\right)\right]$ and between B and A it is $\sum\left[m\left(\frac{1}{OA} - \frac{1}{OB}\right)\right]$, which is independent of the path between B and A. This is the potential difference between A and B. If B is at an infinite distance, $\frac{1}{OB}$ is zero, and $\sum\left(m\frac{1}{OA}\right)$ becomes the potential at A. Similarly, $\sum\left(m\frac{1}{OB}\right)$ is the potential at B.

A neater and simpler demonstration is afforded by the calculus.

Let q be the quantity at O (whether matter, electricity or magnetism), and r the distance from O to the unit quantity that is to be transferred. Then the force upon the unit quantity is $\frac{q}{r^2}$; at P, $F = \frac{q}{OP^2}$, in the direction OP; the component along the path of motion is F multiplied by the cos. of the angle which r makes with the path at the given point; i.e., $F \times \frac{dr}{ds}$, or $\frac{q}{r^2}\frac{dr}{ds}$. The element of work performed in carrying the unit through the distance ds is

$$dW = \frac{q}{r^2}\frac{dr}{ds}ds = q\frac{dr}{r^2}.$$

The difference of potential between A and B is the work of carrying the unit from B to A and is the integral of dW between the limits $r = OB$ and $r = OA$. Designating potential by V we have

$$V_B{}^A = W = \int_{r=OA}^{r=OB} q\frac{dr}{r^2} = \frac{q}{OA} - \frac{q}{OB}$$

and is independent of the path. If B is at an infinite distance from O then $\frac{q}{OA}$ is the potential at A due to q at O; or, in general,

$$V = \frac{q}{r}.$$

156. Application of Potential to Magnetism.—The definitions and propositions thus far presented concerning potential and fields of force are applicable to any system of bodies to which Newton's laws of motion are applicable and in which the law of force is that of the inverse square of the distance. We may proceed to apply them to fields of magnetic force and see what the definitions become and what the principles indicate.

Phenomena.—A magnet attracts iron. Like poles of magnets repel and unlike attract each other. Try two magnets upon a third which is pivoted or suspended, to find which are "like poles," then verify the statement by trying these two upon each other. Usually poles are marked N. and S. and the magnetism of N. is + and that of S. is −; N. is used for the pole which, if the magnet were suspended, would point to the north. In France it is customary to call this pole the south pole, S., and in England and America it is not unusual to call it the *north-seeking* pole.

The strength of a pole can be expressed by the force which it exerts upon another pole, and the unit in terms of which it can be measured is defined thus:

A unit magnetic pole is a pole which exerts upon an equal pole unit force at unit distance. In c.g.s. units this is a force of one dyne at a distance of one centimeter.

Moment of a magnet is the product of the strength of either pole by the distance between the poles.

Force between Two Poles. — The force of attraction or repulsion between two magnetic poles is directly proportional to the product of the strengths of the poles and inversely proportional to the square of the distance between them. This law is deduced from experiment. With the above definition of unit pole, calling the strength of two poles m and m', and the distance between them r, the expression for the force becomes force $= \frac{mm'}{r^2}$. This force is repulsive if the numerator is plus and attractive if minus.

Under the influence of magnetic poles, then, a field of magnetic force exists whose general laws are identical with those of gravitational force. The only change required is to read "unit pole," or particle charged with unit of positive magnetism, in place of the unit of mass there employed to test the field. So our previous definitions will read:

Field of magnetic force is the region surrounding magnetized bodies, within which work has to be done to move a magnetic pole.

Lines of force are lines in the field such that the tangent at each point shows the direction in which a magnetic pole placed there would be urged.

Strength of field at a point, or magnetic force at a point, is the force with which a unit magnetic pole would be urged if placed at that point.

Magnetic potential at a point is the work which would be done in bringing a unit magnetic pole to that point from an infinite distance or from a point outside the field of force. If there be a dis-

tribution of magnetism consisting of quantities $m_1, m_2, \ldots m_n$ at distances $r_1, r_2, \ldots r_n$ from the given point, the measure of the magnetic potential at the point will be

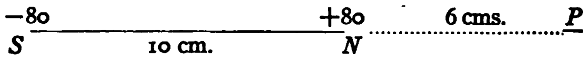
$$\frac{m_1}{r_1} + \frac{m_2}{r_2} + \cdots + \frac{m_n}{r_n},$$

or

$$\sum\left(\frac{m}{r}\right).$$

EXAMPLES. —

1. "Find the magnetic potential due to a bar magnet 10 cm. long, and of strength 80, at a point P, lying in a line with the magnet poles and 6 cm. distant from its north-seeking end."



Potential at $P = \frac{80}{6} - \frac{80}{16} = 8.33$.

2. "A N.-seeking and a S.-seeking pole, whose strengths are respectively $+120$ and -60, are in a plane at a distance of 6 cm. apart. Find the point between them where the potential is 0, and through this point draw the curve of zero potential in the plane."

In Fig. 93, let the poles N and S be at A and B respectively. $AB = 6$ cm. Let C be the point where the potential equals zero. Then

$$\frac{120}{AC} - \frac{60}{BC} = 0; \text{ or } \frac{120}{AC} = \frac{60}{6 - AC}.$$

Whence $AC = 4$ and $BC = 2$.

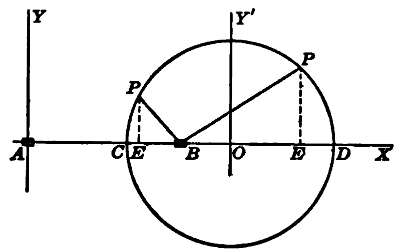


Fig. 93.

For the locus of the point P so that its potential shall everywhere be 0, taking origin at A, and axes X and Y as in the figure, at every position of P we must have $\frac{120}{AP} = \frac{60}{BP}$;

or

$$\frac{120}{\sqrt{x^2 + y^2}} = \frac{60}{\sqrt{(6 - x)^2 + y^2}},$$

whence $x^2 - 16x + y^2 = -48$, the equation of a circle. For the points where it cuts the axis of X, $y = 0$, and $x = 4$ or 12, locating the points C and D; therefore $BD = 6$.

If the origin be transferred to O, midway between C and D, make $x = x' + 8$ and $y = y'$, and substitute in Eq. of locus; it becomes

$x'^2 + y'^2 = 16$, a circle whose center is at O and whose radius is 4.

Experiment No. 67, page 296. — Exhibit lantern slide illustrations of magnetic lines of force; also form and project the lines.

Experiment No. 68, page 296. — To show that the force between two given poles varies inversely as the square of the distance between them.

EXAMPLES. —

1. A magnetic pole of strength 22 is placed in a magnetic field of strength 0.68. What is the force experienced by this pole? *Ans.* 14.96 dynes.

2. What is the strength of a magnetic pole that is urged with a force of 150 dynes when placed in a magnetic field whose is intensity is 2.5?
Ans. 60 c.g.s. units.

3. A and B are successive corners of a regular hexagon whose sides are 50 cm. in length. A has potential $+60$, and B, -40, required the work to move q units from B to A by the shortest path, and also by going along the perimeter of the hexagon. (See Art. 153.) *Ans.* $100\,q$ in either case.

4. A magnetic pole of 140 units' strength is placed at a distance of 15 cm. from a like pole of 30 units' strength. What is the force between them?
Ans. A repulsion of 18.67 dynes.

5. A plus pole of 60 units' strength is placed 5 cm. from a minus pole of 172 units' strength. What force is exerted upon the latter by the former? What upon the former by the latter?
Ans. An attraction of 412.8 dynes.

157. Physical Interpretation of Lines of Force. — Lines of force being defined, as above, by direction simply, and strength of field at any point being defined with reference to the force upon a unit pole at that point, the character of the field can be understood and treated mathematically; but Faraday combined the idea of direction and magnitude of force in one, by giving to lines of force a physical meaning as if they were actual strings stretched to a definite pull of unit force (in c.g.s. units a force of one dyne) and distributed throughout the field in such number as to represent the strength of the field. In a uniform field these lines are distributed uniformly, and in field of varying strength they are packed closely at some places and sparsely at others. In a uniform field of unit strength there is one line per square centimeter of area perpendicular to the line; in a uniform field of ten units' strength there are ten of these dyne lines piercing every square centimeter, and more of these lines can be called into being or put out by any agency of a nature that can vary the strength of a magnetic field.

This mode of depicting a field of force is useful and is applied not only to magnetic but also to electric fields of force, and is further elaborated by the idea of " tubes of force " comprising within them groups of lines, the tubes themselves occupying the entire field instead of leaving gaps as is necessarily done when only the lines are thought of (see *infra*, Art. 168).

158. **Magnetic Substances.** — There are bodies which ordinarily are not magnets but which, under certain influences, may become such, and which may again lose or be deprived of their magnetic character. Substances of which such bodies consist are called magnetic substances. The chief are iron, steel and nickel.

159. **Magnetization by Induction.** — The simplest way to magnetize a body is to place it in contact or to stroke it with a magnet, but this is not the way in which bodies are usually magnetized. Magnetic substances are always magnetized when brought into a magnetic field, without being in contact with any other bodies, and they are then said to be *magnetized by induction*, the extent to which they are magnetized depending upon some molecular quality of their own and upon the strength of the field. A body thus magnetized is called a temporary magnet if it loses its magnetism when removed from the field; a permanent, if it retains its magnetism. Soft iron makes a temporary magnet, hard steel a permanent one. Various magnetic fields may be produced by various arrangements of magnets, and may be partially exhibited by means of iron filings.

The extent to which magnetic substances are affected by magnetic fields of force, and the relation of strength of field to magnetization produced are known by technical terms such as susceptibility, permeability, intensity of magnetization, induction, etc., the consideration of which is deferred until we come to consider magnets and magnetic fields of force produced by electric currents. The direction of a line of force being the direction in which a N.-seeking pole would be urged, the lines are regarded as pointing out from the N. pole of a magnet, and into the S. end. A piece of iron, then, placed in the vicinity of a magnet, will have a S. pole induced nearest the N. end of the

magnet and a N. pole farthest from the N. end of the inducing magnet, and if placed in any magnetic field the direction from its S. to its N. pole will be that of the lines of force in the part of the field in which the iron is placed.

The directive action everywhere exerted upon a magnetic needle shows that there is a magnetic field about the earth, called the earth's magnetic field, and an iron bar becomes magnetized by virtue of its mere presence in this field. If placed in a north and south direction the end towards the north acquires N.-seeking polarity, the other, S.-seeking. The lines of force in the earth's field, however, are not horizontal but incline downward at an angle with the horizontal known as the inclination or dip of the needle.

Experiment No. 69, page 296. — Magnetization by induction in earth's field; measure inclination.

EXAMPLE, p. 904, Watson's *Physics*.

"A magnet is placed horizontally in the magnetic meridian due south of a compass needle. How will its action on the latter be affected if (1) a plate of soft iron is interposed between the two? (2) a rod of soft iron is placed along the line which joins their centers?"

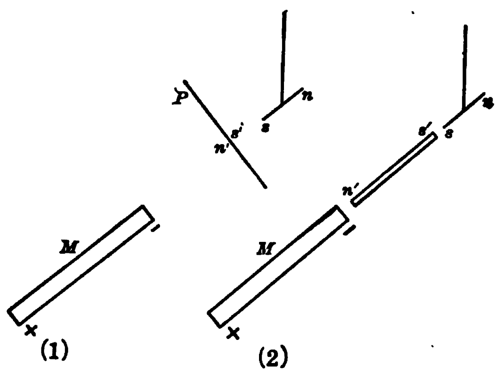


Fig. 94.

In Fig. 94 (1), *ns* is the needle, and it is seen that if the plate *P* is absent *s* is repelled by the magnet *M*. When the thin plate *P* is interposed *M* induces polarity in *P*, but the two faces *n'*, *s'* are so near each other that the effect of either upon *s* is counteracted by the other, so that the plate shields *ns* from *M*. In (2) *M* induces + and − poles *n'* and *s'* in the bar, but they are so far apart that *s'* exerts a strong repulsion upon *s*, and *ns* is deflected more than by *M* alone.

160. Terrestrial Magnetism. — Besides the inclination or dip which we have just considered, there are other elements connected with the magnetic field of the earth, as declination, or, as

it is sometimes called, variation of the compass, and the changes which take place in these angular positions of the magnetic needle as well as the actual strength of field. (See Watson, Arts. 432, 434. For theory of the earth's magnetism, see Barker, p. 605, and Hastings and Beach, Art. 325.)

ELECTRICITY.

161. Static Electricity; Electrification; Ether Stress. — Summarize elementary ideas. Friction between any two heterogeneous substances puts the bodies into two opposite states or conditions called electrification. When a body is in this particular condition it is said to be charged with electricity or to contain electricity. Always two electrifications are produced, equal in amount. The electrifications are of an opposite character and tend to neutralize each other upon a conductor, i.e., a body which permits such readjustment; but a body which does not so permit adjustment upon it is a nonconductor, and a substance of such material, when separating two conductors, is called a "dielectric."

The fundamental phenomena of electricity are attraction and repulsion.

Electrification is either of the sort produced upon glass by rubbing it with silk, and is called vitreous or positive, or it is of the sort produced upon sealing wax or hard rubber by rubbing it with fur or wool, and is called resinous or negative. In these cases, the silk becomes negatively electrified and the fur positively. Each is a nonconductor, and so is dry air, and if the silk and the glass thus electrified are suspended each by a silk cord and separated by air or any other dielectric, they will attract each other by some action transmitted through the dielectric. The action of an electric charge through a dielectric varies with different media, and so the seat of the action is thought to be the ether itself, its facility of action being determined by its association with one form or another of gross matter.

Bodies similarly electrified repel one another, those of unlike electrification attract. When such attraction or repulsion exists

between two bodies not in contact, the ether transmitting the force is said to sustain a stress and to be itself strained, like the spring between the bodies A and B in Fig. 1, Art. 16. The effect of this strain is felt by the material occupying the space between the bodies, and may be severe enough to shatter the dielectric.

But a body electrified either positively or negatively will attract to it light bodies that are apparently unelectrified, as will be explained under Electrification by Induction.

Illustrations. — Paper cylinder rolled along table by electrified sealing wax; pieces of thin paper drawn between sleeve and waist of the coat will adhere to the wall, etc.

As the electrification of a body may proceed to a greater or smaller extent it is said to be charged with a greater or smaller quantity of electricity.

Unit quantity of electricity is such a quantity as will repel an equal quantity of the same kind at a unit distance in air with a unit force; in c.g.s. units, at a distance of one centimeter with a force of one dyne.

The law of force with electric charges is similar to that of magnetism and of gravity, viz.: Force is proportional to the product of the quantities and inversely proportional to the square of the distance, so that with the above definition of unit quantity,

$$F = \frac{q_1 q_2}{d^2}.$$

162. Electric Field of Force; Electric Potential. — An electric field of force is a region in which work has to be done to move a quantity of electricity from one point to another. The electric difference of potential between two points is the work that must be done to transfer a unit of electricity from one of the two points to the other, and the potential at a point is the work that would have to be done in bringing a unit of positive electricity to that point from an infinite distance or from a point without the field of force. There may be a potential at a point whether there is a body there or not, and whether there is electricity there or not.

Since the law of force is like that for magnetism, the expression for the potential at a point at a distance r from a quantity q is $\frac{q}{r}$, and for that due to various quantities at various distances is

$$\Sigma\left(\frac{q}{r}\right).$$

For instance, if charges of 12, 40 and 20 units be placed at the corners A, B, C, of a square $ABCD$ whose side is 50 cm., to calculate the value of the potential V at the point D.

$$V = \frac{12}{50} + \frac{40}{50\sqrt{2}} + \frac{20}{50} = 1.2.$$

Similarly at E, the intersection of the diagonals, the potential $V' = 2.04$. The difference of potential between E and D is $2.04 - 1.2$, or 0.84 erg, and this is the amount of work that would be required to carry a unit of positive electricity from D to E.

EXAMPLES. —

1. Two small bodies are charged respectively with 50 and 75 units of positive electricity. What is the force between them when they are 20 cm. apart? *Ans.* 9.375 dynes, repulsion.

2. If a body having 200 units of positive charge of electricity is attracted by another charged body with a force of 50 dynes at a distance of 16 cm., what is the charge upon the second body? *Ans.* 64 units, negative.

3. How much work is required to carry a charge of 250 units of electricity (*a*) from a place where the potential is 30 to another where it is 80, (*b*) from a place where it is -60 to another where it is $+200$?
Ans. (*a*) 12,500 ergs; (*b*) 65,000 ergs.

4. Charges of 5 units of electricity are placed at each of the four corners of a square whose side is 12 cm. What is the electric potential at the point of intersection of the diagonals? *Ans.* 2.353.

163. Capacity for Electricity. — The potential of the earth is usually assumed arbitrarily as zero. Upon a conductor electricity passes from a point of high to a point of low potential, or distributes itself until the conductor is everywhere at the same potential. A body put to earth comes at once to zero potential, or loses its charge by sharing it with the whole earth. An

insulated conductor rises in potential with an increase of charge, the amount needed to raise its potential by any definite amount depending on the shape and size of the body and its situation relatively to other charges of electricity. By the electrical capacity of a body is meant the quantity of electricity required to change its potential by one unit. (Compare with capacity for heat.)

164. Electrification by Induction. — If AB (Fig. 95) is an insulated conductor and C a body charged with, say, positive electricity, the conductor AB is found to be charged negatively at the end next the positively charged body C and positively at the farther end, as if the + charge on C had attracted a − charge to A and repelled a corresponding + charge to B. With a given quantity on C, and at a certain position of C, this separation on AB is only carried to such an extent as to bring the whole conductor AB to the same potential, but the + charge is repelled to the utmost limit of AB and only stops at B because that is the end of the conductor. If B be connected to the earth by simply touching B with the finger, AB becomes continuous with the earth and the so-called free + electricity is discharged to the earth while the minus remains bound at A by the presence of C. If now, after breaking connection with the earth, C and AB be separated from each other a considerable distance, the minus charge will distribute itself over the whole of AB and this body is then said to be charged by induction, or the charge upon it is an induced charge. If C were originally charged negatively the induced charge on AB would be positive.

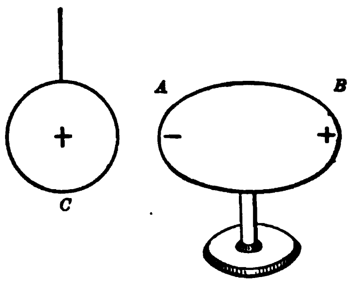


Fig. 95. Induced Electrification.

Observe that although one end of AB has a positive charge, and the other a negative, due to the presence of C, AB will be everywhere at the same potential, which will be higher than if the charge on C were not present.

Experiment No. 70, page 297. — This is a suitable method of charging an electroscope. The inducing body may be a rod of glass or wax held in the hand, for the charge on the end will not be discharged since the rod is a non-conductor.

Experiment No. 71, page 298. — Electrification of water jet.

Experiment No. 72, page 298. — Electrophorus.

165. Electrophorus. — The charging and discharging of the cover of the electrophorus is explained in elementary books, and also in Experiment No. 72, but why the recharging of the cover from the disc can go on indefinitely, thus affording apparently an inexhaustible supply of energy without exhausting the charge on the disc of wax, is not always understood. When the free charge of the upper surface of the cover has been discharged to earth, a definite attraction exists between the disc and its cover. The removal of the cover does not alter the charge upon either, but work must be done to separate them (apart from the work of lifting against gravity) and when they are separated they possess energy of electrical separation equivalent to the work required thus to separate them. The discharge of the spark is the dissipation of this energy, but it is renewed in the next charge, not at the expense of the electricity on the disc, but by the mechanical work done in separating the cover from the disc. If the cover were not first put to earth, no work would have to be done against electrical forces to separate the bodies and the cover would possess no electrical energy.

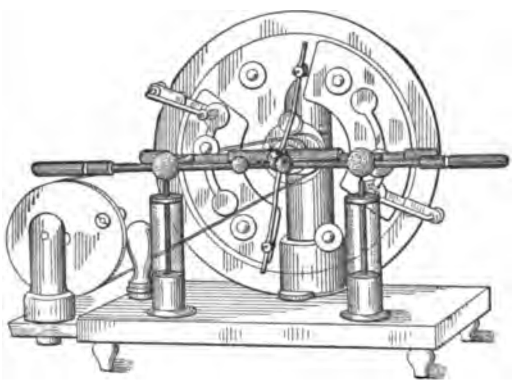


Fig. 96. Toepler-Holtz Electrical Machine.

Replacing the discharged cover upon the charged disc and repeating the transfer of electrification to another body with the electrophorus is an intermittent operation, but by suitable mechanical contrivances the operation may

be made continuous, and we then have an electrical machine. The most approved are the Toepler-Holtz, and the Wimshurst machines, for the description and explanation of which see larger treatises on Electricity.

(Exhibit electrical machines, electrical gas lighter, etc.)

166. Condensers. — Suppose a body be charged positively to a definite potential. This will require a definite quantity of electricity, and means that a definite amount of work is needed to bring a unit of positive electricity up to the body. If, however, a body charged negatively be put near the first body, it is evident that the work of bringing a positive unit to the first body will be less than before, or the potential of that body is lowered by the proximity of the opposite electrical charge. To raise the first body to the same potential as before will now require a larger charge upon it. In Fig. 95 if C is the body with positive charge and AB is brought near it, by putting B to earth the minus charge *induced* at A thus lowers the potential at C, and C must now have a larger charge to restore it to its former potential. Such an arrangement, by which a large quantity of electricity is necessary to produce a small rise of potential, is called a condenser. The actual quantity needed to raise the potential one unit is its capacity, and this depends upon the extent of surfaces opposite to each other, the distance between the surfaces and the nature of the dielectric between them. A common form is two sheets of tin foil with a glass plate between them. The thinner the plate of glass and the larger the sheet of metal the greater the capacity of the apparatus.

Leyden Jar. — If such a plate condenser could be bent into the form of a deep bowl or jar it becomes the apparatus commonly known as a Leyden jar. A condenser is charged by means of an electrical machine by putting one plate to one pole of the machine and the other plate to the other pole or to the earth, — better put both the other plate and other pole to earth. With the Leyden jar, merely holding the external coating in the hand is putting it to earth, and the other coating is brought to either pole of the machine by the knob which is in metallic connection with the

inside coat of tin foil. Care must be taken by the operator not to touch the two coatings of a charged condenser at the same time. For discharging use a discharger, — a bent wire with a knob at each end and an insulating handle in the middle.

Any arrangement of two conductors with a dielectric between them is a condenser in fact; the lecture room itself is such, the surface of any insulated body in it being one condenser surface, and that of the walls and objects in the room constituting the other surface. For fuller discussion of Arts. 165, 166, see Watson, Arts. 453–458 and especially Larden's *Electricity*, pp. 150–152.

Illustrations. — Showing charging and discharging of condensers and Leyden jars; slow discharge; residual charge; human Leyden jar; jar with removable coatings; effects of disruptive discharge, etc.

167. **Energy of Charge.** — If the quantity Q has been applied to a condenser, say a Leyden jar, to raise its potential from zero to V, the work required for this would be $\frac{1}{2} QV$, for if Q units were all raised to the potential V, the work would be QV, but if the first is raised zero, and the last to V, it is the same as if all were raised to the potential $\frac{1}{2} V$, and the work is Q times $\frac{1}{2} V$; and this would be the energy of its discharge. If such a jar, however, were put in connection externally with an equal, uncharged jar, and then its charge were shared with this second jar by connecting the knobs of the internal coatings, a single condenser would be formed of double the capacity of either of the jars. The same quantity of electricity would charge this condenser of double capacity to only half the former potential, or the potential of the large condenser would be $\frac{1}{2} V$, and the energy would be $\frac{1}{2} (Q \times \frac{1}{2} V)$, or $\frac{1}{4} QV$; that is, only one-half as great as before. With no loss of electricity one-half the energy has been lost. This loss of energy was the energy of the spark and noise when the two jars were connected.

For "sparking distance" see *Electrical World*, Dec. 10, 1904. Below 10 cm. it varies with form and nature of electrode; for 10 cm. to 40 cm. the relation of voltage V to sparking distance d is given by the equation $V = 4800 d + 24{,}000$, where d is cm. and V is the maximum difference of potential in volts. At the distance of 10 cm. this gives 72,000 volts, and for 40 cm. 216,000 volts before a spark will pass: from $1\frac{1}{2}$ to 2 mm per kilovolt. *Vide infra* Art. 221.

168. Lines of Force; Tubes of Force. — Arts. 168 to 174 inclusive may be omitted on first reading. In electricity as in magnetism we may represent strength of field by employing one line of force to the square centimeter in a field of unit strength, and with tubes of force a unit tube would be one whose cross section is of such area as corresponds to unit force. If there were one unit line of force to the square centimeter, then a unit tube would have a cross section of one square centimeter; if the field of force had a strength of ten units, there would be ten unit lines of force to the square centimeter, and a unit tube of force would have a sectional area of $\frac{1}{10}$ sq. cm.

But a different method has been adopted. If A and B (Fig. 97) are two electric conductors charged with equal quantities of electricity, A positively and B negatively, all the lines of force (not meaning unit lines) from A inclosing an area on which is a charge of *one unit of electricity* is called a unit tube of force, and this tube, extending to and terminating in the surface B, there incloses an area upon which is one unit of negative electricity. The unit line of force, in this view, is simply the axis line of the unit tube. Evidently there are as many such tubes or lines from any portion of a surface as there are units of electricity on that portion of surface, or as many to the square centimeter as there are units of electricity to the square centimeter. This latter quantity, the ratio of the electric charge to the area over which it is distributed, is called the surface density of the electrification; hence the surface density may be represented by the number of unit lines or the number of tubes of force that proceed from or to the electrified body per unit area. (See further, Watson, pp. 624–634.)

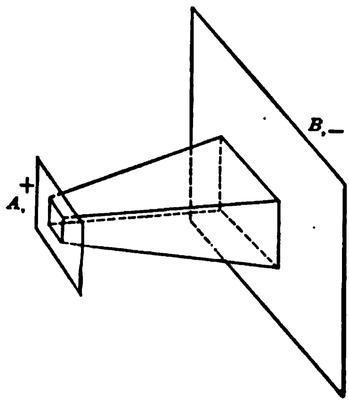


Fig. 97.

Given a small sphere uniformly charged with q units of elec-

tricity, if this sphere is remote from other charged bodies it will have q tubes of force radiating from it and filling the space around it. A spherical surface surrounding this small sphere at a distance r will have $4\pi r^2$ units of area, comprising the ends of q tubes of force; therefore the number of tubes per unit of area will be $\frac{q}{4\pi r^2}$. If the small charged sphere is extremely small we may regard its charge as all at the center of the large sphere, and the force due to it at a distance r is $\frac{q}{r^2}$; this, too, is the strength of field at the distance r, that is, at the surface of the large sphere. Thus, since the number of tubes per square centimeter of this surface is $\frac{q}{4\pi r^2}$ and the strength of field is $\frac{q}{r^2}$, the value of $\frac{q}{r^2}$ may be written $\frac{q}{4\pi r^2} \times 4\pi$. Also if the area of cross section of a tube is s square centimeters, the number of tubes per square centimeter is $\frac{1}{s}$; writing F for the strength of field, or the force at distance r from the small charged sphere, the expression

$$\frac{q}{r^2} = \frac{q}{4\pi r^2} \times 4\pi$$

becomes $$F = \frac{1}{s} \cdot 4\pi, \quad \text{or} \quad Fs = 4\pi = \text{constant}.$$

The cross section of the tube is supposed to be perpendicular to the line of force or on an equipotential surface.

169. **Tubes of Induction.** — The product of the electrical force into the area of a surface perpendicular to the direction of the force is called the "electrical induction" through that surface; and we see that the induction through a normal cross section of a tube of force is constant and equals 4π. If we call a unit tube of induction one in which the induction is unity, the unit tube of force equals 4π unit tubes of induction. Thus on each square centimeter of the surface of a conductor which is charged to a surface density σ, there will be $4\pi\sigma$ unit tubes of induction. (Watson, Art. 458.)

170. No Electrical Force Inside a Hollow Conductor. — It may be shown theoretically and experimentally that no force is exerted upon a charged body within a hollow charged conductor. As there is no field of force in the interior of such conductor, no work is done in moving a charge about within the interior space. (For demonstration, see Watson, Art. 452.)

171. Action of a Uniformly Charged Sphere, Externally. — If we have a sphere of radius R, charged with Q units of positive electricity, and remote from other charged bodies, its lines of force are radial and its tubes of force are cones with their vertices at the center. Q tubes will cover the surface $4\pi R^2$ at a distance R from the center, and the area on the surface of the sphere intercepted by each tube is $\frac{4\pi R^2}{Q}$. If R is the radius of a bounding surface, each tube would include one unit of electricity in this surface, and the force at a point in a normal cross section of such tube, as shown in Art. 168, is $F = \frac{4\pi}{s}$,

or
$$F = 4\pi \div \frac{4\pi R^2}{Q} = \frac{Q}{R^2}.$$

This is the same as the force that would be exerted at that point if the whole charge Q were concentrated at the center of the first (or charged) sphere.

Hence the force at an external point due to a charged sphere is the same as if the charge were all at the center of the sphere.

172. Capacity of a Sphere. — If a quantity of electricity Q is at a point, the potential at a distance R due to Q is $\frac{Q}{R}$. If a sphere is charged with Q units the force exerted by this charge at the surface (and, as may be shown, the potential at the surface) is the same as if the charge Q were at the center; then the potential V of any point on the surface is $V = \frac{Q}{R}$. By the capacity is meant the quantity necessary to raise the potential of the conductor from zero to unity. In this case V can be unity only if

$Q = R$. So the capacity of a sphere is numerically equal to the radius, and unit capacity is the capacity of an isolated sphere, one centimeter in radius (about the size of an ordinary marble).

EXAMPLES. —

1. A charge of 162 units is placed on a sphere of 9 cm. radius, and an equal charge on a sphere of 1 cm. radius. What is the potential of each spherical surface, and what is the energy of each charge?

Ans. 18 and 162; 1458 ergs. and 13,122 ergs.

2. In Ex. 1, what force would be exerted by the charges on the spheres against a unit quantity at a point 1 cm. distant from the surface of the sphere? *Ans.* 1.62 dynes; 40.5 dynes.

173. Distribution of Energy in an Electric Field. — If A and B (Fig. 98) are two conductors constituting a condenser, and B is at zero potential while A has potential V due to charge Q, the total energy of the condenser is $\frac{QV}{2}$ and the number of tubes emerging from one plate and terminating on the other is Q, so that the energy in each tube is $\frac{V}{2}$. It may be shown that of this energy,

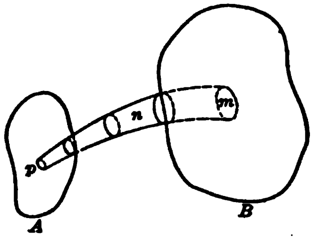


Fig. 98.

the amount per unit length of the tube at any point n is equal to one-half the force F exerted at that point. The cross section of the tube here is $\frac{4\pi}{F}$, and the volume per unit length then is $\frac{4\pi}{F}$ c.c. But the energy per unit length is $\frac{F}{2}$, therefore $\frac{4\pi}{F}$ c.c. of volume contain $\frac{F}{2}$ ergs; or the energy per cubic centimeter, where the force is F, is

$$\frac{F}{2} \div \frac{4\pi}{F}, \quad \text{or} \quad \frac{F^2}{8\pi} \text{ ergs.}$$

Again, suppose the tube to be divided by equipotential surfaces, as in the figure, at every successive unit difference of potential. There will be V cells in the entire tube, and as there are $\frac{V}{2}$

ergs in the entire tube, the energy in the tube for each cell, i.e., for a fall of one unit potential, is $\frac{1}{2}$ erg. (See Watson, Art. 460.)

To account for the actual distribution of the tubes of force in a field, and consequently for the forces to which bodies in the field are subjected, it requires that if F is the force at a point P, then the tension in the air across unit area is $\frac{F^2}{8\pi}$ and at the same time a pressure equally great is exerted at right angles to the lines of force. (See Watson, Arts. 462, 463.)

174. Electrometers. — The principles which we have thus far presented enable us to make many comparisons of potentials, fields of force, etc., but do not determine absolute values. We have not yet seen how to determine the quantity of electricity in a charge or the potential difference between two conductors. An instrument which would measure these in terms of a given unit of quantity or potential would be an electrometer; if it will measure them in terms of mass, length and time, or in mechanical units derived from these, as force, it would be called an absolute electrometer.

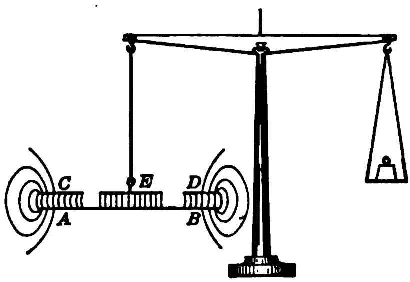


Fig. 99. Guard Ring Electrometer.

Various instruments have been devised for these purposes, of which one that has most commended itself is known as the attracted disc electrometer or guard ring electrometer. It employs directly the fundamental relation of electric attraction or repulsion, and the relation of the force to the distance and quantity of charge. Two circular plates AB and CD (Fig. 99) are placed parallel to each other, and from the upper one a central disc E is cut out so as to be separated from the annular ring around it by a narrow gap. The upper plate, both ring and disc, is connected to a charged body by a conductor, as a wire, and comes to the same potential with this body. The other plate AB

is connected to a body of opposite charge and comes to the same potential with it.

If the distance d between the two plates is small the distribution of the lines of force about the edges will not affect that in the central portions where the field will be uniform, and we may call the charge per unit area, i.e. the surface density, σ. Call the area of the disc $E = S$. This disc will be attracted by the plate AB, and the force f of this attraction may at once be determined by having E suspended from an arm of a balance or from a spring. As there are σ tubes of force for each square centimeter, the area of each tube is $\frac{1}{\sigma}$ sq. cm. Also the electrical force F at any point between the plates is $4\pi\sigma$, and the force exerted by each tube upon the plate E is $\frac{F}{2}$ (Watson, Art. 463). The total number of tubes on E being $S\sigma$, the total attraction of the plate E is

$$f = \frac{FS\sigma}{2}. \quad \text{But } F = 4\pi\sigma,$$

therefore $$f = 2\pi S\sigma^2.$$

Also in the uniform field of strength F or $4\pi\sigma$, the work to carry a unit charge a distance d, i.e., from plate AB to plate E, is $4\pi\sigma d$; but this means the difference of potential between the plates, or $V = 4\pi\sigma d$, hence $\sigma = \frac{V}{4\pi d}$, and this, substituted in the equation for f, gives $f = \frac{V^2S}{8\pi d^2}$,

whence $$V = \sqrt{\frac{8\pi d^2 f}{S}},$$

in which d and S are known in centimeters and square centimeters, and f is weighed in dynes. Then V is electrostatic difference of potential in ergs.

This form of apparatus, variously modified and much elaborated, is a standard means of determining absolute difference of potential. From the dimensions of the plate E and its distance

from AB, its capacity is calculable; therefore the quantity with which it is charged when the other plate is at zero potential (or to earth) is calculable.

Electrokinetics.

175. **Current Electricity.** — Thus far we have considered electric fields of force and distribution of energy only under static electrification, or electrostatics. When, however, a transfer of electrification is going on along a conductor, work is being done and the field is changing unless a supply of electricity is kept up by some agent. Electricity passes by a conductor from a point of higher potential to one of lower potential, and if the difference of potential is to be maintained between the ends of the conductor some instrumentality must act to do it. The measure of its action in maintaining a difference of potential is called the electromotive force, or E.M.F., and the passage of electricity along the conductor is a current of electricity. The science of electricity under these conditions is Electrokinetics, or Current Electricity. Whatever it is that passes from one point to another to constitute what is called a current, work is done, and the energy thus developed appears in the production of heat, or mechanical work, or chemical change. The transfer of electrification is found to be of the same nature as in the case of electrostatics, for, by connecting the two terminals of the current generator (battery or dynamo) to the two plates of a condensing electroscope,* the plates are charged to the same potential as the poles of the generator, and when the latter are disconnected and the upper plate of the electroscope is removed, the instrument is found to be charged electrostatically and to give the same indications of attraction or repulsion when in the vicinity of a body charged with static electricity as if the condenser had been charged by a static machine or by contact with a statically charged body.

* In a condensing electroscope the knob of the ordinary gold-leaf electroscope is replaced by a metal plate, 8 or 10 cm. in diameter, on which rests a similar plate with an insulating handle, like the cover of the electrophorus.

Experiment No. 73, page 299. — Electrification of a condensing electroscope by means of battery cells.

Thus the nature of the electricity transferred by a current is not different from that of electricity at rest, nor does potential difference mean anything different in the two cases, though the field of force around a conductor is found to be greatly altered by the passage of a current.

176. Electric Circuit. — An electric circuit, which is necessary for a continuous flow of electricity or a continuous current, consists of a generator in the form of a battery or a machine to maintain a difference of potential between its terminals, called poles, and a conductor connecting the poles externally. The action of the generator may be chemical, or mechanical, or thermal, and for a current to pass there must be a completely connected series of conductors between the poles externally and through the generator internally. When one pole is raised to a higher potential than the other, if there is no gap in the outer part of the circuit, electrification is transferred from the higher to the lower pole (called respectively + and −) and that would be the end of the process if it were not for the action of the generator in again or continuously building up the potential of the + pole. In doing this the electricity is said to be carried or forced through the machine also. By way of illustration this is compared to the mechanical operation of a pump which continually lifts water from a well (negative pole) to a height of discharge (positive pole), from which it flows back by external circuit to the well. (Read Watson, Art. 472.) The circuit as a whole comprises the conductors that are external to the generator, and those by which the current is carried through the generator. For convenience these are sometimes called the external and the internal circuit respectively.

177. Difference of Potential, and Electromotive Force. — From the experiment with the condensing electroscope, Art. 175, it was seen that the electricity transferred between the poles of a generator is of the same character as that produced by friction, and, therefore, by means of an absolute electrometer the difference

of potential produced by the generator may be determined. The highest difference of potential the machine can produce is that between the terminals when the external circuit is open, and this is the measure of its electromotive force. It is, of course, a quantity of the same sort as potential difference, and therefore potential difference and electromotive force are measured in the same units. Difference of potential is between any two points and is applied to the passage of a current through any specified part of a circuit; electromotive force is at the source of energy and is applied to the passage of a current through the entire circuit.

Of course, to raise a quantity of electricity from a lower to a higher potential means to do work, and when electricity passes from a place of higher to one of lower potential it expends energy.

178. Sources of Electromotive Force. — Electric difference of potential represents energy of electrical separation which is maintained by electromotive force. The chief sources of E.M.F. are:

Chemical combination, in which energy of chemical separation is transformed into energy of electrical separation.

Heat, in which thermal energy is transformed into energy of electrical separation.

Mechanical work, in which mechanical energy is transformed into energy of electrical separation.

The first of these is exhibited in voltaic batteries, the second in thermopiles, and the last in dynamos.

179. Choice of Units. — Certain phenomena associated with the field surrounding a conductor that is carrying a current give a basis for comparing or measuring quantities of electricity, capacities, potentials, or other electrical magnitudes in a different way from that employed in electrostatics, and employing a different set of units. Of course, since the things measured in the two cases are of the same nature, the unit for any of the magnitudes in one case must have a definite and unvarying ratio to that for the same magnitude in the other case, just as the units of the British system of weights and measures have a definite ratio to those of the French system — the pound to the kilogram

or the yard to the meter — although if we are going to confine ourselves to the use of one system we do not need to know anything about its relations to the other. For absolute measurements, however, we still call the erg the unit of work or energy, and then choose such a unit for difference of potential, and such a unit for quantity transferred by the action of a generator, that to transfer that unit quantity through that unit rise of potential will require one erg of work; but the unit quantity here is much larger and the unit of potential much smaller than the corresponding electrostatic units.

180. Oersted's Experiment: Field of Force about a Conductor Carrying a Current. — When a current is flowing in a conductor the region around the conductor is found to be thereby converted into a magnetic field of force. The transfer of energy along a conductor is at once attended by a field of force about the conductor. This field is manifested by the exertion of force upon a magnetic pole placed in it, a north pole being impelled in one direction, and a south pole in the opposite direction, but neither one either directly towards or from the conductor. A magnetic needle with opposite poles to be acted upon places itself in a plane at right angles to the conductor, the direction in which either pole alone would move being in a circle about the conductor. For a plane at right angles to the conductor, this field is readily shown by iron filings that arrange themselves in circular whorls.

Experiment No. 74, page 299. — To illustrate magnetic field around a conductor.

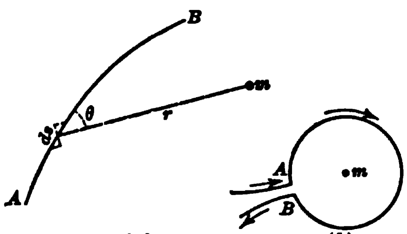
The discovery of this connection between electricity and magnetism is due to Hans Christian Oersted (Danish physicist, 1820), and is shown by placing a magnetic needle above or below or alongside the conductor through which the current is passing.

Experiment No. 75, page 300. — Illustrating the deflection of a magnetic needle by a current.

With the usual convention that assumes the direction of the current to be from the positive to the negative pole of the battery through the external part of the circuit, the direction of the lines

of force or the direction in which the positive pole of the needle would be urged is given by either of several rules; e.g., Ampere's rule: Imagine yourself swimming in the current, with the current, and facing the needle; the north pole will be deflected to your left hand. Also the letters spelling the word SNOW are initials of the words South, North, Over, West; i.e., if the current is from South to North Over the needle, it points West.

The strength of field at any distance r from the conductor carrying the current, i.e., the force which would be exerted upon a unit magnetic pole, is proportional directly to the strength of the current; this is made apparent by placing a second conductor carrying an equal current along with the first, and observing that the force on the pole is then doubled, and so on. The strength of field due to any small element of the conductor is inversely proportional to the square of the distance from the element to the magnetic pole. In Fig. 100(a), if the current C is traversing the conductor AB, the force on a unit pole at m due to the element ds is $\frac{kC\,ds \sin\theta}{r^2}$; $ds \sin\theta$ is the equivalent of the length of the element at right angles to r.



(a) (b)

Fig. 100.

If AB forms the circumference of a circle of radius r, then every element would make an angle $\theta = 90°$ with r, and the force at the center due to each element would be $\frac{kC\,ds}{r^2}$. If the current were flowing in a horizontal plane as indicated in the figure (b), a positive pole at m would be urged downward through the plane of the circuit. The total force on the pole for the entire circumference would be $\frac{kC}{r^2}\sum ds$, or $\frac{kC\,2\pi r}{r^2}$, or $\frac{k\,2\pi C}{r}$. If we choose a circle of one centimeter radius the strength of field at the center is $k\,2\pi C$, and since the effect at m is alike for all equal parts of the circumference, if we choose out one centimeter length of the arc,

that is, $\frac{1}{2\pi}$ part of the circumference, the force at m due to this one centimeter of the arc is $\frac{1}{2\pi}$ part of $k\,2\pi C$, or $F = kC$. Now by choosing for our unit current a current of such strength that, in such a case as this, F shall equal unity, then $k = 1$, and for the force at the center of a conductor in the form of a circle the force on unit pole, or the strength of the field,

$$F = \frac{2\pi C}{r}.$$

181. Unit Current. — *Definition:* In the c.g.s. electromagnetic units, then, unit current is a current of such strength that, flowing in a circle of one centimeter radius, it exerts on a unit magnetic pole at the center a force of one dyne for every centimeter of the circumference. There is no special name for this unit, except the electromagnetic unit of current strength, but a current one-tenth as strong as this is adopted as the "practical unit" current and is named an *ampere*.

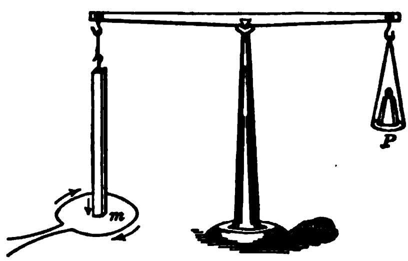


Fig. 101.

If, in Fig. 101, m were the positive pole of a magnet whose other pole was far enough from the plane of the circuit to make the effect of the current upon it negligible, then, with a unit current flowing round the circle of one centimeter radius, it would require a force of 6.2832 (or 2π) dynes at P, on a balance of equal arms for every unit of pole strength at m to displace m upward.

182. The Tangent Galvanometer. — At the center of the circle the lines of force in the magnetic field created by the current are all perpendicular to the plane of the circle, and if the latter is of large radius, say, 20 cm. or more, the field for a distance of several centimeters from the center is practically uniform and of strength $\frac{2\pi C}{r}$. If such a circuit were set up in a vertical position

as in Fig. 102, with its plane in the magnetic meridian and a small needle suspended at its center, this needle, under the influence of the earth's magnetism, would place itself in the plane of the circuit when no current is flowing. The passage of a current in the direction indicated would urge n to the west and s to the east, and the needle would be deflected until the decreasing moment of this deflecting force was counterbalanced by the increasing moment of the restoring force of the earth's magnetism on ns. (Observe, the forces do not change, but their moments do, owing to the change in the angular position of the needle ns.) It can be easily shown that the tangent of the angle of deflection θ is directly proportional to the strength of the current, and if the strength of the earth's field is H and the circular coil has n turns, the current C is

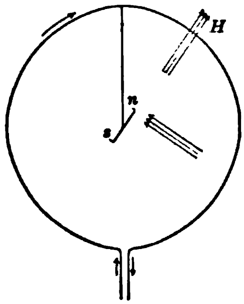


Fig. 102. The Principle of the Tangent Galvanometer.

$$C = \frac{rH}{2\pi n}\tan\theta.$$

Thus if H is known, such an instrument as this enables us to measure the strength of a current flowing through it, in absolute units, and this instrument placed anywhere in the circuit measures the current flowing in the circuit. It is called a tangent galvanometer. (See Watson, Arts. 479, 480, 481.) Even if H were not known, if it is constant the factor $\frac{rH}{2\pi n}$ would be the same with the same instrument wherever it is placed and different currents could be compared by means of it, for the currents would be in just the same proportion as the tangents of the angles of deflection produced by them.

If such a galvanometer is to be very sensitive it would have to show a considerable deflection θ with a very small current C. This would be accomplished by making n very large and r small, which is not always practicable; if r is very small the field of force

in the coil will be uniform only very near the center, making the use of a needle of any considerable size out of the question. The sensitiveness is, however, greatly increased by making the needle nearly astatic.

The purpose is better served by a more recent form of galvanometer, D'Arsonval's, in which a strong magnetic field, practically uniform, is due to the poles of a permanent magnet. Between these poles is suspended by a fine torsion wire or ribbon a coil of many turns of wire through which the current to be observed, or a known fractional part of it, is passed (see Art. 193, Shunts). The plane of the coil is parallel to the plane of the magnetic poles (see Fig. 103).

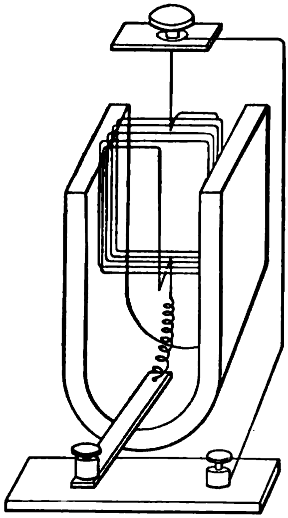


Fig. 103. Principle of the Moving Coil Galvanometer.

With the passage of any current, the tendency is for the poles of the magnet to swing round until the magnetic axis is perpendicular to the plane of the coil carrying the current. But according to the third law of motion, by whatever force the coil pushes the magnet, by just so much force the magnet reacts to push the coil. If the coil is fixed and the magnet pivoted, as in the tangent galvanometer above described, the magnet is deflected; if the magnet is fixed and the coil can swing around, as in the D'Arsonval and other moving coil instruments, it will do so. For small deflections the current is proportional to the deflection.

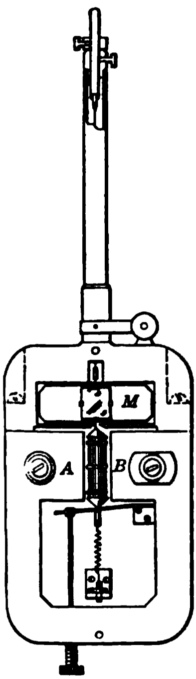
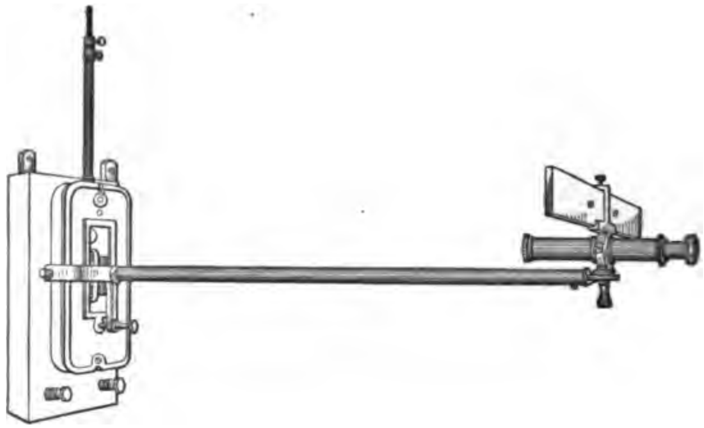


Fig. 104. (a).

Fig. 104 shows the actual construction, *A* and *B* being the

poles of the magnet, and *M* a mirror attached to the coil, in which is viewed the image of a fixed scale, showing the slightest movement of the coil.



(*b*). Fig. 104. The D'Arsonval Galvanometer.

183. Unit Quantity. — With a unit current, as above defined, flowing in a conductor, the quantity of electricity passing a given section of the conductor in one second is, in the c.g.s. system, the unit quantity of electricity. There is no name for this unit, but a quantity one-tenth as great is the "practical" unit of quantity and is known as a *coulomb*.

184. Unit Difference of Potential, or Electromotive Force. — A c.g.s. unit difference of potential between two points is such a difference of potential that to transfer the unit quantity from one point to the other requires one erg of work. This is known simply as the c.g.s. electromagnetic unit, but the "practical" unit is 10^8 (or 100,000,000) times as great as this and is called a *volt*.

185. Unit of Capacity. — A conductor has c.g.s. unit capacity if a charge of one unit quantity raises its potential one unit; if a condenser have one plate at zero potential, then one c.g.s. unit of electricity upon the other plate will produce unit difference of potential between the plates. The "practical" unit of capacity is much smaller, being only 10^{-9} times the c.g.s. unit, and is called a *farad*.

186. Resistance; Ohm's Law. — These units, namely, current strength, quantity, difference of potential, and capacity, serve to measure all the electrical phenomena we have as yet considered. The strength of the current that will flow with a given electromotive force, or the electromotive force that is required to cause a given strength of current, is found to depend upon some quality inherent in the conductor. If two points were connected by a *perfectly conducting* medium, no difference of potential could be maintained between them. The facility with which a conductor will convey a current is known as its conductivity, and the measure of the conductivity of a conductor is its conductance. The maintenance of a given difference of potential at the ends of a conductor is attended by a flow of electricity through it, a large current, if a good conductor; a small current, if a poor conductor. Since a perfect conductor would transmit the electricity perfectly, this defect in conducting power has been called resistance. It is of course the inverse of conductance; the less readily a conductor transmits a current, the greater its resistance.

Dr. G. S. Ohm, in investigating the relations of electromotive force E, current strength C, and resistance R, found that the resistance is directly proportional to the electromotive force (or the difference of potential) between two points, and inversely proportional to the current, i.e., $R = k\frac{E}{C}\cdot$ This is known as Ohm's law and is perhaps the most important generalization in connection with electric currents. If we define our unit resistance to be such a resistance of a conductor that unit E.M.F. already defined will send through it a unit current as already defined, then in the above equation we have $1 = k\frac{1}{1}$ or $k = 1$; and always then, in terms of such units, we shall have $R = \frac{E}{C}$, or $C = \frac{E}{R}$.

187. Unit Resistance. — In c.g.s. system, unit resistance is the resistance of a conductor such that c.g.s. unit electromotive force will send through it c.g.s. unit current.

Now Ohm's law applies to materials regardless of the system of units employed, and if our practical unit of electromotive force is 10^8 times the c.g.s. unit, and the practical unit of current is 10^{-1} times the c.g.s. unit, it follows that the practical unit of resistance is $\frac{10^8}{10^{-1}}$ or 10^9 times the c.g.s. unit. This large unit is called an *ohm*, and R (ohms) $= \frac{E\text{ (volts)}}{C\text{ (amps.)}}$.

188. Practical Units. — We have now pointed out the basis upon which a set of units has been evolved, rationally correlated to one another, and correctly expressing the physical actions they are to measure. In practice it has been found convenient to use units that are definite multiples or fractional parts of the fundamental ones. From here on we shall generally use the practical units already named with the addition of one or two to those that have been mentioned.

Since the work or energy of transferring electricity is measured by the product of the quantity, Q, transferred, into the difference of potential, E, through which it is transferred, we would have, always, work (or energy) $= EQ$. In the c.g.s. system, unit quantity raised unit difference of potential equals one erg; in practical units one coulomb raised one volt equals $10^{-1} \times 10^8$ or 10^7 ergs. This unit is called a *joule*. With a practical unit of current, one ampere, flowing between two points at the practical unit difference of potential, one volt, one coulomb is transferred every second, and one joule of work is done, or one joule of energy is expended, every second. *Rate of doing work* is *power* or *activity*, and an activity of one joule per second is the practical unit called a *watt*.

Although the fundamental units are derived from the electromagnetic effects of a current, the current will produce other effects to an extent whose relations to the current producing them may be determined, and these actions might be taken as the basis of measurement.

These relations are exhibited in the following tables (see Watson's *Physics*, pp. 773, 774):

Quantity.	Name of practical unit.	Equivalent in c. g. s. units.
Current	Ampere	10^{-1} electromag. units.
Quantity	Coulomb	10^{-1} " "
Electromotive force or potential difference	Volt	10^{8} " "
Resistance	Ohm	10^{9} " "
Capacity	Farad	10^{-9} " "
Induction *	Henry	10^{9} " "
Energy or work	Joule	10^{7} ergs.
Power or activity	Watt	10^{7} ergs per second.

* The unit of induction will be explained later; see Art. 206.

Auxiliary units in the practical system.

Name.	Equivalent in the practical system.	Equivalent in c. g. s. units.
Megohm	10^{6} ohms	10^{15} electromag. units.
Microfarad	10^{-6} farads	10^{-15} " "
Microvolt	10^{-6} volts	10^{2} " "
Microampere	10^{-6} amperes	10^{-7} " "
Kilowatt*	10^{3} watts	10^{10} ergs per second.
Millivolt	10^{-3} volts	10^{5} electromag. units.
Milliampere	10^{-3} amperes	10^{-4} " "

* 1 kilowatt equals 1.341 horsepower.

In the effort to determine as accurate standards for the electrical units of measurement as possible, all these varied effects have been examined. The chemical action of a current in depositing metal from a solution of a metallic salt has been carefully compared with the practical unit as above derived from the electromagnetic unit. Also the electromotive force of battery cells constructed according to definite formulas, for standards, has been compared with the practical unit of E.M.F. as derived from the electromagnetic, and in accordance with such comparisons the following definitions have been adopted throughout the world for the practical units. They are known as International Legal Standard Units, and are legalized in the United States by act of Congress. They are as follows:

The ohm, the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, weighing 14.4521 grams, of uniform cross-section, and having a length of 106.3 cm.

The ampere, that current which, under certain specified conditions, should deposit silver from a solution of silver nitrate in water at the rate of $\frac{1118}{1000000}$ of a gram per second.

The volt, the $\frac{1000}{1434}$ of the electromotive force of a Clark cell set up according to certain specified conditions and kept at a temperature of 15° C. For Weston Normal Cell, see Art. 216.

The coulomb, the quantity of electricity conveyed by one ampere during one second.

The farad, the capacity of a condenser charged to the potential of one volt by one coulomb of electricity.

The joule, the quantity of energy expended per second in one ohm by a current of one ampere.

The watt, the rate of work represented by one joule per second.

The henry, the induction of a circuit in which a variation of one ampere per second induces an electromotive force of one volt.

The standard specifications were published in 1895 and may be found, with other details, in the United States Revised Statutes.

We shall not discuss battery cells as sources of current until later, but merely point out that a definite E.M.F. is produced by any combination of different materials between which chemical action occurs, and the E.M.F. in any case depends only on the nature and temperature of the materials constituting the battery cell, and not in any degree upon the size or quantity, so that cells may be constructed for standards of E.M.F., as the Clark cell cited above.

Examples. —

1. If the resistance of a conductor is 250 ohms and the difference of potential at its ends is 110 volts, what current is flowing through it?

Ans. 0.44 amperes.

2. If a battery cell have an internal resistance of 0.07 ohms, and an E.M.F. of 2.4 volts, how strong a current would it give through an external conductor (*a*) of one ohm resistance; (*b*) of 0.1 ohm? If the cell were guaranteed to give 30 amp., what would you understand by that?

Ans. (*a*) 2.243 amps.; (*b*) 14.12 amps.

3. What e.m.f. is needed to send a current of 0.2 amp. through a circuit whose resistance is 2000 ohms? *Ans.* 400 volts.

4. The resistance of the human body in many cases is less than 2000 ohms, and a current of 0.1 amp. may be fatal, what voltage between two terminals of a current generator is dangerous? *Ans.* 200 volts.

189. Specific Resistance. — According to Ohm's law, $R = \frac{E}{C}$, which means that with a conductor of any given material, no matter how the conductor may be varied in size or in shape, so long as it is made of the same material, the resistance will equal the quotient of the difference of potential between the ends of the conductor by the strength of current that flows through it. If, however, two conductors, alike in size and shape, but different in substance, are subject to equal E.M.F., the current will not be alike in the two, one being said to have higher conductivity or lower resistance than the other. Further, with any given material, at a given temperature, the actual resistance varies directly with the length l, and inversely with the area of cross-section s, so that, in general, $R = k\frac{l}{s}$, where k is a constant that pertains to the particular substance of which the conductor is made. If the ratio of $\frac{l}{s}$ is unity, $k = R$, so that if the conductor were one centimeter long and one square centimeter in area of cross-section, called a centimeter-cube (not necessarily the same as one cubic centimeter), $k = R$, and this value of k for any given substance at a temperature of 0° C. is called the specific resistance of the substance. (Conductivity and specific conductivity are the reciprocals of resistance and specific resistance, respectively.)

Pure metals and other solid conductors differ greatly from one another in specific resistance, this being another "constant of nature" of great importance. Extensive tables have been prepared, in which copper has the lowest resistance. A few values are here shown:

Copper,	1.6×10^{-6}	ohms	per	cm.-cube	at 0°.
Platinum,	$8.2 \times$ "	"	"	"	"
Lead,	$19.0 \times$ "	"	"	"	"
Carbon filament,	$4000 \times$ "	"	"	"	"

Experiment No. 76, page 300. — Illustrating conductivity of liquids.

190. Variation of Resistance with Temperature. — The specific resistance of a substance usually increases with a rise of temperature (exceptions are carbon and conducting liquid solutions),

at a rate varying somewhat with different substances, but nearly constant for any given substance.

If the resistance were plotted as ordinates, taking temperatures as abscissas, the curve showing the rise of resistance with rise of

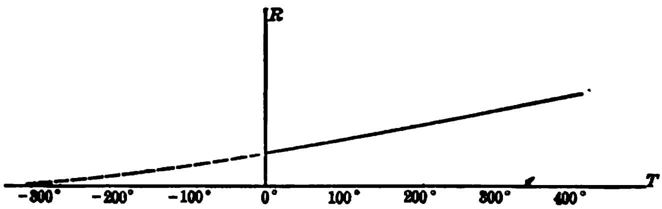


Fig. 105. Curve of Resistance.

temperature resembles that of Fig. 105. The slope is nearly uniform, but the curve is closely represented by an equation of the form

$$R = R_0 (1 + at + bt^2),$$

in which, for metals, a is very nearly 0.0037 and b is very much smaller. If b could be neglected, the equation would make $R = 0$ at $t = -\dfrac{1}{0.0037}$, or $-273°$ C., that is, at the absolute zero of temperature, a singular coincidence if not a significant one with regard to the inherent nature of resistance.

The actual temperature at which the equation for pure metals gives zero resistance is about $-300°$ C. It must be admitted, however, that the coefficients a and b are determined from observations at temperatures too high to justify the application of the equation to extremely low temperatures.

While all pure metals increase in resistance with a rise in temperature, solutions which are conductors and carbon among solids vary in the opposite direction, the coefficient for carbon being about -0.0004 per degree centigrade. The specific resistance of alloys is usually higher than the average resistance of their constituents, and the temperature coefficient is smaller. It has been possible to make an alloy with manganese, whose resistance is practically invariable under varying temperature. Such a substance is especially valuable for constructing standard resistances.

EXAMPLES. —

1. An iron wire, with a resistance of 2.5 ohms at 0° C., just begins to emit a dark red glow when carrying a current of 9.25 amperes. If the difference of potential between its ends then is 90 volts and the temperature of the hot wire is 525° C., what is the temperature coefficient? *Ans.* 0.0055.

2. If the temperature coefficient of the carbon filament in an incandescent electric lamp is −0.00044, the resistance at zero degrees 460 ohms, and the current 0.45 amperes when the lamp is at full glow under 110 volts, what is the temperature of the filament? *Ans.* 1070° C.

191. Resistance Coils. — Lengths of wire of different resistances are wound on spools and assembled in convenient order in boxes, so that they may be inserted in a circuit, and, by connecting them in any combination, may produce any value of resistance up to their sum total. These are called resistance boxes, and such a set of resistances is to the electrician what a set weights is to a chemist. Fig. 106 shows such an arrangement, the terminals of the coils being attached to thick metal bars, each of which can be connected to the next by a metal plug. Any conductor inserted as a part of a circuit to control or vary the strength of the current by its resistance is a "rheostat," whether it has standard values or not.

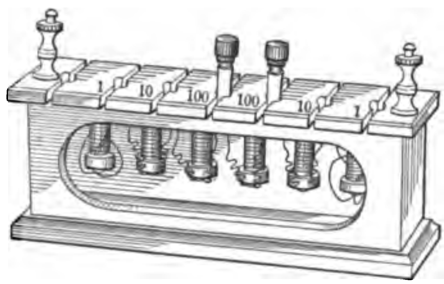


Fig. 106. Resistance Box, Showing Interior.

192. Divided Circuits. — If two resistances r_1 and r_2 are joined tandem or in series, connecting two points A and B of a circuit, their combined resistance is $r_1 + r_2$, but if they are joined in parallel, i.e., each one connecting A to B, as in Fig. 107, evidently there is more opportunity for a current to pass from A to B than by means of either conductor alone if the other were not there,

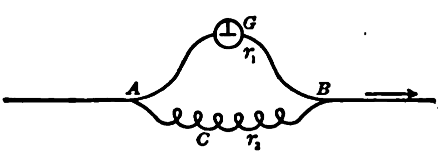


Fig. 107. Divided Circuit.

AEC, are inserted, let V be the difference of potential between A and C. The heavy lines indicate thick metal bars of negligible resistance. For every point along the branch ADC, there is a corresponding point on the branch AEC at the same potential. Suppose E to be at the same potential as D. The four portions of the divided circuit have resistances which we may designate r_1, r_2, r_3 and r_x, as in the figure.

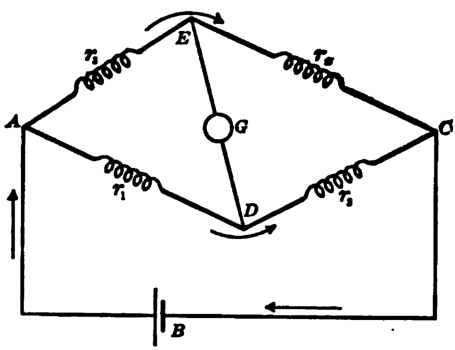


Fig. 108. Wheatstone Bridge Diagram.

The same current, say c_1, is flowing through r_1 as through r_2; and the same strength of current, say c_2, is flowing through r_3 as through r_x. If we call the potential difference between A and D, V_1, this is also the potential difference between A and E. Similarly, if V_2 is the fall of potential from D to C, it is also that from E to C. The value of the current in any branch is the potential difference between its ends divided by its resistance; therefore

$$\frac{V_1}{r_1} = \frac{V_2}{r_2},$$

and

$$\frac{V_1}{r_3} = \frac{V_2}{r_x}.$$

Dividing these two equations, member by member, we get $\frac{r_3}{r_1} = \frac{r_x}{r_2}$. If r_x is unknown we can compute its value from this equation, provided r_1, r_2, and r_3 are known, or if, of these three, either one and the ratio of the other two are known.

To apply the method, two points, as E and D, are joined by a sensitive galvanometer; one branch, as EC, consists of the conductor whose resistance r_x is to be determined. Sets of adjustable resistances are placed in the other three branches.

(1) If r_3 and r_1 are kept at a fixed value, r_2 is varied until no deflection of the galvanometer is produced. E and D are then at the same potential, otherwise a current would pass through the galvanometer. If r_2 is too small, a current passes from D to E; if too large, a current passes from E to D deflecting the galvanometer index in the opposite direction; but when no deflection is produced the equality of ratios holds, *viz.*, $\frac{r_3}{r_1} = \frac{r_x}{r_2}$, in which r_x is the only unknown.

(2) r_1 and r_2 might have been kept at a constant value and r_3 varied until a balance, or no deflection, was obtained and again the same equality of ratios would apply, or

(3) r_3 might be fixed in amount and both r_1 and r_2 varied, one increased as the other is decreased, until the galvanometer shows no deflection. The second is the plan used in the P.O. box bridge, and the third is that of the British Association slide-wire bridge.

195. Variation of Resistance a Means of Measuring Temperature. — If a substance is found to show a consistent change in resistance through a wide range of temperature, the variation in resistance of such a conductor may be used for thermometry. Platinum is such a substance, and as platinum remains solid at a very high temperature, a coil of it suitably mounted on a non-conducting material, as clay or porcelain, may be placed in a furnace or, if suitably encased, may be inserted in a bath of molten metal, and the temperature may thus be determined by observing the increased resistance of the branch circuit of which the coil is a part.

Further, if a fine strip of metal is made to be one arm of a Wheatstone bridge, in which a balance has been obtained with a very sensitive galvanometer, then, so exquisitely susceptible is it that the slightest change of temperature in this strip will disturb the balance. An instrument arranged for this purpose by the late Professor S. P. Langley has been employed by him to detect and measure very low radiant heat; among other instances, that of moonbeams. This instrument is called a bolometer.

196. Current Sheet. — For ordinary thin conductors, such as wires, the fact that the resistance is inversely as the area of cross-section would indicate that the current does not pass over the surface only, but utilizes the entire body of the material alike, the lines of flow being in the interior the same as at the surface. If a current is led into a thin, broad sheet of a conducting substance at one point and out at another, the direction of flow at any point of this conductor can be determined and continuous lines of flow traced out.

Let *MNPQ* (Fig. 109) represent such a conducting sheet, liquid or solid, and suppose the current passes from *A* to *B*. By attaching one end of a conductor (which leads externally through a sensitive galvanometer) at p_1 and moving the other end so as to trace paths along the sheet, a series of positions p_2, p_3, etc., may be found for which no current passes through the external circuit. These points then are at the same potential as p_1, and a line drawn through them is an equipotential line. Any number of such equipotential lines may be determined, and then any line traced continuously from *A* to *B*, crossing the equipotentials at right angles, will be a line of flow. This does not mean that the flow is only on the lines thus traced, but that at any point on such a line the direction of the line is the direction of flow at that point.

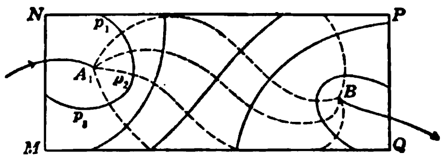


Fig. 109. Lines of Flow in a Current Sheet.

If the conducting sheet be a piece of blotting or absorbent paper soaked with a solution of metallic salt, as, for example, sulphate of zinc, over which are scattered fine zinc filings, a current flowing from *A* to *B* will show lines of flow by the electro-deposition of metallic zinc connecting the zinc particles, much like the iron filings in a magnetic field. Fig. 140 (page 304) shows such lines of zinc deposit. (See Art. 213.)

In mapping equipotential lines, as in Fig. 109 above, the current between *A* and *B* may be a rapidly alternating one, as from

the secondary terminals of an induction coil. In such case the galvanometer should be replaced by a telephone receiver; a rattling noise will be produced in it except when the terminals are at the same potential; then the sound ceases.

197. Measuring Instruments. — The relation of the practical units to the electromagnetic c.g.s. units having been adopted, and standard values for the practical units having been legalized, quantities are now usually measured and expressed in those units, i.e., current in amperes, E.M.F. or potential difference in volts, resistance in ohms, activity in watts (or kilowatts), etc. These terms should not be confounded with one another. To talk of a current as so many volts is as bad as to speak of a period of duration as so many feet.

The following example of misuse of terms is not much worse than can be found in newspaper accounts nearly every day: A man touched the end of a live wire and received a current of 1,000 volts. The potential of his body being about 12,000 ohms, it offered a resistance of a little over 0.08 amperes. The passage of this current through him represented an activity of 80 microfarads. Whether his death was due to the heat produced, which was 80 joules, or to the electrical work of 19.2 calories per second, does not matter, since they are equivalents; and it would not matter to him in any case.

The same quantities would be used in a corrected statement as follows: A man touched the end of a live wire at a potential differing from his own by 1,000 volts. He thereby closed a circuit, and, the resistance of his body being about 12,000 ohms, he received a current of a little over 0.08 amperes. The passage of this current through him represented an activity of 80 watts. Whether his death was due to the heat produced, which was at the rate of 19.2 calories per second, or to the electrical work of 80 joules per second, does not matter since they are equivalents, and it would not matter to him in any case.

Ammeter. — An instrument whose index shows the strength of current in amperes is an ampere-meter, or, as it is more commonly called, an ammeter. To make such an instrument satisfactory it should have a large carrying capacity and a resistance so low that its insertion in a circuit makes the current from a given source of E.M.F. not appreciably different from what it would be if the instrument were not there. In any case, the

ammeter is joined in the circuit in series so that the current that is flowing through the rest of the circuit passes through it also, as at A in Fig. 110. Of the many forms of ammeter that have been devised the most approved is a modified form of the D'Arsonval galvanometer (see Fig. 103), the current or a definite fraction of it going through a thick coil of low resistance (perhaps only a few thousandths of an ohm) that is delicately pivoted between the poles of a strong permanent magnet in the case or box of the instrument. The scale is calibrated so as to read the correct number of amperes for any deflection of the coil. The deflection is shown by a light pointer attached to the coil (see Fig. 111).

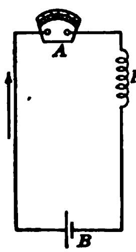


Fig. 110.

Voltmeter. — An instrument that shows the difference of potential between two points directly in volts is a voltmeter (not voltameter). To apply such an instrument in practice its terminals are made to touch the two points whose difference of potential is to be determined, and its needle then moves over the scale to correspond to the actual difference of potential between the two points thus connected. The usual form of this instrument is just the same as that of the ammeter except that the coil suspended in the magnetic field is of very high resistance, sometimes many thousand ohms. To serve its purpose properly the voltmeter, when applied, ought not to disturb the current that is already flowing, if any, or cause any change in the E.M.F. by virtue of its application. But it is seen that its indications themselves depend on the current actually passing through it. This instrument could not be used in series like the ammeter, as such use would at once so greatly increase the resistance of the circuit as greatly to alter the current.

If, in Fig. 112, B is a battery cell sending a current through a circuit $CDEF$, and it is desired to know the difference of potential between E and F, the voltmeter V is connected to these points in parallel, as shown in the figure, and shows its readings while the current is passing. But the line through the voltmeter is a

(*a*).

(*b*) Shows Movable Coil with Pointer.

Fig. 111. Weston Portable Ammeter.

shunt to the part R_2 of the main circuit, and unless the resistance of V is very large, this double conductor from E to F will be appreciably lower in resistance than R_2 alone, the actual current from B will be increased, and the potential difference between E and F at the same time may be less than before the voltmeter was joined in. If, however, the resistance of V is very great compared to R_2 this alteration is negligible.

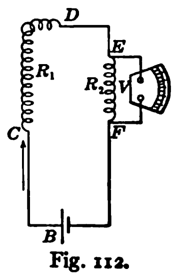


Fig. 112.

By applying the voltmeter directly to the terminals of a voltaic cell it shows very approximately the E.M.F. of the cell; not exactly, for when it is thus joined a small current passes through the instrument and the D.P. between the poles is not exactly equal to the E.M.F. of the cell with the circuit open. To make this plain, suppose, in Fig. 113, the cell AB has an E.M.F. of E, and an internal resistance r, and the circuit is completed by an external resistance R.

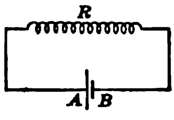


Fig. 113.

For the external circuit alone, the difference of potential between A and B, $V_B{}^A$, is such that $C = \frac{V_B{}^A}{R}$. For the entire circuit the electromotive force E is such that the current $C = \frac{E}{R + r}$. Since these two values of C are identical,

$$\frac{E}{R+r} = \frac{V_B{}^A}{R}.$$

This may be written

$$\frac{E}{V_B{}^A} = \frac{R+r}{R} = 1 + \frac{r}{R}.$$

Here it is seen that if E is to equal $V_B{}^A$, either r must be zero, which is impossible, or R must equal infinity, which is the case only when there is no conductor joining A and B. If, however, R, representing the voltmeter, is sufficiently large the ratio of E to $V_B{}^A$ can be brought as nearly equal to unity as we please. (Voltmeter should be shown along with this discussion.)

Wattmeter. — If a current of A amperes is flowing between two points whose D.P. is V volts, then A coulombs are transferred every second and $V \times A$ is the number of volt-coulombs, or the number of joules per second of work done electrically on the part of the circuit in which the fall of potential is V. An activity of one joule per second is called a watt, or one volt-ampere. An instrument which will indicate at once not the volts measuring the D.P. between two points of a circuit, nor the amperes measuring the current passing, but the product of these two quantities, will show the activity of the current, and is called a wattmeter. For further account of wattmeters, as well as of other less common meters, refer to technical electrical engineering works.

Resistance Coils and some other means of measuring resistances have already been described. Instruments which read off resistance directly in ohms are called ohmmeters; they need not be described here.

Standard condensers of tested capacity are mounted in boxes like resistance coils and are used to test capacities by special methods of comparison.

198. Heating Action of a Current; Joule's Law. — The difference of potential between two points of a circuit is the measure of the amount of work done in carrying a unit quantity of electricity from one point to the other. If a current C is flowing, then C units per second are transferred from one point to the other, and if V is the difference of potential, VC is the work done per second. If V and C are expressed in electromagnetic units, VC is ergs per second. When no other work is done in this part of the circuit except to overcome the resistance of the conductor, the energy expended there is manifested altogether as heat. If R is the resistance between the two points, by Ohm's law $C = \frac{V}{R}$, or $V = CR$, and the work (or heat) $VC = C^2R$ per second. In t seconds the total heat produced is, in mechanical units, $H = C^2Rt$. This fact was discovered experimentally by Joule and the equation is a statement of Joule's law. We see here that it was deducible from the principles of potential, but its experimental determination was one of the things that established the theory of potential, and particularly that brought out the fact that electric potential and heat were measurable in like terms, and that both were ultimately reducible to mechanical work.

In the equation $VC = C^2R$, if C is electromagnetic unit current and R is electromagnetic unit resistance, the performance of work represented by VC or C^2R is one erg per second. If the current C is one ampere, i.e., $\frac{1}{10}$ an electromagnetic unit; and if R is one ohm, i.e., 10^9 electromagnetic units of resistance, the product C^2R will be the work of carrying one ampere through one ohm resistance, $10^{-2} \times 10^9$, or 10^7 ergs per second; and so would be the product VC if V is one volt, which is 10^8 electromagnetic units of potential, for $10^8 \times 10^{-1}$ makes 10^7 ergs per second as before. 10^7 ergs is an amount of work called a joule, which may also be expressed in calories, or electrically as one volt-coulomb, and when work is done electrically at the rate of one joule

per second it is one volt-ampere, or an "activity" of one watt.

Under the Mechanical Equivalent of Heat, Art. 109, we found that one calorie is equal to 4.19×10^7 ergs, and therefore we should find that one calorie, when produced by an electric current, should equal 4.19 joules, or, 1 joule $= \frac{1}{4.19} = 0.2387$ calories. By immersing a coiled conductor in water, measuring carefully the current C flowing through the conductor, and the potential difference V that is maintained between its ends, we have the rate at which electrical work is being done, which is VC joules per second. By measuring the mass m of the water heated, and the rise of temperature $t°$ that is produced in T seconds, we have the number of calories $mt°$ developed; this divided by T gives the calories produced in one second, and this again divided by VC (the number of joules per second) gives the calories equal to one joule. Taking from our definitions the mechanical value of one joule as 10^7 ergs, we can thus arrive at the determination of the mechanical equivalent of heat from an electric current. This work, carried out by Dr. Joule (1852–4) in determining the mechanical equivalent of heat directly by the work of descending weights, and then indirectly by electrical work, before electricity itself was on a scientific basis or any systematic electrical measurements were known, was the beginning of the correlation of various branches of physics in one general scheme of energy and was the foundation of modern physics.

199. Localizing Work in a Circuit; Electric Lights. — If a circuit has in it a source of constant E.M.F., and the agent developing the E.M.F. has small resistance, the external part of the circuit may be so made up as to make the resistance of any given portion of it great in comparison with the rest. As the same strength of current is flowing everywhere through the circuit, the difference of potential between any two points will be exactly in proportion to the resistance in that portion. If the conductor at one point has small mass with high resistance, as a thin fiber or wire, the work, VC or C^2R, expended there per sec-

ond, will be large and the heating will be great. The ordinary incandescent electric lamp is a carbon filament with a resistance of about 400 ohms when cold, or about 200 ohms when glowing, and, with heavy good conductors for the mains or leading wires, the loss of potential in them is so small that with a dynamo maintaining 115 volts at its terminals there is a potential difference of about 110 volts at the ends of the lamp filament. This gives an activity of 55 watts in the lamp of 16 candle power, or $3\frac{1}{2}$ watts per candle. The removal of the air from the bulb is to prevent combustion of the carbon filament by the oxygen of the air when the lamp is white hot, or to avoid any other deleterious effects of the gases upon the filament. Efforts are continually being made to improve upon carbon filaments by the use of tungsten, thorium, tantalum, or other substances. A modification of tungsten has been prepared which somewhat toughens the fiber and gives not only much more light in proportion to the electric energy expended ($1\frac{1}{2}$ to 2 watts per candle power), but a light that is whiter and more agreeable than that of the carbon. By introducing nitrogen into the lamp bulb, the pressure upon the filament seems to check its disintegration, thus permitting a higher temperature and increased brightness.

The Nernst lamp is a calcium compound, which becomes sufficiently conducting when hot to give a strong white light.

If a circuit carrying a current is broken, the resistance in the gap becomes great, and any considerable air gap would be practically nonconducting. At the instant of separating, however, the connection is momentarily only so weakened as to make a high resistance without actually destroying the current. Great heat is at that moment developed, even to the melting and volatilizing of the metal at the ends of the broken conductor. Separating the ends slightly further gives a space filled with incandescent gas of high resistance between terminals that are glowing white hot. The heat here is intense and the light is brilliant. This is an arc light, so called because the incandescent vapor between the terminals has a curved or arched form. To maintain an arc requires a difference of potential of thirty volts

or more. A voltage of 40 with a current of 10 amperes between carbon terminals would have an activity of 400 watts and would give a light of 600 to 1000 candle power, or about one-half or two-thirds of a watt per candle.

Experiment No. 77, page 300. — Illustrating localization of energy by chain of alternate links of, say, copper and platinum.

Experiment No. 78, page 300. — Heating a wire by electric current.

Since an excessive current in a circuit may be destructive, if not actually dangerous, precautions to prevent this are taken in any scheme of wiring. The maximum current allowable in any part of a circuit is determined, and then that part of the circuit is connected to the other portion by a piece of easily fusible metal which will be so heated by the maximum current as to melt, thus breaking the circuit automatically. Such a piece of metal is called a fuse plug. It is prepared in sizes suitable to melt under currents of from one-half an ampere to fifty amperes. (Lecturer exhibit and illustrate.)

The electric arc possesses a threefold interest in that it displays much of a purely electrical nature as to potentials and currents; it produces the highest known artificial heat, even to the degree of volatilizing diamond, and it is the most powerful light that can be produced by man. (Exhibit.)

Examples. — One joule produces 0.24 calorie, and an activity of one watt produces one joule per second.

1. An electric lamp has a current of 0.4 ampere with a potential difference of 115 volts; what is the activity of the current? At what rate is heat supplied to the lamp? *Ans.* 46 watts; 11.04 cals. per sec.

2. A wire of resistance 25 ohms carries a current of 5 amperes; at what rate is it absorbing heat? *Ans.* 150 cals. per sec.

200. Systems of Electric Lighting. — The electric incandescent lamp has a low illuminating power, having been adjusted to a convenient unit for distribution, of about 16 c.p. This is sometimes reduced to 8 c.p., or raised to 32 c.p., but these can all be employed upon the same system if they give their rated illumination at the same potential difference between their terminals. The form that has come into general use requires about 110 volts,

and such a voltage is produced and maintained by dynamo-electric machines to be described later. Since, however, a difference of potential of over 500 volts is dangerous to life, not more than five such lamps could be put in series without requiring a voltage so dangerous as to preclude its use in households or where the lamps are to be manipulated frequently. Accordingly the incandescent lamps are arranged in parallel (illustrate), between mains that are large enough to require small E.M.F. for driving a large current through them, and the current is supplied by a dynamo machine so constructed as to maintain practically a constant electromotive force under a varying load.

The arc light, on the other hand, cannot be maintained at all without producing illumination many times as intense as that from the 16 c.p. incandescent lamp. It is not, therefore, suitable for lighting in small rooms, but is better adapted to large halls or outdoor lighting. Also it requires at best only about fifty volts, and therefore a considerable number of such lamps can be put on one circuit in series, all taking the same strength of current. The dynamo used commercially for this purpose will generate ten to fifteen amperes with an E.M.F. of 1500 volts, and will therefore supply thirty arc lamps. In the "inclosed arc," the supply of air is almost cut off, and a much longer service is obtained from the carbons, with a current as small as five amperes and a potential of eighty volts. (Further details of electric lighting as well as of distribution, three-wire system, etc., are more in place as features of electrical engineering. A few lantern slides might be exhibited.)

201. Attraction and Repulsion of Currents. — We have seen that a conductor carrying a current is encircled by magnetic lines of force, showing a magnetic field of force in the vicinity of the conductor. If another conductor, also carrying a current, be placed alongside the first, the two will attract or repel each other according as the currents are in like or unlike directions. In general it is said that two currents flowing both towards or both away from a given point attract each other, but if one flow toward and the other away from the same point they repel each other.

The circuits endeavor so to place themselves as to bring the lines of force parallel to one another and in the same direction.

Experiment No. 79, page 301. — Roget's Spiral.
Experiment No. 80, page 301. — Attraction and repulsion of currents.

201. Induced Currents; Lenz's Law. — If two circuits are near each other, any variation of the current in one is attended by a variation of the current in the other. If neither circuit has a current flowing, then the producing of a current in one induces a current in the other *so long as the first current is growing*, but if the inducing current presently reaches a value at which it holds steady, then the induced current ceases. If there is a current in the first circuit and it falls off or stops, this decrease or cessation induces a current in the second circuit *during the time that the first is decreasing*, which of course is momentary if the first is suddenly interrupted. If a current is flowing in both circuits, any variation in one is attended by a variation in the other *while the change is in progress*. This induction is ascribed to the change in the magnetic field about the circuit in which the current is induced, for if AB (Fig. 114) is part of a conductor carrying a current in the direction AB, lines of force encircle it as in the figure, some of which circles will be large enough in circumference to include the neighboring conductor $A'B'$. The action between the electric current and the magnetic whorls is a mutual one. The establishing of the current creates the magnetic field, and the creation of such a magnetic field will produce the current, but with this remarkable difference, that the magnetic field persists so long as the current continues but the current due to the magnetic excitation exists only while the field is changing. *This change in the strength of the magnetic field is evidently the source of the E.M.F. that sends the induced current.*

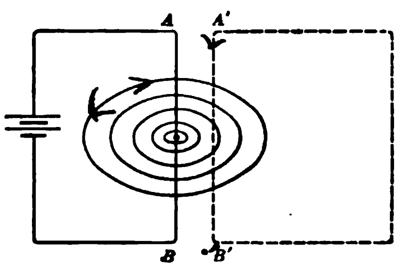


Fig. 114.

In the figure, an increase in the current in AB causes more lines to encircle $A'B'$ and a current is induced in the latter. The current thus induced is always in a direction such that it tends to oppose the action that produced it; thus, in the case here mentioned, starting a current in AB, or increasing it, sets up a current in $A'B'$ in the opposite direction, for with currents in opposite directions the conductors would tend to separate or move so as to diminish the field about $A'B'$. On the other hand, if a current is flowing in AB, say, in the direction AB, and is stopped, with that stoppage the lines of force about both conductors will vanish, but a current is instantly established in $A'B'$ in the direction $A'B'$, since this is the direction of a current in this conductor which would make an attraction of it toward AB, and a movement of it toward AB would be of a sort to prevent a diminution of the lines of force encircling $A'B'$. Similar effects are produced upon moving the circuits toward or from each other. If AB carry a current and $A'B'$ is brought up to it, a current is induced in the latter in the direction $B'A'$, as such a current gives repulsion from the former, or opposes bringing the two toward each other; but if the second conductor is removed from the first it has a current induced in it in the same direction as that in the first, since then the two exercise an attraction which resists their separation. These relations between induced currents and the change of magnetic fields are generalized in Lenz's law as follows: *The currents which are induced in consequence of movements or other changes in an electromagnetic system are invariably in such a direction that they tend to oppose these movements or changes.*

Both AB and $A'B'$ above are supposed to be parts of complete circuits, for it is only in such case that a current can flow. In the case of a circuit around which a current is flowing, all the magnetic lines of force due to that current penetrate the plane of the circuit in the same direction. In the illustrations mentioned, an increase in the lines of force about one of the conductors is attended by an increase in the number penetrating the other circuit and *vice versa*. That circuit in which the change is made arbitrarily is called "the primary circuit," or simply "the primary," and

that in which the change is induced is called " the secondary circuit " or " the secondary." If the latter is not closed, then a change in the current of the primary will set up an electromotive force in the secondary resulting in a corresponding potential difference between A' and B' (Fig. 115).

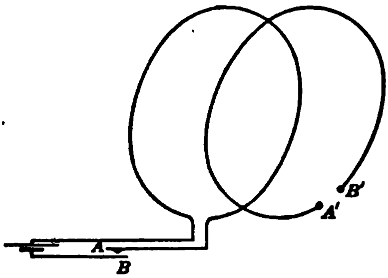


Fig. 115.

The amount of this potential difference is determined by the rate at which the magnetic field in $A'B'$ is altered, i.e., by the number of lines of force per second introduced into or withdrawn from the partial circuit $A'B'$. This rate is great and the induced E.M.F., therefore, high if the current in AB is made or broken very suddenly. While we perceive the induction effects in the form of currents, it is better to estimate them by the induced E.M.F., since this depends only on the rate of change in the magnetic field, while the current depends not only on the E.M.F. that is induced, but also upon the resistance of the secondary circuit. This sort of induction is called electromagnetic induction to distinguish it from that due to a static charge of electricity, or simply electric induction.

Experiment No. 81, page 302. — Electromagnetic induction of currents.

Note. — We use the term "line of force" in measuring the strength of field to mean *unit* line of force, and understand the numerical strength of field to be represented by the number of such lines per square centimeter. More recently the phrase "tubes of force" is being used to make the nomenclature in electromagnetic induction conform to that of electric induction. When this term is used a unit tube is a tube whose surface is composed of linear elements every one of which is a line of force in meaning but not in intensity as above described, and the sectional area of a tube of force is such that the strength of field in the tube multiplied by the area of cross-section is unity. There are then as many tubes to the square centimeter as there are units in the strength of field. This makes "tube of force" correspond exactly to the "unit line of force" and the tubes of force just as numerous as the unit lines of force, and distributed in the same way. The advantage in its use is that it represents a filled-up field instead of one of threads with vacant spaces between them. See Arts. 168, 169 and 173, also Watson, p. 707.

203. Magnitude and Direction of Induced E.M.F. — A change of one unit line of force per second through a circuit corresponds to one unit, electromagnetic, of E.M.F., or the total E.M.F. developed is the total change in the number of lines of force divided by the time in which the change is effected. In the notation of the calculus, at any instant it is $\frac{dN}{dt}$.

If a conductor $AAAA$ (Fig. 116) inclose a space one centimeter wide perpendicular to the lines of force in a field of unit strength, one line of force (or tube of force) will pass through

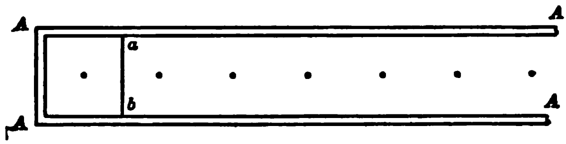


Fig. 116. Induced E.M.F.

this space in every centimeter of its length. If a conductor ab in contact with AA extends across the space and is moved along from left to right at the rate of one centimeter per second, the inclosed circuit $aAAb$ will be penetrated by lines of force increasing in number by one per second, and an electromagnetic unit of E.M.F. will be generated in this circuit and will send a current around it. If the lines of force are positive upward, the current will flow from a to b. Also if $AAAA$ is very heavy so as to be virtually of no resistance, there will be virtually one unit D.P. between a and b, and if ab have unit resistance the circuit will have unit current.

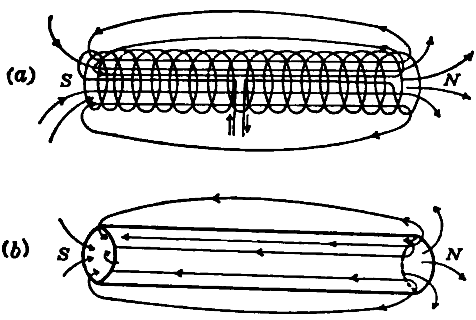
Note. — If ab have a resistance of one ohm, 10^9 c.g.s. units, it would have to be moved across the unit field with a velocity of 10^9 cm. per sec. to send a unit c.g.s. current. With current recognized by its various effects without reference to resistance or to Ohm's law, the resistance of a conductor might be represented by the velocity with which it must be moved across a given field to produce the current.

When a conductor is moved in a magnetic field of force so as to cut the lines of force, an E.M.F. is developed and, if possible, a current is produced whose direction may be determined by this rule (Fleming):

Point the *forefinger* of the right hand along the line of *force*; (i.e. in the direction a north pole would be urged); point the thumb in the direction the conductor is to be moved (i.e., across the lines of force); point the middle finger at right angles to the plane of the thumb and forefinger; it will indicate the direction of the induced current.

Fore	thuMb	mIddle
Force	Motion	Induced

204. Magnetic Field of Force in a Solenoid. — When a series of turns of wire are wound on a cylinder, the leading-in and leading-back wires being led along or parallel to the axis of the cylinder, a current through this apparatus is virtually a series of parallel plane circuits perpendicular to the axis, carrying equal currents all in the same direction. This gives rise to a field of force represented by a bundle of lines along the interior of the solenoid, constituting within the solenoid a strong magnetic field which would induce magnetism in a magnetic substance placed there. The lines of force external to the solenoid are like those external to a bar magnet, but within the solenoid they are in the opposite direction to those within the material of the magnet, as shown in Fig. 117 (*a*) and (*b*). See Watson, Arts. 515, 516.



(*a*) Field of Force in a Solenoid.
(*b*) Field of Force in a Solid Cylindrical Magnet.

Fig. 117.

Experiment No. 82, page 302. — Lines of force in a solenoid.

205. Strength of Field in a Solenoid; Electromagnets. — We have learned that a current in a conductor forming one turn around a circle produces within the circle a magnetic field of force perpendicular to its plane; and in a solenoidal coil of many turns,

a field whose lines thread through the coil parallel to its axis; also that a change in the strength of the current changes the strength of the field; also that with a given current the strength varies with the number of turns of the conductor; but we have not yet determined the actual strength of field due to a given current or given number of turns. The demonstration for this in the case of a solenoid is rather advanced for this course, but when the turns are virtually all in one plane with virtually the same radius for all, we saw, Art. 180, that at the center the strength of field is $\frac{2\pi C}{r}$ in c.g.s. units for one turn and for n turns it is $\frac{2\pi n C}{r}$. If the current is in amperes, A, then since one ampere is only one-tenth of the unit C the number to express C is only one-tenth of that to express A, and C in the formula is replaced by $\frac{1}{10}A$, so the strength of field is $\frac{\pi n A}{5r}$. Within a solenoid of n turns and current C (electromagnetic) it may be shown that the strength of field in c.g.s. units is $4\pi n C$. (Watson, Art. 516, or Carhart's *College Physics*, Art. 609; or Carhart's *University Physics*, Part II, Art. 322.) In any case it is seen to be directly proportional to the strength of the current and to the number of turns of the coil. The actual strength given by the formulæ above is that produced when the space inclosed by the coil is air. Of course such a magnetic field would magnetize by induction a piece of iron placed within it, and how strong the magnetic induction would be depends upon the magnetic properties of the iron, — its permeability and susceptibility, but the very presence of iron in the space results in a much larger number of lines of force threaded through it than were in the space when it was filled with air instead of iron. Such an iron core greatly strengthens the magnetic field inclosed by the electric currents, and itself becomes a powerful magnet. It is called an electromagnet. A break of the current or removal of the lines of induction at once destroys the induced magnetism. Nearly all magnetic effects in practice nowadays are produced by electromagnets, which can be varied in strength, or made or un-

made, or even reversed in polarity, at will. (Observe the current is not sent through the iron core but around it in insulated wire.)

Instead of fuse plugs to prevent overloading a circuit, as described in Art. 199, the winding of an electromagnet is sometimes made part of the circuit, the armature being attached to a switch by which the circuit may be opened or closed. When the current is at the maximum allowable strength, the magnet is powerful enough to pull the armature to it and thus open the circuit. Such an arrangement is known as a circuit breaker.

Illustrations. — Electromagnets, straight or yoke form; electric bells, etc.

206. Self-Induction. — An electric current manifests something like inertia, comparable to the flow of water in a pipe, in its tendency to persist, or to react against change. Not only does a change in the current in one conductor induce a current in a neighboring conductor of a nature to oppose the action that brings it about, but the same thing occurs in the circuit in which the change is first made *while the change is going on.* If in the circuit $ACDB$ (Fig. 118 (a)), the action of the generator AB is increasing so as to make the current rise, a counter E.M.F. is set up in the circuit, tending to oppose this change; on the other hand, if the current already flowing is rapidly diminished or is interrupted, an E.M.F. is set up tending to continue the flow in its original direction. This effect is most readily seen when the circuit is actually broken, making a sudden cessation of the current, i.e., an extremely rapid removal of the lines of force from the circuit. Such an act, even when the current is not very strong and the E.M.F. producing it is perhaps only that of a single battery cell, will show a spark across the gap when the circuit is broken. This indicates momentarily a high difference of potential at the broken termini. It is as if the current around

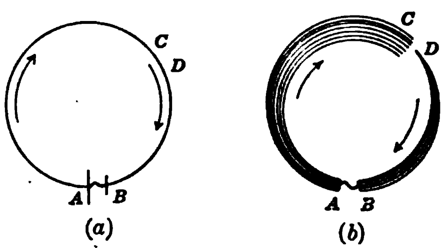


Fig. 118. Self-induction.

from A to B would not stop at once when the conductor was broken, say between C and D (b), but continued to drain off electricity from D and pile it up at C sufficiently to break across the gap while it is very narrow. If by any means the strength of magnetic field within the circuit is great, and is destroyed by a break in the circuit, the E.M.F. of self-induction is great. If $ACDB$, therefore, were a coil of many turns the self-induction would be greater than for a single circuit, and if the coil had an iron core it would be further heightened.

Experiment No. 83, page 303. — Self-Induction.

The practical unit of self-induction is called a henry, and is defined as "the induction of a circuit in which a variation of one ampere per second induces an electromotive force of one volt," i.e., changes the lines of force in number 10^8 per second.

207. The Electric Dynamo. — Electric currents on a large scale for commercial uses are obtained by electromagnetic induction. Let *abcd* (Fig. 119) be a single turn of wire in a magnetic field, directed as in the figure and mounted so as to be rotated in this field. Suppose the ends of the wire to be attached to the two halves of a slotted ring AB, called a commutator, and the coil and slotted ring set rotating in the direction shown (top to the right). When in the position shown in the figure no E.M.F. is produced, for at that instant the number of lines through the coil *abcd* is not changing, and even for a considerable angular movement the change in the number penetrating the circuit

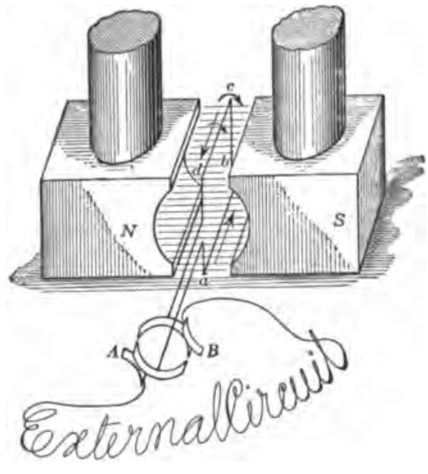


Fig. 119. Principle of the Dynamo.

is small. In one degree there may be a decrease of, say, ten lines, though the total number through the circuit may be large. When the coil has moved nearly a quarter of a revolution, say $89\frac{1}{2}°$, or to nearly a horizontal position, the total number of lines penetrating it will be small, perhaps not exceeding fifty in all, but in the next degree of angular motion, i.e. to $90\frac{1}{2}°$, the number will have decreased to nothing and risen to fifty in the opposite direction, making a *change of one hundred*, and in that time an electromotive force due to a change of one hundred lines will be acting to send a current round the circuit. The magnitude of E.M.F. developed is determined by the rate at which the field is altered. If the above change of one hundred lines were effected in one-thousandth of a second, it would be at the rate of one hundred thousand per second, which would produce one-thousandth of a volt. It would be vastly greater if instead of a single turn of wire the coil had many turns, the rotation were much more rapid, and the strength of magnetic field between the poles many times greater. An increase of lines through in the opposite direction causes a current around *abcd* in the same direction as when they were first decreasing, but this ceases when *dc* comes into the place of *ab*. After that the number decreases, slowly at first, but this sends a current in the opposite direction through *abcd*, and it would also be reversed through the external circuit but at this instant the sliding brush *B* comes into contact with the other half of the divided ring, and so does *A*, and consequently in the external part of the circuit the direction of the current is unaltered. That is the function of the commutator and also the meaning of the name. If, instead of connecting with semicylinders, the ends of the coil were connected each with a slender bar that was in contact one with the brush *A* and the other with *B* at the time the coil was generating its highest E.M.F., and then these bars passed out of contact with the brushes, and if another coil at an angle with the plane of *abcd* connected with another pair of commutator bars, that would come into contact with the brushes while this coil passed through the position of highest E.M.F., this, in turn, to be succeeded by a considerable number of such

coils in planes radiating from the common axis of rotation, each with its own pair of commutator bars coming successively into contact with the brushes in their fixed position, the current would be all the time produced from a maximum E.M.F. The effect is multiplied by making a coil of many turns instead of a single turn of wire *abcd*.

If, instead of having a slotted cylinder as a commutator, *d* is connected with a continuous metal ring or collar which is in sliding contact with *B*, and *a* and *A* are similarly connected with another ring insulated from the first, then the external circuit will undergo the same alternations of current as does the armature circuit or rotating coil.

The magnetic field might be due to a permanent magnet, or, as is commonly the case, a shunt circuit connecting *A* and *B* might carry a part of the entire current around the limbs of a horseshoe of iron making a strong electro-magnet. This would be a shunt dynamo. If the entire current is carried round the branches of the magnet in series with the armature it makes a series dynamo. The former is employed when a constant E.M.F. is desired with a varying load (current), the latter with a constant current but varying E.M.F. A combination of series and shunt constitutes a compound wound dynamo,

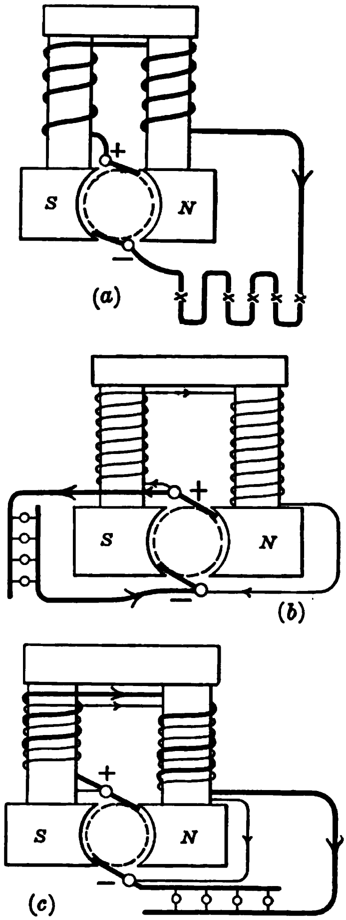


Fig. 120.
(*a*) The Series-wound Dynamo.
(*b*) The Shunt-wound Dynamo.
(*c*) The Compound-wound Dynamo.

which affords a more perfect adjustment of E.M.F. to current. The three types are shown in diagram in Fig. 120 (*a*), (*b*), (*c*).

Except for small or isolated plants, the compound winding is almost always employed for dynamos, the other two modes being employed in motors (Art. 208).

When the coils are wound longitudinally about an iron core rotating with the shaft, they form a "drum armature." Sometimes they encircle a ring of iron, as in Fig. 121, their ends being led to commutator bars, but the coils themselves being connected in series so that the current in the armature coils is continuous from brush to brush. This constitutes a "ring armature" or a Gramme ring. The field of magnetic force then follows chiefly the iron of the revolving ring, and the change of lines of force through the coils varies as in the preceding case. The polarity of the iron ring is fixed in position though the ring itself revolves with the coils. As one coil departs from its maximum generating position its contact with the brushes ceases and the next coil takes its place. Fig. 121 (from *Wüllner's Experimentalphysik*) shows this diagrammatically.

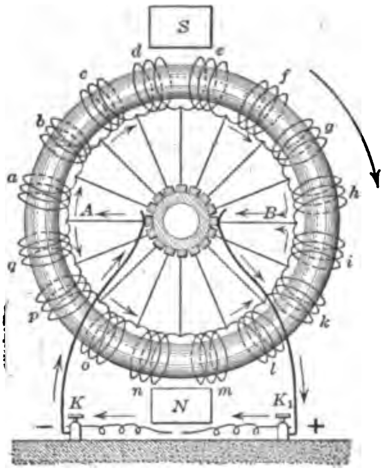


Fig. 121. The Gramme Ring.

208. The Electric Motor. — In either form of armature of a dynamo, when a current flows in a coil of the armature the iron core is magnetized, its poles keeping approximately a constant position relatively to the poles of the field magnets, a position due to that of the armature coil at its maximum generating position. By so adjusting the position of the brushes upon the commutator that when a given coil has its terminals in contact with the brushes the poles of the armature core are in a position to be

strongly drawn by the poles of the field magnets, then a current sent through the winding of the field magnets and the armature from an external source will produce a strong pull, or "torque" (turning moment), upon the armature. When the latter has turned a short distance its poles are no longer in the most effective position; the coil carrying the current breaks contact with the brushes, but the succeeding one makes contact and takes the place of the preceding one and the first condition of polarity and torque is restored; this continues in indefinite succession, and the machine is an electric motor. For instance in Fig. 121, if current is led into the apparatus at K_1 and out at K, there is always a N. pole in the iron ring at the left and a S. pole at the right, and the ring will revolve clockwise.

The dynamo and motor are interchangeable. The dynamo, driven by mechanical power, is called a generator, and acquires an E.M.F. which will send a current if the circuit through the armature is completed externally. If a current is led into the dynamo from an external source it will drive the armature and furnish mechanical power as a motor. The late Professor Clerk Maxwell is credited with saying that the greatest discovery of the third quarter of the nineteenth century was that the Gramme machine was reversible.

Experiment No. 84, page 303. — Cycle of Energy-Changes.

209. The Induction Coil. — A strong current may be produced by a low electromotive force in a circuit of low resistance, and if this is a coil as PP (Fig. 122) with an iron core, it will be penetrated by a large number of lines of force. If the same coil is enveloped by another coil, as SS, having a large number of turns, insulated from the first coil, the break of the current in the first or so-called "primary" circuit will induce a great E.M.F. in the second or "secondary" coil.

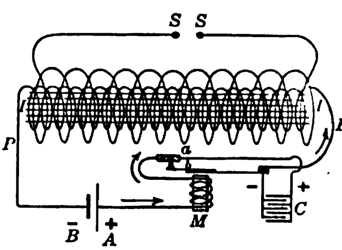


Fig. 122. The Induction Coil.

The sudden closing of the primary induces an E.M.F. in the opposite direction. The primary may be so arranged with a circuit breaker that the closing of the circuit and thus establishing of a current actuates an electromagnet M which attracts an armature in the form of a spring so as to open the circuit at ab; then, the attraction of the magnet ceasing, the spring again closes the circuit, the electromagnet again acts, and so the make and break are continued automatically, precisely as in an electric bell, and the secondary has a high E.M.F. produced in it alternately in opposite directions. Such an apparatus is an induction coil. Instead of the magnetic automatic interruption the make and break is sometimes accomplished by an independent mechanical action. A form of interrupter, Wehnelt's, is sometimes employed in which the interruptions are produced by the formation of gas bubbles due to the passage of the primary current through an electrolytic cell.

If the terminals SS of the secondary are separated, the difference of potential between them may be great enough to cause a spark of considerable length to pass between them. Coils are in use capable of producing a spark a meter in length.

As we have seen, the self-induction in the primary, as well as the induction in the secondary, tends to delay the rising of the current to its full strength when the circuit is being closed and to hinder the stoppage of the current when the circuit is being opened, and, especially in the break, a spark carries on the current so as to make the interruption tardy. This is overcome by joining a condenser across the gap of the primary as at C in the figure. When the gap at ab is closed the current flows through the primary PP in the direction of the arrow. The magnet M at once pulls the armature b and opens a gap between a and b. The flow of self-induction which would expend itself in a spark bridging over the gap ab now goes to charge the condenser C, and the interruption of the primary is made more sudden and complete, as is seen by the diminution or almost complete extinction of the spark between a and b. On the return of b to close the circuit, the discharge of the condenser acts with the battery in

supplying energy to the primary and again helps the action of the coil, besides preventing the loss of energy that would have otherwise been wasted in the spark. The presence of the iron core *II* greatly increases the number of lines of force introduced into the circuit or taken out of it and, of course, vastly heightens the E.M.F. induced in the secondary *SS*. Whatever the value of this E.M.F. for a single turn of the secondary, it becomes n times as great for n turns, so that by making this number enormously large a high E.M.F. may be developed from a low E.M.F. in the primary circuit. But this effect is interfered with if the self-induction in the secondary is great; the secondary, therefore, must have fine wire and its resistance becomes enormously greater than the resistance of the primary, so that, although the E.M.F. is large, the actual current in the discharge of the secondary is small. This condition would be recognized in any case, since the energy of the induced current could not be greater than that of the current inducing it. The product of the E.M.F. by the quantity of electricity transferred measures this energy. In the primary there is a large current with a small E.M.F.; in the secondary, a large E.M.F. and a proportionally small current. Or, again, the magnetic field due to the primary is proportional to the ampere-turns of the primary current; the induced current is due to the formation or destruction of this same field, and if the dimensions of the two coils were alike this field would bear the same proportion to the ampere-turns of the secondary. The turns being large in number the current is small.

The nature of the discharge from the induction coil, its charging of Leyden jars or other condensers, and other features of static electricity are like those produced by static electric machines.

Illustration. — Exhibition and operation of induction coil.

210. **Transformers.** — With two different sets of windings around an iron core, either one may be used as a primary and the other as a secondary. If one has, say, ten times as many turns as the other and the same change of magnetic field is effected in both, then a break of the current in the shorter circuit

will induce an E.M.F. ten times as great in the other, and *vice versa.* A contrivance so arranged is called a transformer, and is a "step up" or a "step down" transformer according as the induced E.M.F. is higher or lower than that of the primary. For commercial transformers an alternating current is best adapted since that accomplishes the greatest change of field at a rapid rate. Electricity generated at a high potential is transmitted over a small conductor since the current is small even for a large transference of energy. Then by suitable transformers it is reduced to a voltage proper or safe for introduction into houses or shops. For heating and lighting the alternate current may be used as well as the direct; for power purposes, however, the motor that will work with an alternating current is not as satisfactory as the direct current motor, and therefore when electric power is sent out in alternating form it is used to drive one large alternate current motor which in turn drives the armature of a direct current dynamo, either on the same shaft as its own, or by gearing to it.

211. The Telephone and Telegraph. — Without going into particulars of central-station connections the principles of the telephone are shown in Fig. 123. A disk D is set vibrating

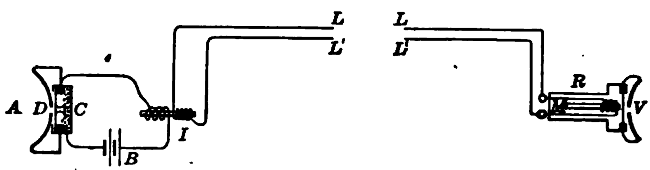


Fig. 123. Local Battery Telephone Circuit.

by the voice spoken into the transmitter A. This disk makes contact with one or more granules of carbon in the back of the instrument as at C. Varying pressure between D and C causes varying resistance in the circuit of the battery B through C, D, and the primary of a small induction coil I. The secondary of this induction coil forms a circuit to the distant receiver R. In this receiver a permanent magnet M has one pole encircled by a coil of fine wire which is part of the circuit with the secondary

of the coil at the transmitting station. Variations of the current in the primary circuit of *I* cause induced currents in the line to the receiving station. The magnet *M* attracts an iron disk *V* and the varying current through the line *L* causes corresponding variation in the attraction of *M* upon *V*, so that the latter vibrates in exact response to the vibrations of *D*. A duplicate arrangement in the sending and receiving stations makes the apparatus to work both ways. The circuit from *B* is open until the removal of the receiver from the hook closes the circuit. In most cases, now, the local battery *B* is replaced by a larger battery at the central station. The telephone is essentially an apparatus of induced currents.

The telegraph, on the other hand, is operated by electromagnets, the signals being due to the making or unmaking of magnets by closing or opening an electric circuit by means of a key. Nearly all electric apparatus for mechanical purposes is operated in a similar manner by electromagnets.

212. Thermoelectricity. — If a circuit be composed of different metals, say, two for simplicity, joined in series, and one junction is heated while the other junction is not changed in temperature, a current flows around the circuit. The electromotive force thus set up is called thermoelectromotive force, and two metals so arranged are called a thermocouple. Suppose the couple to consist of copper and iron. If one junction is kept at the ordinary temperature while the other is gradually heated, the current will pass from copper to iron through the hot junction and increase as the temperature is raised until it reaches a maximum. With further rise of temperature the E.M.F. falls off and the current decreases. The temperature at which this maximum E.M.F. is reached is called the neutral temperature for those two metals. The decreasing current beyond this ceases when a temperature is reached that is as high above the neutral temperature as the unheated junction is below it. At a still higher temperature the direction of the current is reversed. The rate at which the E.M.F. rises per degree of increase in temperature is called the thermoelectric power of the metals. It is, of course, different

at different temperatures, being zero at the neutral temperature. It is different also for each different pair of metals.

Antimony, iron, copper, silver, tin, lead, nickel, bismuth, form a series such that if a junction of any two be heated a current will flow from the earlier in the list to the later through *the external circuit*, provided the mean temperature of the two junctions is below their neutral temperature. The actual E.M.F. set up is low. The thermoelectric power and the E.M.F. for the various metals may be obtained from suitable tables and diagrams. Antimony and bismuth give the highest E.M.F. in the list. Taking lead as a basis to compare with other metals, the thermoelectric power of selenium is about *thirty times* that of antimony. In a copper-iron couple, as described above, if one junction is at 20° C. and the other at 100° C., the E.M.F. is 524 microvolts, (0.000524 volts). That is, a thousand such couples in series would have about one-half the E.M.F. of a gravity cell. (See Watson, Arts. 501–503.)

Experiment No. 85, page 303. — Thermoelectric current.

213. Chemical Effect of a Current; Electrolysis. — When a current passes through a liquid conductor other than metals, the liquid is decomposed. This effect of the current is called *electrolysis*, and the liquids *electrolytes*. They are usually dilute acids or aqueous solutions of metallic salts. The solid conductors by which the current is supposed to enter and to leave the liquid are termed *electrodes;* that by which the current enters is connected with the positive pole of the source of current and is called the *anode*, the other, by which the current leaves, is the *cathode*.

Suppose the current to pass through the series of four vessels (Fig. 124), in which *C* contains dilute sulphuric acid with the electrodes inserted in two inverted tubes that are filled with the same liquid, *D* and *E* contain a solution of copper sulphate, and *F* a solution of silver nitrate. The electrodes in *D* are plates of copper, and those in *C*, *E* and *F* are sheets of platinum. When the current has been flowing for some minutes, equally strong everywhere, the tube over the anode in *C* will be found to contain

oxygen gas which has risen to the top of the tube, and that over the cathode will contain hydrogen gas in quantity twice as much as the oxygen, in *D* the copper anode will have been partially eaten away while the cathode will have metallic copper deposited upon it, the solution remaining little altered in concentration; in *E* the anode will have given off oxygen, copper will be deposited on the cathode, and the concentration of the solution will be diminished; in *F* oxygen will have been given off at the anode and silver deposited on the cathode, while the solution will be

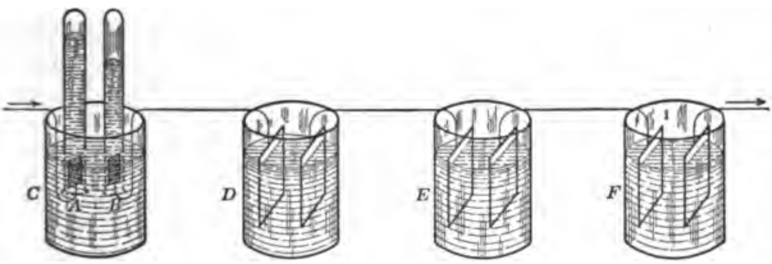


Fig. 124. Electrolysis.

weakened. Electrolysis will have occurred in each vessel. Where the electrolyte is acidulated water, the result of dissociation is simply hydrogen liberated at the cathode and oxygen at the anode; in the case of many electrolytes, however, secondary chemical reactions occur in the electrolytic cell, but the final result is the liberating of one substance at one electrode and a different substance at the other. These final products in their elementary form are called *ions*, those which appear at the anode being anions, and those at the cathode cations. In the electrolysis of a metallic salt the cation is the metal that is in the compound. Its deposition on the negative electrode is electroplating. In the experiment just described the cathode in *D* and *E* is plated with copper, and that in *F* with silver.

Experiment No. 86, page 304. — Electrolysis.
Experiment No. 87, page 304. — Current Sheet (see *Art. 196*).

The laws of electrolysis, as determined by Faraday, are as follows:

(1) "The mass of an electrolyte set free by a current of electricity is directly proportional to the quantity of electricity which has passed through the electrolyte." If m grm. of a substance are deposited by the passage of q units of electricity, then m is proportional to q, or the ratio of $\frac{m}{q}$ is constant for that substance. The mass deposited is the same by a weak current kept up a long time as by a strong current in a short time if the quantity of electricity transferred is the same.

(2) "If the same quantity of electricity passes through different electrolytes, the masses of the different ions deposited will be proportional to the chemical equivalents of the ions." If the same current passes through acidulated water and solution of copper sulphate, for every gram of hydrogen liberated there will be 8 grm. of oxygen and 31.6 grm. of copper.

The relation is simplified by referring at once to the *electrochemical equivalent* of a substance, by which is to be understood the mass of the substance deposited by the passage of one unit of electricity. If we call the electrochemical equivalent g, and m grm. are deposited by q units of electricity, then the mass deposited per unit of electricity is $\frac{m}{q}$, or $g = \frac{m}{q}$. Now if the current strength is c and the time it has been in action is t, then $q = ct$, and $g = \frac{m}{ct}$. Thus by observing the mass deposited in any length of time by a known current, the electrochemical equivalent of any substance may be determined. An instrument for such purpose is called a *voltameter*. By making such determinations with various substances it is found that their electrochemical equivalents are proportional to their chemical equivalents.

If the current is measured in amperes, the unit quantity of electricity is the quantity conveyed by a current of one ampere in one second, or a coulomb, and this quantity liberates 0.000010366 grm. of hydrogen, which is therefore the electrochemical equivalent of hydrogen. Calling the chemical equivalent of hydrogen

unity, that of silver is 107.88, and accordingly the electrochemical equivalent of silver is 0.000010366 × 107.88, or 0.0011183. This rate of deposition of silver, it will be remembered, was made the basis of the international unit of current strength. (See Art. 188.)

As many grams of a substance as the number expressing its chemical equivalent are called a gram equivalent of that substance. A gram equivalent of silver is 107.88 grm., and to deposit one gram equivalent of silver would require $\frac{107.88}{0.0011183}$, or 96,470 coulombs, and this is the quantity of electricity that will cause the separation of one gram equivalent of any kind of ion.

EXAMPLES. —

1. How many grams of hydrogen will be liberated in one minute by a current of 5 amperes through acidulated water? *Ans.* 0.003108 g.
2. How much silver will be deposited from a silver solution in two hours by the passage of a current of 2 amperes? *Ans.* 16.1 g.
3. If a current of 3 amperes through a solution of copper sulphate deposits 3.52 grams of copper in one hour, what is the electrochemical equivalent of copper? *Ans.* 0.000326.

214. Migration of the Ions. — It is supposed that electricity passes through an electrolyte by a sort of convection, being carried by the ions, each cation carrying a definite positive charge in the direction of the current, and each anion a definite negative charge in the opposite direction. The theory requires the supposition that there are always a number of free ions in the electrolyte, but it is possible to derive a pretty definite value for the velocity with which the ions travel. For example, in HCl at 18° C., with a potential gradient of one volt per centimeter, the migration velocity u of the cation H is 311×10^{-5} cm. per sec.; and for the anion Cl it is $v = 78 \times 10^{-5}$ cm./sec., a slow travel. A fluid in this condition is said to be ionized. Hydrogen behaves as a metal. The whole theory of ionization including solution pressure and osmotic pressure is much too elaborate for presentation here. It plays an important part, however, in the theory of the voltaic cell. It is extensively treated in Watson's *Physics*, Book V, Part VIII.

215. Polarization. — One result of the carrying of charges by ions is to electrify the cathode in the liquid positively and the anode negatively, and to that extent to introduce a tendency to send a current in the reverse direction (sometimes called counter-electromotive force). In the simple acid-water voltameter, the cathode quickly becomes covered with bubbles of hydrogen and the anode with oxygen, the former being electropositive and the latter electronegative, and the difference of potential between the plates on that account may rise to more than two volts. Unless the external E.M.F. exceeds this, the current will weaken and finally cease. This effect is termed polarization. If such a cell be disconnected externally from its source of E.M.F., it is then itself capable of being discharged, something like a condenser, and if its terminals be connected through a galvanometer a current is shown for a brief time by the latter, in the opposite direction to that by which the cell was charged.

216. The Voltaic Cell. — Two unlike substances, when brought into contact, come to a different potential, called contact potential difference. If two solids dip into a liquid conductor which acts chemically upon either of them, the final difference of potential between the terminals of the metals, called "poles," will usually be higher than that when there is no chemical action.

The typical arrangement for this is a plate of copper and one of zinc, not touching each other, in dilute sulphuric acid. If the zinc is pure (or if it is amalgamated with mercury), it is attacked by the acid only momentarily until the copper pole, outside the acid, rises to a potential 1.08 volts higher than the zinc pole. Internally an equilibrium is established between the solution pressure of the metals in the acid and the electric forces due to the charged ions, and chemical action then ceases. When, however, the poles are joined by an external conductor, electricity carried through it from copper to zinc destroys the equilibrium in the cell, and chemical action is resumed, the energy that is supplied being directly that of the chemical combinations taking place within the cell. Such an apparatus is a voltaic cell or element. In the course of its action the current passing from zinc

to copper within the liquid polarizes the cell just as in electrolysis, unless some means, chemical or mechanical, are devised to take up or get rid of the hydrogen that appears at the copper plate, or to neutralize the polarizing action. Some forms of battery cells are made of materials which so act as to depolarize at the same time that they polarize. Such cells are called constant, and are of especial service where the action of the battery is to continue a long time without interruption. The best type is the Daniell or gravity cell. Others depolarize slowly, and regain their normal condition if allowed to rest after they have been used for a brief time. They are used on what is called open circuit work, i.e., for intermittent service, as telegraph, telephone, or call-bell service. Dry cells are all of this type. Innumerable forms of cells have been used, which must be studied in special works. Such as do not first require the action of an electric current to put them into condition for producing a current are called primary cells.

Experiment No. 88, page 305. — Polarization and depolarization.

In the Weston Normal Cell, one pole is of pure mercury covered with a paste of mercurous sulphate. On this lie crystals of cadmium sulphate which are covered by a saturated solution of cadmium sulphate. The other pole is cadmium amalgam covered by crystals of cadmium sulphate and those by saturated solution of cadmium sulphate. The internal resistance of such a cell is about 900 ohms. Its E.M.F. at 20° C. is 1.0183 volts and at any temperature, t, is given by the equation

$$E_t = E_{20} - 0.0000406(t - 20) - 0.00000095(t - 20)^2 + 0.00000001(t-20).^3$$

As a standard of E.M.F. this has taken the place of the Clark cell mentioned in Art. 188. The volt, then may be defined as the $\frac{10000}{10183}$ part of the E.M.F. of the Weston Normal Cell, at 20° C.

217. The Storage or Secondary Battery. — It was pointed out (Art. 215) that an electrolytic cell becomes a source of current on its own account after it has been in action. If a cell is made by putting into dilute sulphuric acid two lead plates that are coated with lead peroxide, the coating becomes a paste of lead sulphate. If a current is passed through this cell one plate, by loss of oxygen, becomes covered with pure

spongy lead, and the other is coated with lead peroxide. These have a high difference of potential, and, moroever, their condition chemically is unstable. A continuance of the charging will finally produce a difference of potential of about 2.5 volts between the terminals, and chemical energy will have been expended to a definite amount in charging the cell. If, now, the source of the charging current is disconnected, the storage cell is in condition for use. Connecting its terminals to form an external circuit, reverse chemical action occurs in the cell, and a current is produced which will continue until the energy of the inverse chemical action has been expended electrically. Various other forms of secondary batteries have been devised.

The electromotive force of any cell rises to an amount that is determined, not by the amount, but by the kind of chemical reactions that go with it. The E.M.F. of any cell, then, is determined by the nature of the materials that compose it and not in any degree by their quantity or size.

218. Joining Cells to Form a Battery. — Properly speaking, one combination of plates and liquid constitutes a cell or an element; a battery is a combination of two or more cells, though in careless speech it is not uncommon to call a single cell a battery. When several cells are joined with the positive pole of one to the negative of the next, successively, they are said to be joined tandem or in series; the E.M.F. of the combination is the sum of them all, and the battery resistance is increased in the same way. When the positive poles are joined together they have a common potential, and so will the negative poles if so joined, and when so connected they are said to be joined parallel, or in multiple. The E.M.F. of the combination then is the same as that of a single cell, and if n such cells are thus joined, the internal resistance becomes $\frac{1}{n}$ part of that of a single cell. It may be shown that the maximum current from a voltaic battery will be produced when the cells are so combined that the total external resistance equals the total internal resistance.

219. Displacement Currents. — (See Watson, Art. 577 and Arts. 585–595, also Glazebrook's *Electricity and Magnetism*, Art. 255.)

When two plates are separated by a dielectric, as A, B (Fig. 125), they constitute a condenser and may be brought to a high difference of potential by being charged from a source of E.M.F., as at E. The effect of E in charging the plates has been regarded as a displacing of positive electricity in the direction of the current through the dielectric, and of negative electricity in the opposite direction, and this displacement evokes a condition of strain in the dielectric. This strained condition may rise to the extent of overcoming the ability of the dielectric to withstand it, and discharge the condenser across the dielectric from A to B. Without going so far as that, however, if E is replaced by a continuous conductor from A to B, or even if there is a small gap there, occupied by a dielectric that is not able to withstand the difference of potential between A and B, then the condenser will discharge suddenly through AEB. This is the way a Leyden jar is discharged by means of a discharger. Instead of the discharge being final in a single action, however, it is found that B rises to a higher potential than A as if the electricity had possessed inertia, and this is followed by a surging back of the discharge from B to A through BEA, a process which is repeated thousands of times in the small fraction of a second that is occupied in the apparent discharge of the condenser. This oscillatory discharge is accompanied by corresponding alternations of stress in the dielectric between A and B, and also in that at E if there be one there. These alternations of stress in the dielectric are ascribed to what Maxwell has termed "displacement currents." Every such alternation of stress between A and B due to a displacement

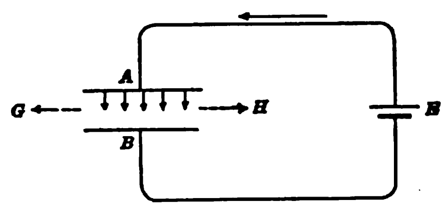


Fig. 125. Displacement Currents.

current, no matter how brief, is extended outward through the ether of space in the form of electric waves at right angles to the direction of the displacement current, much as water waves would move out from a vertical rod that is moved up and down through the surface of the water. Not only so, but as every electric current has a magnetic field of force represented by lines of force encircling it, in the figure, if displacement currents pass between *A* and *B*, electric waves move out toward *G* and *H*, and at *G* and *H* there will be magnetic force perpendicular to the plane of the paper, and alternating in direction. These last alternations are electromagnetic waves which traverse space simultaneously with the electric waves and always have a direction at right angles to them.

220. Wireless Telegraphy. — The electric waves discussed in the preceding article are the means of wireless signaling through space. An account of the refinements of theory and practice by which results are obtained on a large scale is beyond the scope of these lectures, but the essential principles are, briefly, as follows:

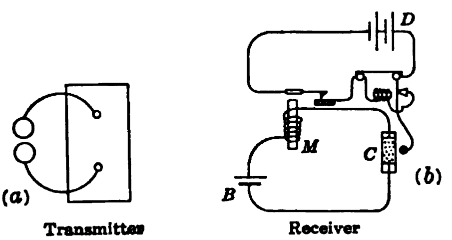


Fig. 126. The Principle of Wireless Telegraphy.

A pair of metal balls (Fig. 126 (*a*)) several centimeters in diameter are placed near each other and are connected with the terminals of an induction coil. This is the oscillator or transmitter. The induction coil being set in action, sparks of high energy pass between the balls, and this intermittent discharge sends out electric waves.

The receiver (Fig. 126 (*b*)) in the earlier form consisted of a circuit of one or two battery cells *B*, the external circuit containing a relay magnet *M* and another portion of poor conductivity. This was a tube *C* containing metallic filings. The current through this circuit is too feeble to actuate the magnet of the relay, but under the impact of electric waves the

filings become a much better conductor. Then the local current actuates the magnet which closes a circuit from a stronger battery *D* that may ring a bell or work a recording apparatus. The filings do not decohere readily when the waves cease, so some contrivance is needed to accomplish this. The tube is struck automatically by the hammer of the bell or other apparatus which is giving the signal, and thus the decohering is effected and the current through *C* ceases until another wave or train of waves excites the coherer. The action thus may be made to continue a long or short time by manipulating a key at the induction coil of the transmitting station, and the message will be perceived by the intervals, as in telegraphy.

In later practice the coherer, the relay magnet, and battery *D* have been dispensed with, and a different form of detector is employed. Several varieties are in use, known as crystal, or electrolytic, or audion detectors. In each case the detector is connected in series with the battery *B* and a sensitive telephone, one end of the circuit leading to an aerial wire and the other to the ground. In some instances the battery *B* also is omitted, proper connections being made to the two sets of plates of a condenser, and the telephone receiver is connected across the condenser. The "aerial" is a long conductor of several wires, suspended in mid air, one end being connected to one pole of the transmitter (or of the receiver). The other pole of the transmitter is then grounded. The aerial sends the alternating waves through or from the apparatus. The detector has the peculiar effect of transmitting the impulses in one direction only, making an intermittent effect in the telephone. Electric waves travel through space with the velocity of light.

(Should be illustrated with students' or larger, wireless outfit.)

221. Discharge of Electricity through Gases. — If two metal conductors are separated by a gas, a high difference of potential is necessary to overcome the resistance of the dielectric between them for a gap of a few centimeters. The necessary potential difference depends somewhat upon the form of the terminals between which the discharge occurs, but with the ends of the second-

ary of an induction coil separated by one centimeter of air that has not been electrified, a difference of potential of from 10,000 to 30,000 volts is necessary to cause a spark discharge across the opening. For a gap of 10 cm., 72,000 volts are required, and for distances greater than this the voltage required before a spark will pass is given approximately by the equation

$$V = 4800\,d + 24{,}000,$$

where d is centimeters and V is the maximum difference of potential in volts. (*Electrical World*, Dec. 10, 1914; see also Art. 167, Note.)

If the gas through which the discharge occurs is inclosed in a tube, into which lead the terminals of an induction coil or an electrostatic machine, and the tube can be connected with an air pump, so that the gas can be extracted and a more or less perfect vacuum formed, the electric discharge between the terminals undergoes remarkable changes in character as the pressure is reduced.

If the apparatus is capable of producing a spark of, say, 10 cm. in open air, and the terminals in the tube are, say, 20 cm. apart, there will at first be no discharge in the tube when the coil is set in operation. As the pressure is reduced there will presently appear between the electrodes a thin quivering streak of light surrounded by a violet sheath; with further diminution of pressure to, say, 2 mm. of mercury, the tube is filled with a diffused violet light, and when the pressure is as low as one millimeter this light breaks up into laminæ or striæ in planes perpendicular to the line of discharge between the electrodes. The tube of gas in this condition is known as a Geissler tube. A much smaller potential difference here will produce the discharge. A potential that will bridge a gap of four or five millimeters in air will produce a discharge through a Geissler tube of 20 cm. or more. Such tubes are prepared with various gases in them, for the purpose of studying the light that is peculiar to each gas on its own account. In a Geissler tube the line of discharge follows the shape of the tube from one electrode to the other, whatever may be the number or variety of turns it may have.

As the vacuum is made more complete, say, to a pressure as low as one-thousandth of a millimeter, the gas is so rarefied that the average distance traversed by the molecules between impacts is comparable to the dimensions of the bulb or tube containing the gas. This is the condition described in Art. 95 (1) as a "Crookes' layer," and the tube or bulb is called a Crookes' tube. When this state is reached, the potential difference required for the electric discharge between the terminals within the tube is much greater than in the Geissler tube. The discharge from the negative terminal, or the cathode, now displays new peculiarities. It is said to consist of "cathode rays." These comprise electrified particles of the rarefied gas, and also an order of radiation from the surface of the cathode itself, all in straight lines normal to the surface. The rays carry energy, and where they strike upon the walls of the tube, or upon material within the tube, they heat it, or they produce chemical or mechanical effects.

If the cathode is a concave disk the rays may be focused from it and thus the energy be concentrated. The rays will not follow the shape of the tube, but they may be deflected from a straight path by a magnet. Among their most striking properties is that of causing certain substances to fluoresce; i.e., when they are directed upon glass, or certain minerals, they cause those substances to emit a light of their own. To a slight extent the special cathode rays (not the electrified gas) penetrate the walls of the tube or even a thin sheet of metal, as aluminum.

Illustrations. — Geissler tubes and Crookes' tubes.

The cathode rays are now regarded as minute, negatively electrified particles, moving with great velocity, comparable to the velocity of light.* They are called ions, though such an ion differs in mass and velocity from the ion of electrolysis. A gas is not ordinarily a conductor of electricity, but any gas upon which cathode rays are directed acquires the power to conduct electricity, its particles becoming themselves so electrified that

* The velocity varies with the square root of the potential difference, and is about $\frac{1}{30}$ that of light, or 10,000,000 meters/sec. with a $PD = 300$ volts.

in their freedom of movement they will carry away a charge from a statically electrified body, or facilitate the discharge between two electrodes. This power may be conferred upon gases by other means as well as by cathode rays, but whenever a gas is thus made conducting, it is said to be *ionized*.

From elaborate investigations, notably by Prof. J. J. Thomson, of Cambridge University, England, and later by many other distinguished physicists both in Europe and America, the theory results that the electric charge, e, carried by a hydrogen ion (cation) in electrolysis is equal to that on an ion discharged from the cathode in a rarefied gas, but that the mass of the latter particle is only about one eighteen-hundredth as great as the mass of the hydrogen ion. Professor Thomson called this minute particle a corpuscle, but it is now termed an *electron*, the name ion being more commonly applied to the carrier of electricity in electrolysis.

Note. — See Art. 213. For the hydrogen ion the ratio of quantity to mass, $\frac{e}{m} = 9647$, where e is electromagnetic units, and m is grams. For cathode ray particles the corresponding ratio has been found to be $\frac{e}{m} = 1.772 \times 10^7$. If e is the same in both instances, i.e., if the charge on a hydrogen ion in electrolysis equals that on a cathode-ray particle, the mass of the hydrogen ion must be $\frac{1.772 \times 10^7}{9647}$ times that of the gaseous ion, or the mass of the hydrogen ion is 1840 times the mass of the electron.

222. Röntgen (or) X Rays. — When cathode rays strike upon bodies these bodies emit a species of radiation known as Röntgen rays, from their discoverer, Professor Röntgen, or as X rays as he himself termed it. While the nature of X rays is not certainly known it is pretty certain that they are not material particles like those constituting cathode rays. They are probably wave motion of extreme rapidity, set up in the ether by the impact of cathode particles, and proceeding from the surface of impact. In a Crookes' tube this would mean that the X rays proceed from the walls of the tube; or if the cathode rays are concentrated upon any special part of the tube or upon the body within the tube, then

that part or body becomes the source of X rays. In the commonest form of tube for X-ray service, the anode either is a small sheet of platinum inclined at about 45° to the line of discharge between the electrodes, or it is directly connected to it, and the cathode is of aluminum, cup-shaped, which brings the cathode rays to a focus upon the platinum anode. The latter then emits X rays (see Fig. 127). X rays, like cathode rays, cause strong fluorescence, but penetrate many substances that are impervious

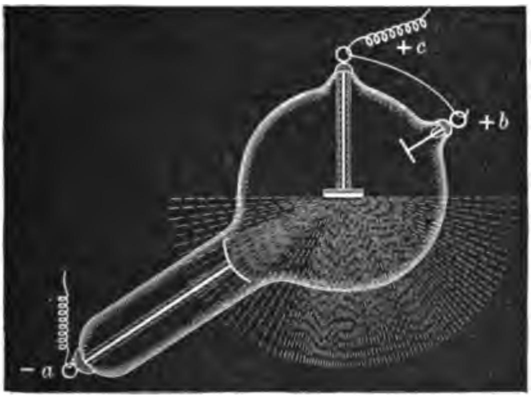


Fig. 127. An X-ray Tube.

to the cathode rays. They also produce strong photographic action, and they ionize gases. They penetrate glass with considerable readiness but are intercepted by metals, while various organic substances vary in the degree to which they permit passage through them. In the tube of Fig. 127, the X rays are emitted from one side of a plate, upon which the cathode rays from the cup-shaped terminal of the cathode *a* impinge, and proceed in all directions on that side, so that the portion of the glass bulb on that side of the plate fluoresces a rich green or blue, while the other part is unexcited and remains dark. The action by which objects may be viewed by means of X rays is not at all like that by which light makes objects visible. Usually a screen of cardboard coated with a fine layer of some fluorescent substance, as tungstate of calcium, for example, forms one end of a

dark box into which the eye can look while all light is excluded. The fluorescent surface is on the inside. When X rays, penetrating the cardboard, fall upon the mineral coating within, the inner surface becomes luminous as an effect of the X rays, and the light from that surface is not X rays but common light. If an object, as the hand, is placed against the outside of the screen, it intercepts in some measure the X rays; the flesh permits the rays to pass through without much hindrance and therefore to cause slightly diminished fluorescence; the bones are more impervious, and consequently the part of the screen covered by them is sheltered and the intercepting object looks dark; thus this shadow picture reveals the bones dark, in a hazy envelope of lighter tissue, upon a still brighter field. The box for viewing it is a fluoroscope. (Exhibit X rays.)

223. Radioactivity. — From the mineral pitchblende have been derived several compounds, notably salts of the metals uranium, thorium, and more recently radium, which emit a species of radiation that possesses many of the properties of cathode rays. The most striking effects are photographic action, the causing of fluorescence, and especially the ionization of gases. The power of giving out rays that ionize gases is called radioactivity. The rays from radioactive bodies are separable into three kinds called respectively α rays, β rays and γ rays, each of which has properties peculiar to itself.

The more intensely radioactive substances, notably radium, are continually giving off material, termed vaguely an "emanation," which is itself like a radioactive gas, and which, after a time, *becomes helium*. The energy of radiation is great, and as the radiation seems to proceed from within the substances the bodies emitting it are affected by its passage so as to be kept at a temperature several degrees higher than that of the atmosphere around them. In the course of the transmutation which the substances undergo, they liberate an enormous supply of energy. Special works on the subject are *Radioactivity*, by E. Rutherford, *Conduction of Electricity through Gases and Radio-Activity*, by R. K. McClung,—Blakiston & Co., Philadelphia; and Chapter XV of *The*

Electron Theory, by Fournier, — Longmans, Green and Company.

Professor Crookes devised an instrument by which the fluorescent effect of the radiation from radium is beautifully shown. It is called a spinthariscope. By an eye piece *E* (Fig. 128), in one end of a short metal tube, is viewed a small screen of paper *S*, coated with zinc sulphide. Close above *S* is a small strip of metal, the head *M* of which has been dipped into a solution of radium bromide, and is thus coated with a minute quantity of this radioactive substance, which remains on the metal after it is dry. The radiation from it causes the zinc sulphide to scintillate with constantly changing, intermittent flashes of points of light, due to the bombardment by the particles emitted from *M*.

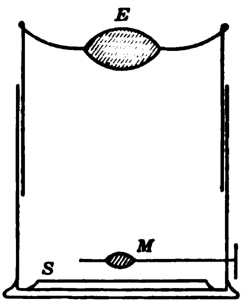


Fig. 128. The Spinthariscope.

224. Electron Theory of Electricity. — (An interesting statement of both theory and experimental facts concerning electrons is given by J. A. Fleming in *Popular Science Monthly* for May, 1902; also see *The Electron Theory*, by Fournier, — Longmans, Green and Company.) The theory is summarized as follows, by Professor Glazebrook (Glazebrook's *Electricity and Magnetism*, Art. 270): "According to the electron theory a neutral atom consists of an electron or series of electrons each carrying its negative charge together with a positively charged nucleus, the total positive charge being equal to the sum of the negative charges on the electrons.

"It is possible in various ways to attach one or more electrons to such an atom; it then becomes negatively charged; it is also by hypothesis possible to detach one or more electrons, the remainder — the coelectron as Professor Fleming has called it — remains positively electrified.

"A univalent atom, like hydrogen, is one which can receive or give up one electron and no more. A divalent atom can receive or give up two electrons, and so on.

"A current of electricity is a stream of electrons; a body through which the electrons pass freely is a conductor; within a nonconductor they cannot move about readily. A gas may be nonconducting; because of the absence of electrons; if they are introduced it gains conductivity. All the phenomena of electric discharge and current are convection phenomena.

"When electromotive force is applied to a conductor, the electrons are urged through the conductor; if it be a gas at low pressure they stream from the cathode as the cathode rays.

"In an electrolyte in solution, some of the free ions are positive; they are coelectrons, and the electrons which have left them have joined on to other ions, making them negative; there is probably a continual interchange going on, but on the average the above statement represents the position.

"The negative ions are driven by the E.M.F. to the anode, the positive ions travel to the cathode.

"In a solid conductor the same kind of separation and combination of ions and electrons is taking place, but the ions are not free to move; the current is conveyed by the electrons moving on from ion to ion through the solid; the solid is porous to them but not to the ions."

If the quantity of electricity constituting the charge upon an ion is called e and the mass of the electron m, the ratio of $\frac{e}{m}$ for an electron is about eighteen hundred times as great as for a hydrogen ion in electrolysis, and if the charge e may be assumed to be alike in both ions, it follows that the mass of an electron is only about one eighteen-hundredth that of a hydrogen ion. The actual amount of the elemental charge e has been determined by various methods with fairly consistent results. Among other important conclusions, these give for accepted values at present, $e=4.7\times10^{-10}$ electrostatic units, this would equal $4.7\times10^{-10}\div(3\times10^{10})$ or 1.57×10^{-20} electromagnetic units. The number of molecules in one cubic centimeter of gas at 0° C. and 76 cm. pressure is 2750×10^{19}; the

number of molecules in a gram molecule* is 6.16×10^{23}; the kinetic energy of agitation in ergs of a molecule at 0° C., 76 cm. pressure is 5.57×10^{-14}; the mass in grams of an atom (half a molecule) of hydrogen is 1.63×10^{-24}. See Kaye and Laby's Tables, pp. 97, 98.

225. Electric Actions Summarized. — In our survey of current electricity we have come to recognize

As sources of electromotive force,

Friction or cleavage.

Motion of a conductor in a magnetic field.

Contact of different substances.

Chemical Combination.

Heat.

As effects of currents,

Heat.

Magnetization.

Attraction and repulsion of conductors.

Chemical decomposition.

Secondary effects of cathodic discharge.

As the nature of a current, the transfer of electrification, which may be the

Propagation of a strain (or relaxation) through the ether of space or of a dielectric; or

The travel of electrons through solids, liquids and gases.

* A gram molecule is the weight of gas whose volume at 0°, 76 cm. is 22,400 c.c.; i.e., the volume of 32 g. of oxygen. Molecular weight is the weight in grams of a gram molecule; i.e., it is 2.016 grams multiplied by density of the gas compared to hydrogen. For hydrogen it is 2.016; for oxygen it is 2.016×15.88 (= 32). In round numbers it is usual to take density of hydrogen, 1; of oxygen, 16; molecular weight of hydrogen, 2; of oxygen, 32.

EXPERIMENTS TO ILLUSTRATE CHAPTER V.

Experiment No. 67, Art. 156. Formation of Magnetic Lines of Force.

Place a pane of dry glass over one or more magnets, bar or horseshoe, and sprinkle over the glass dry, fine iron filings by sifting them through a piece of gauze. On tapping or jarring the glass the particles of iron arrange themselves along lines of force in the plane of the glass. Project upon the screen.

Experiment No. 68, Art. 156. Law of Magnetic Force.

NS (Fig. 129) is a long powerful magnet, placed horizontally at right angles to the plane of the magnetic meridian, and *ns* a magnetic needle with its pole, say, 30 cm. from the opposite pole of *NS*. If *ns* is slightly deflected and released it will oscillate with a period due to the force of *N*, the direction of which is in the line of the two magnets. The conditions of oscillation of *ns* are similar to those of a pendulum under the force of gravity. For such a pendulum it was seen, Art. 26, that the time of oscillation varies inversely as the square root of the force. Observe the period T of oscillation of *ns* with N at various distances d from s. It is found that $T \propto d$. If the force is F, by the law of the pendulum, $T \propto \frac{1}{\sqrt{F}}$.

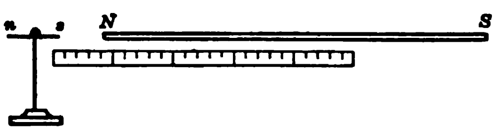


Fig. 129.

Therefore $$\frac{1}{\sqrt{F}} \propto d; \text{ or } \sqrt{F} \propto \frac{1}{d},$$

or $$F \propto \frac{1}{d^2}.$$ Q.E.D.

Experiment No. 69, Art. 159. Induction in Earth's Magnetic Field.

Hold an iron rod *AB* (Fig. 130), 70 or 80 cm. long and 10 to 15 mm. thick, in an east and west position opposite the center of the compass needle *ns*, at a distance of about 10 cm. If *AB* is not magnetized it will cause no deflection of *ns*. Turn it into a position parallel to *ns*, *B* to the north; *A* will now be found to attract *n* and repel *s*, showing that it is a south pole. Move *BA* along horizontally until *B* is in the place of *A*, and its north

polarity will be evidenced by its attraction of *s* and repulsion of *n*. *AB* has become magnetized by induction in the earth's magnetic field. If it is struck two or three times sharply by a hammer while in the north and south position, the magnetism is considerably heightened.

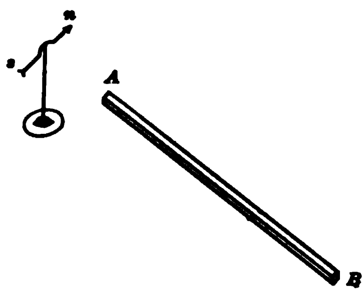


Fig. 130.

Now place the bar in a vertical position, *B* downward and again give it a few sharp taps in this position. While held vertically the end *A* will now show a much stronger attraction of *n* and repulsion of *s*, and when the bar is moved up until *B* is opposite the center of *ns*, the reverse polarity is strongly shown. This indicates that the vertical component of the earth's magnetism is stronger than the horizontal. Finally, the strongest magnetization is obtained by holding the bar in the plane of the magnetic meridian, and dipping downward at an angle of about 70° below the horizontal (at New York), i.e., in the direction of the earth's magnetic lines of force.

With a lantern arranged for vertical projection, *ns* may be projected on the screen, and the influence of the magnetism induced in *AB* shown on a large scale.

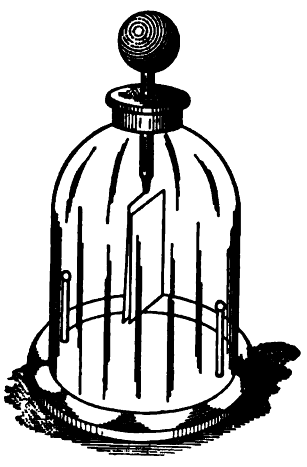


Fig. 131. The Gold Leaf Electroscope.

Experiment No. 70, Art. 164. (*Nos. 70 to 72, incl., illustrate Electrification by Induction.*)

Rub a piece of sealing wax with wool or fur, and bring it near the knob of a gold leaf electroscope (Fig. 131). Negative electrification is driven to the gold leaves which diverge in consequence of their repulsion for each other. While they are thus divergent, owing to the proximity of the sealing wax, touch the knob of the electroscope with the finger. The negative charge escapes while the positive electrification is held by the presence of the negative sealing wax, and the leaves collapse. Take away the finger and then remove the sealing wax; the positive charge distributes itself over the leaves as well as the knob and the leaves diverge with a positive charge. On the approach of a negatively electrified body to the electroscope the leaves sink

together, but the approach of a positively charged body causes them to separate further.

Experiment No. 71, Art. 164.

A striking example of electrical induction is shown in the electrification of a water jet.

Arrange a small nozzle, as A (Fig. 132), so as to give a jet of water slightly inclined to the vertical. If the water pressure in the lecture room is sufficient, the tube T may be connected to a faucet at the lecture table; otherwise it may be used as a siphon from the vessel V, mounted upon a stand.

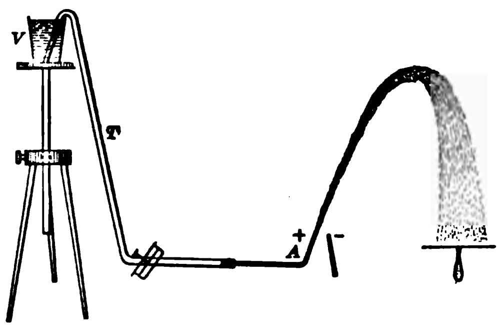


Fig. 132. Electrification of a Water Jet.

The jet separates into drops a short distance above A. If an excited rod, say of sealing wax, is brought close to A, the jet breaks into a broad, fine spray; but if the rod is very feebly excited, or is held a couple of meters beyond the falling drops, they coalesce and form a more compact stream.

In the first case, negative electricity is repelled to earth or to V, and the jet at A is electrified positively before breaking into drops, and these drops repel one another. In the second case, the falling drops are all charged positively on the side next to the sealing wax and negatively on the farther side, and they attract one another, besides having their rotary motion checked.

If the spray falls upon an insulated metal plate, as, e.g., the cover of the electrophorus, the plate becomes charged, and on presenting it to the electroscope, as in the preceding experiment, its charge is found to be of the opposite character to that of the excited rod. On the other hand, if a wire is led from the water in V to the knob of the electroscope it gives the latter an electrification of the same sort as that of the rod. Except for this last test, it is well to connect the water in V by a wire to the earth.

Experiment No. 72, Art. 164.

A thin cake of sealing wax or hard rubber, w (Fig. 133), in a shallow metal case c, is supported by an insulating stand S. A cover plate p of thin metal, e.g., tin plate, a little smaller than w has an insulating handle S'. c should be put in connection with the earth. With p removed, electrify w by rubbing with cat's fur; place p upon w; positive charge is induced on lower side of p and negative on upper; discharge latter by touching p with the finger.

By means of the handle S' remove p; it is charged positively and when brought near the hand or any other body, it communicates its positive charge with a slight spark. p may then be replaced on w and the operation repeated.

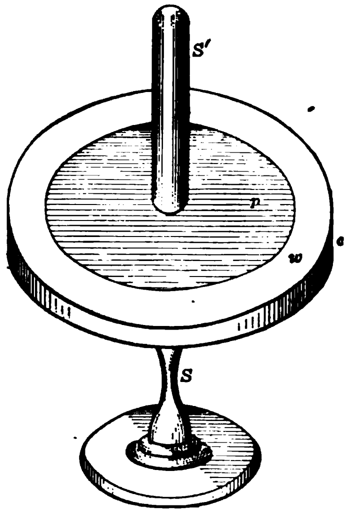


Fig. 133. The Electrophorus.

Experiment No. 73, Art. 175. Static Electrification from Voltaic Battery.

The plates of a condensing electroscope are insulated from each other by a coat of shellac varnish on the surfaces next each other. If the other surfaces are not thus varnished, they are conducting and may be charged by contact with another charged body. Touch the top of the upper plate with the end of an insulated wire from one pole of a voltaic battery, and the under surface of the lower plate with a wire from the other pole of the battery. The plates acquire a charge, their potential difference being that of the battery terminals. The quantity in the charge may be considerable but no effect is seen in the gold leaves. Remove the upper plate and the leaves diverge. The charge may now be tested as to whether it is positive or negative by bringing near the electroscope an excited rod of glass or sealing wax. The electrification from the battery is identical in character with that from friction. The effect is perceptible in a charge from a battery of as little as five or six volts. The experiment may readily be projected on the screen.

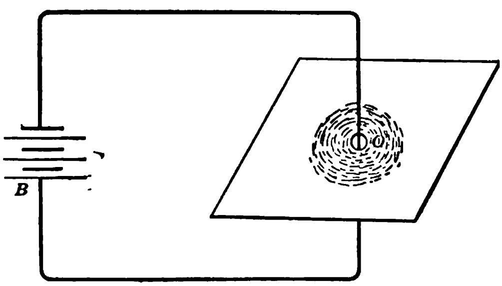


Fig. 134. Magnetic Lines of Force Produced by an Electric Current.

Experiment No. 74, Art. 180. To show the magnetic field around a current.

Drill a small hole in a pane of glass as at O (Fig. 134), and perpendicularly through this lead a wire about No. 16 or 18 in size, that is part of an electric circuit. Pass a current of 8 or 10 amperes through the wire and

sprinkle iron filings on the glass. On tapping the latter the filings will arrange themselves in circular whorls around the wire. Project on the screen.

Experiment No. 75, Art. 180. Oersted's Experiment.

Suspend a slender magnetic needle and let it come to rest in the magnetic meridian. If a straight portion of a wire carrying a current is held in a parallel direction above the needle the latter is deflected; if the current is placed below the needle the deflection is in the opposite direction.

Experiment No. 76, Art. 189. Conductivity of a Liquid.

Solder a disk of sheet copper, as *A* or *B* (Fig. 135), about 3 cm. in diameter, to the end of each of two insulated wires, and place the wires in a tall glass vessel about 4 cm. in diameter, e.g., a graduate glass. Connect the terminals *C* and *D* so as to complete a circuit through an electric bell and two battery cells. Fill the jar with water. Its conductivity is poor and little or no current flows. Add a few cubic centimeters of sulphuric acid, and the resistance of the water is so greatly reduced that the bell rings. By raising or lowering one of the electrodes, as *A*, the variation in the current shows the variation of the resistance of the water column with change of length.

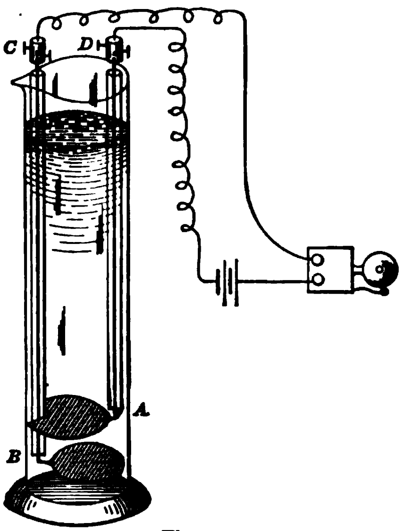


Fig. 135.

By using other liquids in the jar, some idea is obtained of their various conductivities.

Experiment No. 77, Art. 199. Localization of Energy in a Conductor.

Connect in series three or four links of thin platinum wire (about No. 30), 4 or 5 cm. in length, alternately with similar links of bare copper wire, and through the chain pass a current of 5 to 10 amperes, controlled by a rheostat. The same current traverses the copper and the platinum links, but, owing to the greater resistance of the latter, they glow and become white hot while the copper remains dark.

Experiment No. 78, Art. 199.

Through a rheostat connect a No. 26 or 28 iron wire, about 150 cms. long, with the terminals of the 110-volt electric light system, and apply the

current to heat the wire. Gradually increase the current until the metal is white hot. Owing to its expansion by heat the wire sags at the middle as much as 12 cm. From this sag the elongation of the wire may be determined, and from the coefficient of expansion an approximate value of the temperature may be computed, 1200° C. to 1500° C.

Experiment No. 79, Art. 201. (Nos. 79 and 80 show Attraction and Repulsion of Currents.)

Fig. 136. Roget's Spiral.

Roget's Spiral (Fig. 136) is a coil of 30 or 40 turns of copper wire suspended so that its lower end just touches the surface of mercury in a cup. The mercury and the upper end of the coil are in circuit with a battery. The end thus in contact with the mercury should be a tip of platinum wire attached to the copper. The spires should be about 4 cm. in diameter and about 2 mm. apart. With an E.M.F. of 8 or 10 volts a strong current will traverse the wire, and the attraction of the parallel turns will cause a shortening of the coil and a break in the circuit at the mercury surface. Then the coil elongates, contact is again closed, the current is restored and the process is repeated and continued in a steady vertical oscillation of the coil.

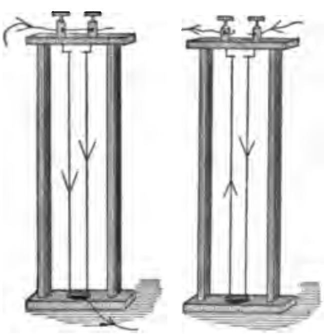


Fig. 137.

Experiment No. 80, Art. 201.

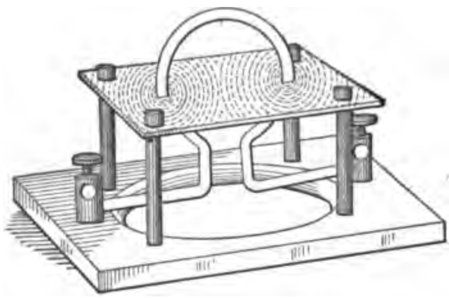
Two wires a meter or more in length (Fig. 137) suspended from binding posts at a distance from each other of 1 or 2 cm., dip into a cup of mercury at their lower end. If the binding posts are connected to the terminals of a battery of, say, 10 or 12 volts, the currents in the two wires are in opposite directions, and the wires separate by repulsion.

If the binding posts are connected to each other and one terminal of the battery is connected to them and the other to the mercury, the currents through both wires are in the same direction and the wires move up closer to each other by attraction.

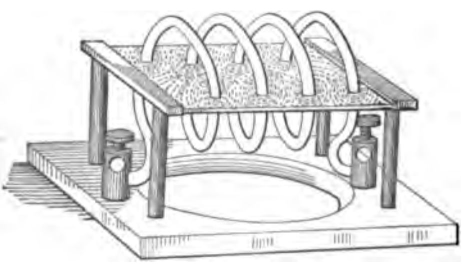
If the wires hang in the light of the lantern, some distance from the screen, their shadows on the screen, and the attraction and repulsion, may be more easily observed throughout the room.

Experiment No. 81, Art. 202.

Currents due to electromagnetic induction may be shown by turning a coil in a field no stronger than that of the earth. Use a spool of many turns, several thousand, of fine wire. (The secondary of a small induction coil may be used.) Attach the ends of the wire to a sensitive galvanometer (preferably a D'Arsonval in which a beam of light reflected from the mirror is focused on a scale). Holding the coil with its axis parallel to the earth's magnetic lines of force, suddenly invert it. *During the turning* a current is instituted, as shown by the deflection of the spot of light. When the coil is suddenly reversed to its first position the light is deflected in the opposite direction. By numerous repetitions, timing the alternating motion of the coil with the movement of the light, the swing of the latter may be made very large. The effect is heightened by placing an iron core in the spool.



(a)



(b)

Fig. 138. Magnetic Field of Force within a Coiled Conductor.

Instead of induction from the earth's field, let the coil remain stationary in any position, and suddenly insert a bar magnet in it; a current is momentarily produced. Suddenly drawing the magnet out of the coil induces a current in the opposite direction.

Experiment No. 82, Art. 204.

The magnetic field of a circular conductor (Fig. 138(*a*)), or of a solenoid (Fig. 138(*b*)), may be shown by winding a wire spirally several turns, 5 or 6 mm. apart and about 2 cm. in diameter, through punctures in a piece of mica. On passing a current of 8 or 10 amperes through this coil and sprinkling iron filings upon the plate, the lines of force are at once

seen. With vertical projection apparatus this and similar experiments may be shown on the screen.

Experiment No. 83, Art. 206. Self-Induction.

Connect a wire to one pole of a battery of 10 or 12 volts, and touch the free end momentarily to the other pole; on suddenly separating them a spark ensues. If now the end of the wire is connected to a rough metal surface, say, a coarse file, and the current is led by the other terminal through a coil of several hundred turns of insulated copper wire, about No. 18, wound upon a lead pencil, and the free end of this coil is drawn along the file, the flashes resulting from the breaks of contact are more intense than before; if the pencil core of the coil is replaced by a corresponding piece of iron the effect is still further heightened, and the sparks emitted on drawing the end of the wire along the file are white.

Instead of a specially wound coil, the primary of a small induction coil may be used.

Experiment No. 84, Art. 208. Cycle of Energy Changes.

If the commercial current at 110 volts, an electric motor of as much as ¼ H.P., and a small dynamo capable of generating 30 volts or more are available, it is an instructive experiment to couple the motor to the dynamo, and from the latter to operate a small arc light, and then trace the complete cycle of energy changes, as, e.g.:

Energy of LIGHT (of the sun) ages ago produced luxuriant vegetation that afterwards formed beds of coal; energy of chemical separation, transformed into heat by combustion of coal in the furnaces under steam boilers; heat, transformed into energy of gas (steam) in the boilers; energy of steam, transformed into mechanical energy of steam engine operating dynamos in power house; mechanical energy of steam engine, transformed into electrical energy of dynamos supplying current; this electrical energy, transmitted along the mains, transformed into mechanical energy in the electric motor in the lecture room; this mechanical energy again transformed into electrical energy in the small dynamo, and this electrical energy again transformed into that of LIGHT (of the arc).

Experiment No. 85, Art. 212. Thermoelectric Current.

Cut a strip of sheet lead 1 cm. wide and 30 cm. long, scrape one end bright and hammer into the bright lead a similarly cleaned end of an iron wire, making an intimate junction of lead and iron. From the other ends of the lead and the iron complete a circuit by any kind of wire through a *sensitive* galvanometer. If the junction of lead and iron is warmed simply by holding it between the thumb and finger, a current through the circuit will be

indicated by a slight deflection of the galvanometer needle or spot of light, and this will much increase if the flame of a match or burner is applied to the junction.

Experiment No. 86, Art. 213.

The electrolysis of water is shown by means of an apparatus like that of Fig. 139. The two tubes are filled with water to which has been added a little sulphuric acid to give it conductivity. An electromotive force of over 2.5 volts is needed from a battery in circuit with the binding posts at the base.

Fig. 139. Apparatus for Decomposition of Water.

With platinum electrodes both dipping into a flat-sided glass vessel of acidulated water, and the latter mounted on the stand of the lantern, the process of electrolysis and the liberation of gas at each electrode may be projected upon the screen.

Experiment No. 87, Art. 213. Lines of Flow in a Current Sheet.

A sheet of coarse filter paper or absorbent paper, about 10 cm. by 15 cm., moistened with a solution of zinc sulphate, is laid on a pane of glass and fine zinc filings are sprinkled over it.

The terminals of an ordinary electric lighting circuit are applied to the sheet thus prepared, 10 or 15 cm. apart, and an electric current passes as from A to B (Fig. 140).

Fig. 140. A Current Sheet.

Lines of dark metallic zinc grow by steady deposition which begins at that point of one or another filing which is toward the anode or positive terminal, producing a set of "lines of flow" resembling the lines as shown by iron filings in a magnetic field of force. Figure 140 is a reproduction of

the lines as they appeared after the paper was dried and *the filings were brushed off.* A T-shaped piece of lead was laid between the electrodes, and as this was a better conductor than the solution, the lines of flow, both from A to the point of the T and from the head of the T to B, are very significant. There is no magnetic action or magnetic substance in the experiment.

Experiment No. 88.— To Illustrate Polarization and Depolarization, Art. 216.

A vessel 5 to 10 cm. in diameter (Fig. 141) (a common table tumbler will answer) contains a layer of mercury. On this is a layer of strong solution of common salt (sodium chloride). A copper wire Cu is dipped into the mercury, the copper being insulated where it is in the salt solution, and a zinc plate Zn is supported in the solution, near but not touching the mercury. If the circuit is completed externally through an electric bell the cell will cause the bell to ring at first strongly, but rapidly diminishing in intensity, as a film, probably sodium amalgam, forms on the mercury. This is cleared up promptly by putting into the liquid a very few grains of mercuric chloride (corrosive sublimate), which reacts to produce sodium chloride and mercury; the depolarization is immediately accomplished, and the bell again rings until the current is again checked by polarization. If the mercury surface is not bright and clean at first, a few granules of the mercuric chloride are needed to start the action.

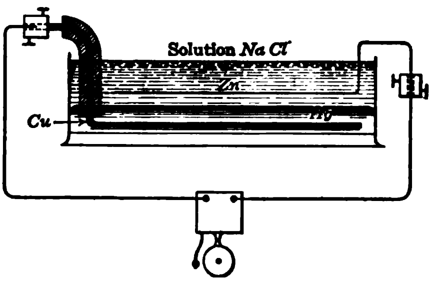


Fig. 141. Polarizing and Depolarizing Cell.

CHAPTER VI.

LIGHT.

226. Nature of Light. — (*a*) *Periodicity.* — In Art. 112 it was stated that "to prove that any phenomenon is due to wave motion it is sufficient to show, first, that it is periodic; second, that it is propagated with a finite velocity." Periodicity in light is most readily evidenced by phenomena of interference. Interference phenomena, which show regions of reinforcement and of destruction that do not shift, could only arise from periodic action in the agent, and such phenomena are easily seen to characterize light, in the succession of light and dark bands from two glass plates separated by a thin film of transparent substance reflecting (or transmitting) monochromatic light, or from the successive spectra when composite light is so reflected (or transmitted). Nothing is concerned in the phenomenon but light itself, and periodicity is at once established.

Newton's rings, and other interference bands are examples.

(*b*) *Velocity.* — That light is propagated with a finite velocity, i.e., that it requires time to proceed from one place to another, has been established by at least four methods, two of which are astronomical and two terrestrial.

(1) In 1676 the Danish astronomer Roemer observed that eclipses of Jupiter's moons recurred at times earlier than the average, when the earth was at the position in its orbit nearest to Jupiter, as at A, Fig. 142; and later than the average when the earth was at the opposite side of its orbit from Jupiter, as at B. In the former case the eclipse is observed after light travels from m to A; in the latter not until light has traveled from m to B, and the difference of time, or the time by which the eclipse is delayed, represents the time required for light to traverse the axis of the earth's orbit, or the distance AB. It amounts to almost

exactly 1000 seconds (998). As the axis of the orbit is about 186,000,000 miles, this gives for the velocity of light about 186,000 miles per second. This method, dealing with long distances, involved quantities that were thought to give the results a high degree of credibility and probability; but the distance of the

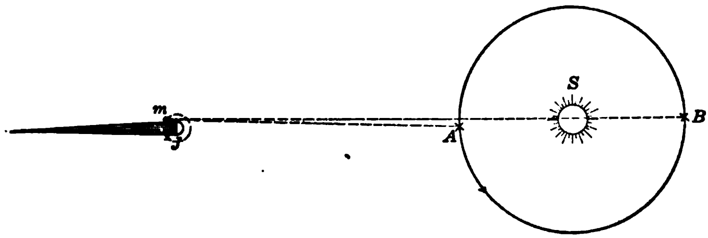


Fig. 142. Eclipse of Jupiter's Moon.

earth from the sun is computed by the use of an angular quantity known as the solar parallax, which is the angle at the center of the sun formed by a line to the center of the earth and one tangent to the earth. This quantity is more accurately known now than it was in Roemer's day, but is still given with a considerable probable error as 8″.80.

(2) About fifty years after the application of Roemer's method came another method due to the discovery of the aberration of light by the English astronomer Bradley. This is also an astronomical problem and depends upon the velocity with which the earth moves in its orbit, and this, in turn, is deduced from the size of the orbit or the distance from the earth to the sun, the calculation of which, as before, depends upon the sun's parallax. But as an independent astronomical method it may be presented in outline. In the first place let us deal with quantities for illustration, more readily within reach of our conceptions than the velocity of light.

Imagine a perfectly calm day with rain falling, and, because of calmness, falling vertically. If we stand quietly and observe it, it will seem to us to fall from the zenith. But if we move onward at a given velocity the rain will seem to approach us or fall in a slanting direction. A tube, which at rest permitted rain to

fall through it while in a vertical position, must now be inclined with the top advanced toward the point approached if the rain is to go through without coming in contact with the sides of the tube. The rain will seem to fall from a point displaced from the zenith in the direction in which the observer is moving. This changed direction is the resultant of compounding the downward velocity of the rain with a *backward* velocity equal to that with which the observer is advancing. A similar effect occurs if we travel to meet or cross the path of any other moving object. In Fig. 143, if ZO represent the actual motion of the rain, and OF that of the observer, ZB will represent the apparent motion of the rain.

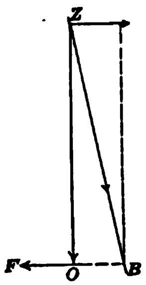


Fig. 143. Aberration.

Now the earth is moving around the sun in nearly a circular orbit at about 20 miles per second. The light coming to us from a fixed star makes the star appear to us always somewhat out of its true position, displaced towards a point 90° in advance of the heliocentric position of the earth. As the earth completes its revolution about the sun the apparent position of the star describes a corresponding, but very small, ellipse against the dome of the sky, about its true position. The angular displacement, however, may be pretty accurately determined, and from that the ratio of the velocity of the earth to that of light itself. "The *maximum* angular displacement thus observed is only about $20\frac{1}{4}''$ . . . or such an angle as would be subtended by a six-inch rule at the distance of one mile." (Stokes, *Lectures on Light*, First Series, p. 10.) The best results give for the ratio of these velocities, 0.0000994. But the final value of the velocity of light depends here upon the motion of the earth in its orbit; taking the mean distance of the sun to the earth as 93,000,000 miles, the velocity of light, by eclipse of Jupiter's moons, is 302,300,000 meters per second; by aberration of light, 299,300,000 meters per second.

(3) *Fizeau's Method.* — The methods of determining the velocity by dealing with terrestrial magnitudes only, are those origi-

nally employed by Fizeau in 1849 and Foucault about 1854. Improved determinations upon these methods have been made in recent years. The method adopted by Fizeau was essentially as follows:

Light from a slit *s* (Fig. 144) is reflected by a mirror *m* to a distant mirror *M*. In the path of this light is a toothed wheel *W*. If the wheel is slowly rotated, light passing between two teeth can go to *M* and return before it is obscured by the next tooth,

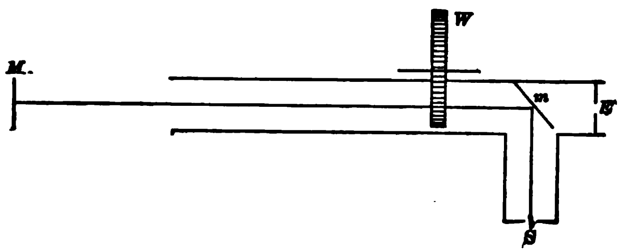


Fig. 144. Velocity of Light by Fizeau's Method.

and therefore the eye at *E* can perceive the light from *S*. If *W* is rotated at such a speed that any light from *m* passing between the teeth can go from *W* to *M* and back to *W* in just the time for the opaque teeth to move into the path of the returning light, i.e., to where the open spaces were, the returning light is intercepted and the eye cannot perceive it; by increasing the speed of *W* light again appears dimly and increases to a maximum when the returning light is perceived not through the same spaces through which it went out from *W* to *M* but through the next ones. Still higher speed of *W* again obscures the light. By careful determination of the speed of *W* and consequently of the time to turn through the space from one tooth to another, and by measuring the distance from *m* to *M*, the velocity of light was determined. With this distance equal to 8333 meters (a little over five miles), the wheel having 720 teeth, it was making 12.6 revolutions per second at the first eclipsing of the light. This gives for *V*, 312,274,000 meters per second.

(4) *Foucault's Method.* — In Fig. 145, *s* is a slit perpendicular to the plane of the paper, *R* a rotating plane mirror, *L* a lens of

considerable focal length. For a certain position of *R*, light coming from *s* will form an image of *s* on the plane mirror *M*, and the axis of the pencil of light will retrace itself, and the whole pencil will be reversed, *M* being placed at right angles to *RL*.

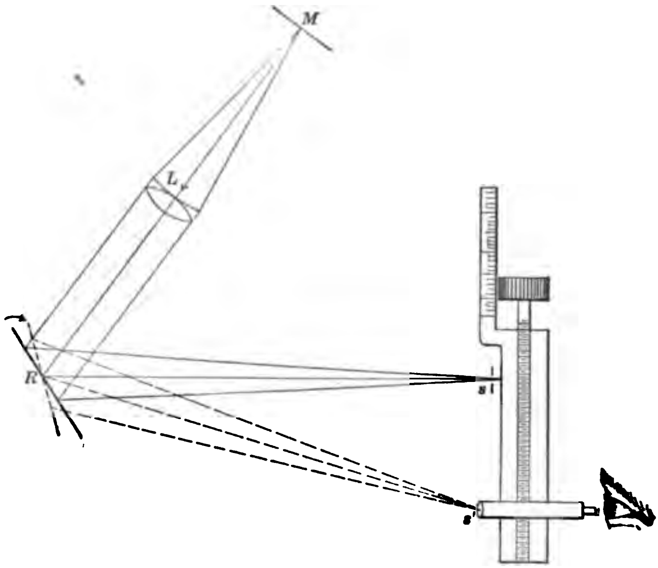


Fig. 145. Velocity of Light by Foucault's Method.

Now if "in the time occupied by the light in traveling from *R* to *M* and back, the revolving mirror has not appreciably altered its position, the return pencil will be reflected back to *s* and an image of the slit formed which will coincide with *s*. If, however, the mirror *R* be revolving rapidly in the direction indicated by the arrow, so that by the time the pencil reflected by *M* reaches it, it occupies the dotted position, the image of the slit formed by reflection will no longer coincide with *s* but will be displaced to some point *s′* to the left of *s*; and the amount of this displacement will depend on the rate of rotation of the mirror and the time occupied by the light in traveling from *R* to *M* and back." (Glazebrook, *Physical Optics.*)

To determine the velocity of light, measure *ss′*, *sR*, *RM*, and the rate of rotation of *R*.

Let $ss' = \delta$, $sR = a$, $RM = c$, V = velocity of light.

τ = time for light to traverse $2\,RM$. θ = angle turned by R in time τ, N = number of complete revolutions of R in one second. Then in one second the mirror turns through the angle $2\,\pi N$, and in τ seconds, $2\,\pi N\tau$. Thus, $\theta = 2\,\pi N\tau$.

By turning the mirror through an angle θ the reflected ray has been deviated from Rs to Rs'; therefore, angle $sRs' = 2\,\theta$; also $ss' = sR.\tan sRs'$, or $\delta = a.\tan 2\,\theta$, so that $\delta = a.\tan 4\,\pi N\tau$. But τ = the time for light to travel the distance $2\,RM$,

$$\therefore\quad \tau = \frac{2\,c}{V}. \qquad \therefore\quad \delta = a.\tan\frac{8\,\pi Nc}{V};$$

but we may take $2\,\theta = \tan 2\,\theta$ and consequently

$$\delta = a.\frac{8\,\pi Nc}{V}, \qquad \text{whence} \quad V = \frac{8\,\pi Nac}{\delta}.$$

Professor Michelson employed values a = 8.58 meters, c = 605 meters, N = 257. The observed value of δ was 0.113 meters, whence V = 296,500,000 m./sec. His final result, making all corrections, gave for velocity *in vacuo*, 299,940,000 meters per second.

By Fizeau's method, repeated by Cornu in 1876,

$$V = 300{,}400{,}000.$$

Fizeau's method, by Forbes and Young, later, gave

$$V = 301{,}382{,}000.$$

The mean of reduced values gives as the best result a velocity of 300,574,000 meters per second for white light in a vacuum. This is 186,770 miles per second.

As there is a small margin of error even in this mean result it is usual to take the even figure 300,000,000 meters per second as the velocity of light. This is 186,400 miles per second. (An account of the four methods is given briefly in Watson's *Physics*, Arts. 365–369, and in Hastings and Beach, Art. 543.) The velocity is different in different media.

The periodicity of light and its velocity having been estab-

lished, those phenomena which characterize wave motion generally, are readily recognized in connection with light.

(*c*) *Character of Vibration.* — It will be seen, however, that certain phenomena can only be accounted for by supposing that the medium transmitting the waves is put in oscillation transversely to the direction in which the disturbance progresses; i.e., the vibrations of the ether are said to be transverse.

It is difficult to understand how a fluid, as the universal ether is usually called, and especially a fluid so exceedingly rare as the ether is, can sustain a shearing stress, which is necessary in the ordinary view of elastic transverse vibrations. Even with small density the elasticity must be very high to give so great a velocity, which, as we have seen, depends upon the ratio of the elasticity to the density. So great is this stumblingblock that the ether has been boldly treated as *an elastic solid* in explaining light. (See especially Sir Wm. Thomson's (Lord Kelvin's) lecture on The Wave Theory of Light, in *Popular Lectures and Addresses*, Vol. I, Macmillan Company.)

But Clerk Maxwell evolved a theory of the propagation of electromagnetic waves that provides for a disturbance of magnetic character in one direction simultaneously with one of an electric character in a direction at right angles to this, and which includes light as one form of electromagnetic disturbance. This is known as the electromagnetic theory of light.

227. Waves and Rays. — Light proceeding from a point through any medium will always progress a definite distance in a given time, and if the medium is homogeneous, this distance will be the same in all directions and the boundary of the region through which the disturbance has progressed will be the surface of a sphere. This surface will comprise the wave front from the source out in any direction. At a given point of the surface a normal line will be the direction of the progress of light at that point. A line traced from the source continuously perpendicular to the wave fronts is a *ray* of light. In a homogeneous medium, or in one throughout which light travels with the same velocity, the ray is straight, and in such a medium light travels in straight

ines, but only in such media. If the medium is one in which light travels at different rates in different directions, then the ray may be curved. If Fig. 146 represents a vertical plane section of a medium in which light from O goes in the horizontal direction twice as fast as in the vertical, then the wave fronts would not be circles, and horizontally and vertically the ray would be straight, but along other paths it would be curved.

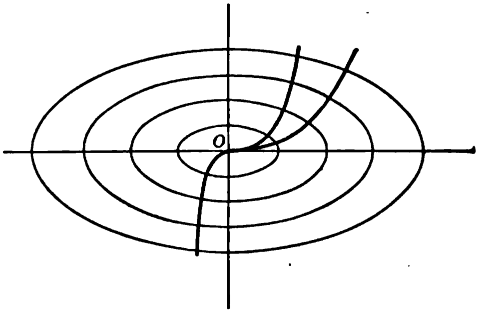


Fig. 146. Curved Rays of Light.

228. Reflection and Refraction. — Light arriving at the surface of any medium is either absorbed by the medium, in which case its energy is usually changed into some other form, or it is transmitted through the medium, or it is reflected. It may be affected in all three of these ways. That part that is reflected conforms to the laws generally applicable to waves.

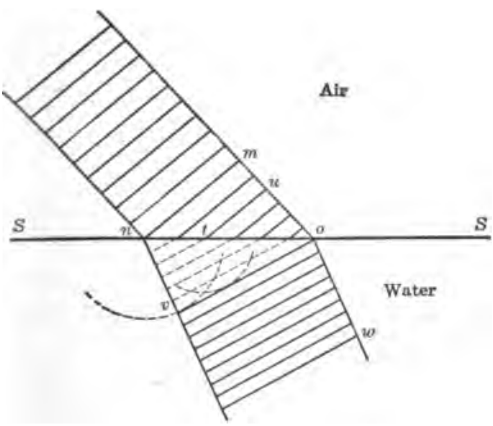


Fig. 147. Change in Position of Wave Front by Refraction.

For that which is transmitted, suppose *mn* (Fig. 147) to be a plane wave front proceeding in air and arriving at a surface *SS* of water. If the velocity of light in water is less than that in air, say, three-fourths as great, by the time the wave front in air would progress the distance *mo*, or the disturbance at *m* would reach *o*, that at *n* would advance into *W* a distance three-fourths

as great as mo, and would therefore be somewhere on a semi-circumference described about n with a radius nv equal to $\frac{3}{4}$ mo. In the same way, while the disturbance is going from u to o that from t goes a distance $\frac{3}{4}$ uo, and in the water the wave front will have the position ov, and the direction of the light will change from mo to ow.

A change in the direction occurring when light passes through media in which its velocity is different is called refraction; the angle made by the wave front before refraction with the surface at which the refraction occurs is called the angle of incidence i, and that by the wave front after refraction is called the angle of refraction r. For two given media, no matter what may be the angle of incidence, if the light enters the second medium at all the refraction will be such that $\frac{\sin i}{\sin r} = \mu$, where μ is constant for the same two media. This is seen from the figure; for $\frac{mo}{no} = \sin i$, and $\frac{nv}{no} = \sin r$, therefore $\frac{mo}{nv} = \frac{\sin i}{\sin r}$; but $\frac{mo}{nv} = \frac{\text{velocity in air}}{\text{velocity in water}}$, a constant ratio; so that $\frac{\sin i}{\sin r}$ is constant and equal to the ratio of the velocities in the two media. This ratio is called the index of refraction for the two media.

When the direction of the light is considered by means of rays, the direction of the incident ray is perpendicular to the incident wave front, that of the refracted ray is perpendicular to the refracted wave front, and the angle of incidence is the angle between the incident ray and a line perpendicular to the interface of the two media, while the angle of refraction is that between the refracted ray and the normal to the interface. Evidently the sines of these angles are in the same ratio as before.

This "law of sines," sometimes called Snell's law and sometimes the law of Descartes, was determined in the first place experimentally, and was part of the evidence to establish the wave theory of light when taken in connection with the fact, also determined experimentally (by Foucault), that light travels more slowly in the more highly refractive medium.

In a medium of the same nature the refractive power increases with the density, and usually, of different substances, the denser is the more highly refractive, so that it is often stated that a ray of light in passing from a rarer into a denser medium is bent towards the normal; but this is not always true, and especially is it not true for gases. So the term "optically denser" has been employed to designate a medium which has a greater refractive power, whether its mass per unit volume is greater or less than that of the substance with which it is compared.

If light is traversing a medium of the same nature but of changing density, it will undergo continual refraction and therefore continual change of direction. This is familiarly seen in the effect of the atmosphere upon the sun's rays, by which the sun appears above the horizon when it is actually below it. (For curved path of light through a solution of varying density, see Wood, *Physical Optics*, p. 90.)

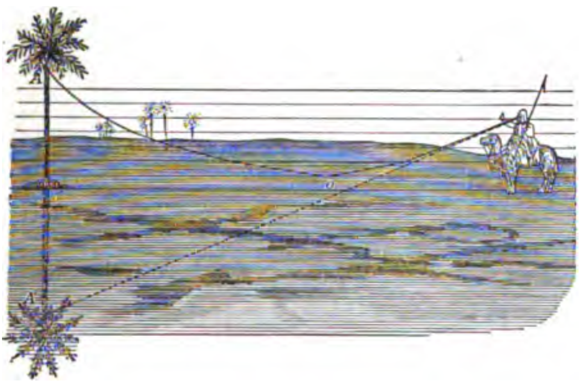


Fig. 148. Refraction of Light produces the Mirage.

In a homogeneous medium, however, light travels at the same rate in all directions, the wave front is spherical, and a ray drawn perpendicular to a wave front at any point is perpendicular to all the successive wave fronts, its prolongation being then a straight line.

A limited portion of a spherical wave front at a great distance from the source of light or the center of curvature is so nearly

plane that the normals to it or the rays are so nearly parallel as to be called parallel light. Light from a source at an infinite distance is parallel light, and the light from the heavenly bodies is practically so at the earth.

Change in the direction of light by diffraction will be considered later.

229. Optics, Geometrical and Physical. — That branch of Physical Science that treats of Light is Optics. For the purposes of specialization it is sometimes divided into Geometrical Optics and Physical Optics, but we shall not follow such distinction further than to define it. Geometrical optics "treats of the propagation, reflection, and refraction of light according to definite laws; and the utilization of such reflection or refraction in various optical instruments." Physical optics "deduces those laws as consequences of a certain hypothesis as to the nature of light, and in addition explains numerous phenomena which geometrical optics leaves unaccounted for." (Glazebrook, *Physical Optics*, Preface.)

230. Light Invisible. — Light is the agent external to ourselves that gives us vision, but we must not forget that, although it is by means of light that objects become visible to us, light itself is not visible. If light were corporeal it ought to be visible, but when we fancy we see the rays it is not so. Light is not a thing, but a revealer of things. It is itself and by itself absolutely invisible. View the heavens on a clear winter night when the land is covered with gleaming snow and the dome of the sky is studded with glittering stars. Each star is a sun like our own, sending its light throughout space to all the other stars, and visible to us by the light it is sending to us, yet when we look sideways at the light thus passing from star to star, we cannot see it; space is black though flooded with light. Even the light of our own sun that is then streaming past the earth cannot be seen.

231. The Seeing of Objects. — Objects that are not primarily luminous, i.e., that do not of themselves emit light, are seen by scattered reflection. A perfectly reflecting surface would reflect all the light that fell upon it according to the laws of reflection,

and light so reflected would reveal to the eye the source of the light before reflection, and the reflecting surface would not be visible. This is called specular reflection. With surfaces not well polished light is reflected in various directions, and in countless variety of condition; every point of the surface becomes a luminous point from which light proceeds in all directions, and if by the eye or any other means we collect and converge these rays we form an image of these innumerable points that are secondary sources of light, and so form an image of the surface of the object. The difficulty of seeing a surface is in direct proportion to the perfection of its polish; hence the frequent blunders and deceptions arising from the use of fine mirrors in rooms or halls. (Examples.)

No matter what may be the position of the source from which light has proceeded in the first place, when the light enters the eye we locate the source by the direction of the line by which the light entered the eye, so that the apparent position of the source may be very different from its real position; it may be exactly reversed, as when, by the use of two mirrors, one sees the back of his head in front of him. (Here the change of direction of light is due to reflection. Fig. 148 shows how, by means of refraction, one might perceive the inverted image of vegetation above a glaring surface, producing the mirage of trees upon the edge of water.)

232. Images, Real and Virtual. — While the propagation of light is a wave phenomenon, the representation of it is sometimes simpler by means of rays than by wave forms. We shall have recourse to either as may seem better suited to our immediate purpose.

By Waves.

When a disturbance is progressing through a medium in the form of waves, the wave front may be either concave, convex, or plane. If the disturbance proceeds from a point, the wave front is necessarily convex. This form may be changed, however, by encountering a different medium so that it may become plane or even concave, and in the latter case its further progress will be to converge upon a point. Such a point is said to be the *real* image of the point from which the light proceeded.

If, upon the passage from one medium to the other, the wave front becomes less convex, the disturbance will seem to come from a point behind the starting point. If the wave front becomes more convex, the disturbance will seem to come from a point in front of the starting point. In either of these cases, this second point is said to be the *virtual* image of the point from which the disturbance actually proceeded.

By Rays.

When rays proceed from a point in all directions they are divergent, but upon encountering a different medium they may be rendered parallel, or convergent upon another point. This point is said to be the *real* image of the point from which they proceeded.

If, when the rays pass from one medium to another, they are rendered less divergent than at first, they appear to come from a point behind the actual starting point. If they are rendered more divergent they appear to come from a point in front of the starting point. In either of these cases, this second point is said to be the *virtual* image of the point from which the rays actually proceeded.

These ideas may be illustrated by the following diagram (Fig. 149) to trace the progress of a disturbance proceeding from P towards A through several media:

(a) In Spherical Waves.—The diagram shows the intersection of the spherical waves by the plane of the paper.

Suppose DAD to be the interface between two media X and Y. When the disturbance reaches A, the wave front is CAC. If, now, Y is such a medium that a disturbance will progress in it a distance AB while it will go a distance CD in X, then the unmodified wave front that would have passed through DD in a curve, with P as a center, is now modified so as to pass through

DBD, which may be a straight line, and the wave front will have become plane, and will move on as such if the medium Y is homogeneous. Whether this will be a plane front is found to depend on the nature of the two media, the curvature of the incident wave front and that of the interface DAD.

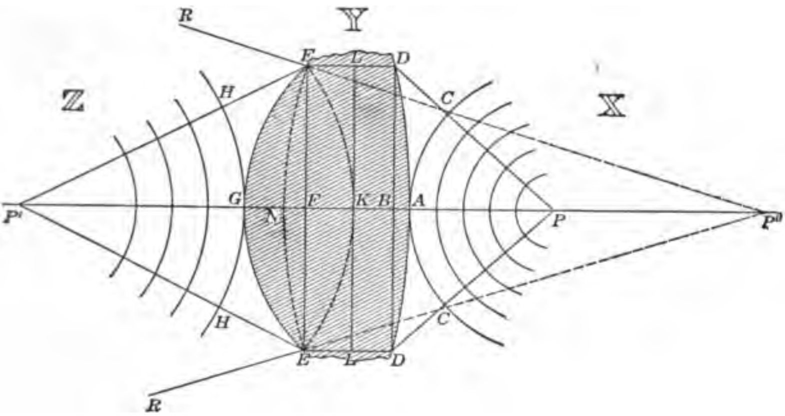


Fig. 149.

If the next boundary of Y be EGE, the wave front EFE will undergo a change of curvature, and if the third medium Z is such that a disturbance may progress from E a distance EH in the time that a disturbance goes from F to G, the wave front will pass through HGH, and thereafter advance in waves converging upon P' which is the image of P.

The interface between Y and Z might have a form like EKE, such that a disturbance from a point L on the wave front LKL could go as far as LE in the medium Y while one would go in the medium Z from K the greater distance KN. Then the wave emerging beyond E would have the form ENE, and would proceed in expanding spheres *as if* proceeding from a center at P''. P'' is then the virtual image of P.

At either surface also the form of wave front may be so modified by reflection as to cause an image by the waves being reversed and made to return in the same medium through which they advanced. This image will be real if the reflected waves

really converge upon it, and virtual if they appear to proceed from it. The virtual image will be behind the reflecting surface, if this surface is plane or convex.

(*b*) The corresponding constructions with rays would be as follows: A ray from P normal to the interface at A or G continues without deviation. An oblique ray, as PD, will be deflected as along DE, the change of direction depending upon the obliquity of the ray and the nature of the two media. At E, on again changing the medium of propagation, the course is again altered and the refracted rays intersect at P', which is the image of P. The second interface might have a form EKE such that the emergent rays would diverge instead of converging, and then they would appear to proceed from a point P'' which is the virtual image of P.

Either mode of construction is justifiable (by waves or by rays), and if they are both made for the same media, the same forms of surface and the same position of P, they would give the same resulting images when applied to reflection as well as to refraction. The positions of a point of light and its image are said to be *conjugate* to each other; i.e., if the former is put in the position of the latter, the latter will occupy the position of the former.

233. Critical Angle; Total Reflection. — As light emerging from an optically denser to an optically rarer medium is refracted from the direction normal to the surface between the two media, making the angle of refraction greater than the angle of incidence, it follows that while the angle of incidence is still less than a right angle the angle of refraction may equal 90° and the refracted ray lie in the surface between the two media. If the angle of incidence be now further increased, to pass into the second medium, the law of refraction would result in the emergent ray making an angle with the normal greater than 90°, so that emergence is impossible.

Suppose, in Fig. 150, NQ is such a surface of separation, and light from P makes an angle of incidence NPQ such that the refracted ray QR lies in the surface NQR; the angle NPQ then is called the critical angle for those two media. Light from P in a

direction at an angle with PN greater than the critical angle will not pass into the second medium, but will be totally reflected into the first medium, giving the most perfect reflection possible.

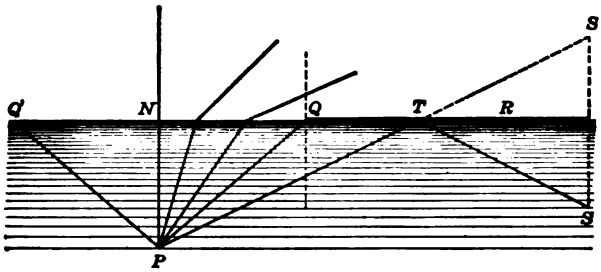


Fig. 150. Total Reflection.

The angle of incidence at this critical stage is obviously determined by the condition

$$\text{Index of refraction} = \frac{\sin i}{\sin r} = \frac{\sin \text{critical angle}}{\sin 90^\circ}.$$

Calling the index of refraction μ,

$$\text{Critical angle} = \sin^{-1} \mu.$$

For light passing from water to air,

$$\mu = \frac{3}{4} = 0.75 = \sin 48^\circ\, 35'.$$

From crown glass to air,

$$\mu = \frac{2}{3} = 0.667 = \sin 41^\circ\, 48'.$$

From bisulphide of carbon to air,

$$\mu = \frac{1}{1.63} = 0.6135 = \sin 37^\circ\, 51'.$$

If an eye at P looking up could see in the first medium by direct view only objects within the cone QPQ', it could see in the second medium objects within the whole range of the horizon. It would have no difficulty in seeing round the corner at Q. (See the "fish-eye" views by Professor R. W. Wood, Wood's *Physical*

Optics, p. 67.) An object at S would appear brilliantly by total reflection at T to be at S'.

Experiment No. 89, page 349. — Total Reflection by Luminous Jet.

234. Apparent Depth of a Transparent Medium. — It may be shown (see Hastings and Beach, *General Physics*, Arts. 546, 547) that, in general, when the wave form is modified by reflection or refraction at a surface, if

γ = curvature of surface at which wave is modified,

$$\left(\text{curvature} = \frac{1}{\text{radius}}\right),$$

C = curvature of the incident wave front,

C_1 = curvature of the modified wave front, and ρ = ratio of velocity of light after modification to velocity before modification,

then $C_1 = \rho C + (1-\rho)\gamma$. (An important formula.)

Let P (Fig. 151*a*) be a point in a transparent medium, as water, to be viewed by the eye vertically above it in a different medium, as air. If the velocity of light in the medium above N were the same as below, the wave fronts would be spherical surfaces about P as a center, and the eye would see P in its true position. But if the velocity in air is, say, four-thirds that in water, then the emergent wave instead of having the unmodified form $o'no'$ will have the greater curvature of $o'n'o'$, for while the disturbance goes from o to o', that from N will go to n' where Nn' is ρ times Nn, and the center of the new wave system will be at P', and to the eye the light will seem to come from P', or P will appear to be at P'. If P is at the bottom of the lower medium the apparent depth will be NP' instead of NP. In the case of a plane surface, γ in the above equation is zero and $C_1 = \rho C$; for water and air $\rho = \frac{4}{3}$, or the curvature of the wave entering the upper medium at N is four-thirds that of oNo or the radius $P'N$ is $\frac{3}{4}\,PN$, or the apparent depth is $1/\mu$ times the real depth, μ being the index of refraction from the second medium into the first.

The corresponding construction by rays would be as follows: If ME (Fig. 151*b*) is the limiting ray from P that can enter the eye after refraction at the surface MN, to the eye the light will appear to come from P', and the apparent depth will be

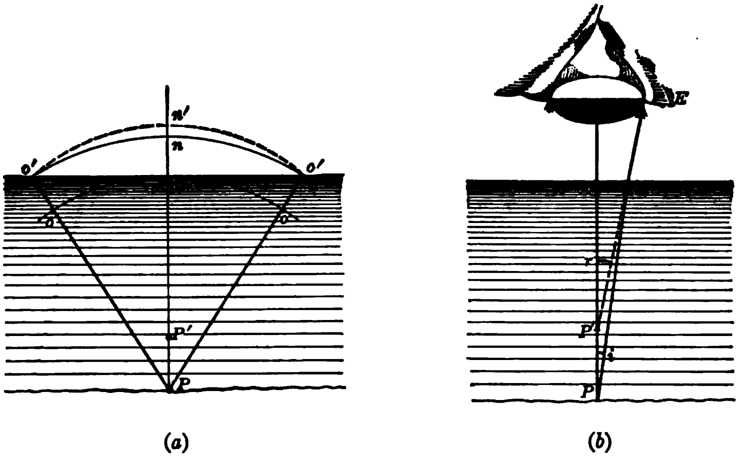


Fig. 151. Thickness of a Transparent Layer apparently reduced by Refraction.

NP' instead of NP. The angle MPN is the angle of incidence i, and $MP'N$ the angle of refraction r. In the triangle MPP', $\frac{MP}{MP'} = \frac{\sin MP'P}{\sin MPP'}$. But $\sin MP'P = \sin MP'N = \sin r$, therefore $\frac{MP}{MP'} = \frac{\sin r}{\sin i} = \mu$, if μ is the index of refraction from the upper medium into the lower.

The pencil of light entering the eye is very narrow, both i and r are small, and the ratio of $\frac{MP}{MP'}$ is very nearly that of $\frac{NP}{NP'}$. It becomes more and more nearly equal to it for light that is more and more nearly along the normal line PN. Thus, for perpendicular viewing, $\frac{NP}{NP'} = \mu$. In the case of water and air, μ is $\frac{4}{3}$, and the apparent depth is three-fourths the actual depth.

EXAMPLES. —

1. If the index of refraction for light passing from air into water is $\frac{4}{3}$, and the velocity of light in air is 299,000,000 meters per second, what is its velocity in water? *Ans.* 224,250,000 m./sec.

2. If a right-angled glass prism have all three faces polished, trace a ray of light that falls perpendicularly upon one of the faces forming the right angle, the index of refraction of the glass being 1.5.

3. A plate of clear glass 1 cm. thick, whose refractive index for light from air into the glass is 1.5625, is laid upon a line of printing. How far below the upper surface of the glass will the letters appear to an eye looking vertically down upon them? (*NP'*, Fig. 151.) *Ans.* 6.4 mm.

235. Foci. — When light diverging from any point is so changed in direction by reflection or refraction as to pass really or apparently through another point, the second point is the image of the first. The positions of the source and the image are interchangeable. The point to which the light converges or from which it appears to diverge is called a focus.

The usual means of producing images are mirrors and lenses, and commonly these have plane or spherical surfaces. A plane surface may be treated as that of a sphere whose radius is infinite or whose curvature is zero. A line passing through the center of the sphere and perpendicular to the plane cutting the lens or mirror surface from the sphere of which it is part is the principal axis. Light from a point on this axis has its focus also on this axis, and when the light is parallel or proceeds from an infinite distance, and is parallel to the principal axis, the focus is called the principal focus of the mirror or lens. Light from a luminous point at the principal focus would become parallel light after modification by the apparatus (lens or mirror). Light from an object at a finite distance comes to a focus, that is, it forms an image, at a position such that if the object were placed there its image would then be where the object had been. These are said to be "conjugate positions," or a luminous point and its image are *conjugate foci*. For spherical mirrors of small angular aperture and radius R, the principal focal length f is $\frac{R}{2}$, and the relation of conjugate foci is $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{R} = \frac{1}{f}$.

DEMONSTRATION. (*a*) *By Rays.* — In Fig. 152 (*a*), let PP'' represent a concave section of a spherical mirror with center at C and radius of curvature $CA = R$. Let F be the middle point of CA. Light rays parallel to the axis FA striking the mirror at any point as P' are reflected, making the angle of reflection $CP'F'$ equal to the angle of incidence $CP'L$. Since LP' and CA are parallel $\angle\, LP'C = \angle\, P'CA. \therefore \angle\, P'CF' = \angle\, CP'F'$, and $P'F' = CF'$. As the portion of the arc AP by which the rays are reflected is taken smaller and smaller, $P'F'$ becomes more nearly identical with AF', and F' becomes more nearly identical with F, so that for a small angular aperture AP', the principal focus is practically midway between

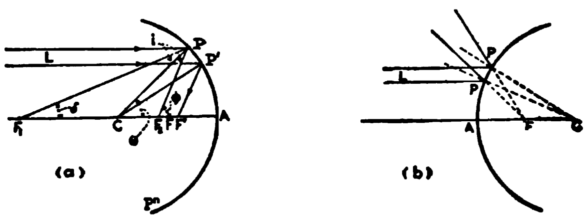


FIG. 152.

C and A, or at F. Rays which proceed from F as an origin are reflected parallel to the axis CA.

If the rays L fell upon the convex side of the mirror as in Fig. 152 (*b*), at the points P, they would, after reflection, not pass through F at all, but would diverge as if they had come from F behind the reflecting surface. The point F is then a virtual focus, and the focal length of the mirror is $\frac{R}{2}$ as before.

For conjugate foci, suppose rays to proceed from F_1, Fig. 152 (*a*). Such a ray is reflected at P to F_2, the angles of incidence and reflection F_1PC and CPF_2 being equal, say $= i$. Call the distance $F_1A = f_1$, $F_2A = f_2$, $\angle\, PF_1A = \delta$, $\angle\, PCA = \theta$, and $\angle\, PF_2A = \phi$. Then

$$\phi = \theta + i,$$
$$\theta = \delta + i;$$

$$\phi - \theta = \theta - \delta;$$

or

$$\delta + \phi = 2\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If arc AP is small, $\delta = \frac{AP}{f_1}$, $\phi = \frac{AP}{f_2}$, and $\theta = \frac{AP}{R}$, and these substituted in (1) give $\frac{AP}{f_1} + \frac{AP}{f_2} = \frac{2AP}{R}$ or $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{R}$; and if $f_1 = \infty$ the incident rays are parallel, f_2 = principal focal distance f, and $\frac{1}{f} = \frac{2}{R}$. *Q. E. D.*

(*b*) *By waves.* — By applying the general formula at the beginning of Art. 234, waves proceeding from a point at a distance f_1 have a curvature at the mirror such that $C = \frac{1}{f_1}$; on reflection they meet really or apparently in a point at a distance f_2, so that $C_1 = \frac{1}{f_2}$; also $\sim = \frac{1}{R}$, and $\rho = -1$, and the formula, becomes

$$\frac{1}{f_2} = -\frac{1}{f_1} + \frac{2}{R}, \quad \text{or} \quad \frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{R}. \quad . \quad . \quad . \quad (2)$$

If the incident light is parallel light, the distance f of the principal focus is called the focal length of the mirror or lens. In such case the incident wave front is plane, $C = O$, f_2 becomes f, and we have $\frac{1}{f} = \frac{2}{R}$, so that for conjugate foci, $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2}{R} = \frac{1}{f}$. *Q. E. D.*

If the distance f measured in front of the mirror is positive, either f_1 or f_2 in the equation might be negative, which would mean that it was measured back of the mirror.

For lenses the principal focal distance is given by the equation

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where μ is the index of refraction from air into the glass of which which the lens is made (also the ratio of the velocity of light in air to that in the glass) and r_1 and r_2 are the radii of cur-

vature of the lens surfaces, with opposite signs if the faces of the lens are curved in opposite directions. (For demonstration of this see Watson's *Physics*, Arts. 340, 351, and 352.)

The conjugate focal distances, however, may be expressed in terms of the principal focal length by the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ where f is + for a converging and − for a diverging lens. (See p. 330, and for demonstration see Mumper's *Physics*, Art. 206.)

As an illustration, suppose in Fig. 153, LL is a lens of negligible thickness whose one face is plane and the other is spherical

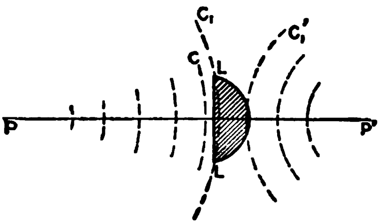


Fig. 153.

with a radius of curvature of 5 cms. Light proceeds from a point P on the axis at a distance of 30 cms. from the lens. The velocity of light in air is $\frac{3}{2}$ the velocity in the glass. From the formula of Art. 234, $C_1 = \rho C + (1 - \rho)\gamma$. Here, for the light passing from P into the lens, $\rho = \frac{2}{3}$, $C = \frac{1}{30}$, and $\gamma = 0$. These in the formula give $C_1 = \frac{2}{3} \times \frac{1}{30} + \left(1 - \frac{2}{3}\right)0 = \frac{1}{45}$. If the thickness of the lens is negligible, we may suppose the waves immediately to emerge from the lens into the air through the curved surface of the glass. In the general formula this value of C_1 now takes the place of C. The wave has the curvature $\frac{1}{45}$, $\rho = \frac{3}{2}$, and $\gamma = \frac{1}{5}$. Then the emergent wave will have curvature $C'_1 = \frac{3}{2} \times \frac{1}{45} + \left(1 - \frac{3}{2}\right)\frac{1}{5} = -\frac{1}{15}$. The direction of curvature is reversed and the diverging light from P con-

verges upon P' on the other side of the lens, at a distance of 15 cms. from it.

Or thus: From Eq. (3) we may find for the lens LL:

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} + \frac{1}{5}\right) = \frac{1}{10}.$$

Then for the conjugate focal distances f_1 and f_2 we have $f_1 = 30$ and

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{or} \quad \frac{1}{10} = \frac{1}{30} + \frac{1}{f_2},$$

whence

$$\frac{1}{f_2} = \frac{1}{15},$$

or P' is 15 cms. to right of LL.

In applied optics, if f is measured in meters, the quantity $\frac{1}{f}$ measures the "power" of the lens or mirror in *diopters*, one diopter being the power of a lens whose focal length is one meter.

236. Refraction of Light through a Plate with Parallel Sides. — If light pass from, say, air to glass in the direction RI (Fig. 154), it is refracted in the direction IE such that $\frac{\sin \alpha}{\sin \beta} = \mu$. At E it emerges in a direction ER' such that $\frac{\sin \beta'}{\sin \alpha'} = \frac{1}{\mu}$. If GN is parallel to AM, $\beta' = \beta$; then $\alpha' = \alpha$, and ER' is parallel to RI, a result verified by experiment.

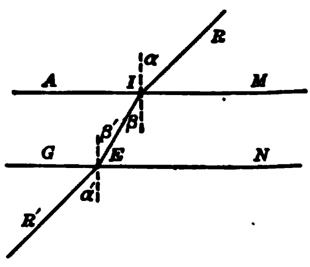


Fig. 154. Passage of a Ray through a Plate with Parallel Sides.

237. Images of Objects as Formed by Lenses. — The surfaces of a lens at points on the principal axis, as at M and N (Fig. 155), are parallel; also for a point P on one surface there is usually a corresponding point Q on the other surface such that the tangent planes at P and Q are parallel. A line joining the points P and Q cuts the principal axis in a point O called the optical center

of the lens. Any incident ray RP, falling upon the surface PN in such a direction that by refraction it takes the path PQ, will emerge at Q in the direction QR' parallel to RP, and if the thickness of the lens is negligible, a point on RP will have its image on QR'. All rays passing through O are undeviated in direction. With a thin lens and points at a considerable distance, a line drawn through O gives, with sufficient accuracy, the direction of the undeviated ray. With a double convex lens of like curvature on both sides, O is at the middle of the glass.

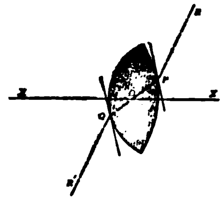


Fig. 155. Optical Center of a Lens.

A line through O other than the principal axis is called a secondary axis, and the image of any point that is not on the principal axis is found on the secondary axis through the point. Thus an image may be constructed if the values of μ and f are known. In Fig. 156, the image of A is on the line AO; that of B on BO at B' where $\frac{1}{BO}+\frac{1}{OB'}=\frac{1}{f}$. The image of C is on CO. These relations, however, are only applicable to thin lenses with small angular aperture. With large angular opening from m to n, the rays from any point, as A, will not all intersect in the same point after reflection or refraction, and a sharp image will not be produced. This failure to come to a common focus on account of the sphericity of the lens surfaces is called spherical aberration. In photography, as in optical work with lenses generally, sharpness of image is gained by using a diaphragm so as to permit light to pass only through the central part of the lens; but this sharpness of definition is gained at the expense of the quantity of light, and consequently of brightness.

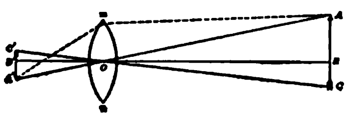


Fig. 156. Formation of an Image by a Convex Lens.

The object and its image are at conjugate focal distances from

the lens, like the point of light and its image in Art. 232, and their sizes are as these focal distances.

A converging lens (convex) is thickest at the middle and causes light which passes through it to be more convergent (or less divergent) than before; a divergent lens (concave) is thinnest at the middle, and causes light which passes through it to be more divergent (or less convergent) than before. Since these two types of lenses change the direction of the light in opposite ways, the former are sometimes called positive and the latter negative lenses.

It is shown in higher optics that if two thin lenses are placed together the "power" of the combination, i.e., the reciprocal of the focal length, equals the algebraic sum of the powers of the two lenses separately. If f_1 and f_2 are the focal lengths of the two lenses, and F' that of the combination,

$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{f_2};$$

if f_1 or f_2 is for a divergent lens the sign of its reciprocal in this equation is minus.

EXAMPLES. —

1. If the focal length of a camera lens is 10 cm., how far behind the lens will be the ground glass to show a sharp image of an object that is one meter distant in front of the lens? How far if the object is 15 meters away? If it is 100 meters? *Ans.* 11.11 cm.; 10.06 cm.; 10.01 cm.

2. In the second case, what would be the size of the picture of a house front 20 ft. × 30 ft.? *Ans.* 1.6 in. × 2.4 in.

3. The distance of an object from a convergent lens is twice the focal length of the lens; show that the image and object are of the same size.

At the front of the eyeball is a crystalline lens which, together with the humors of the eye, forms images on the retina, a membrane at the back of the eyeball (see Fig. 157).

The distance from the lens to the retina is about 2.3 cm. The normal eye gives a distinct image of ordinary print which is at a distance of 25 cm. in front of the lens, but the latter, along with the rest of the eye, can so adapt itself as to make on the retina the image of objects that are at a greater or smaller distance than this. This change of the eye is called *accommodation*. If, however, the lens becomes too flat its focal length is

increased (or,what is the same thing, its power is diminished) and the position of the image would be behind the retina except for distant objects. Such an eye is said to be far-sighted. If the lens is too much curved, the image is in front of the retina except for objects near the eye. Such an eye is near-sighted. These positions are illustrated in Fig. 157. If the image of an object at P is distinct on the retina at p, the eye is normal; but if the image is indistinct when the object is at P and is distinct only when the object is brought nearer, as at P', the eye is near-sighted. In that case a sharp *image* of P is formed in front of the retina, as at p'. If the image of an object at P is indistinct, but becomes distinct from a greater distance as

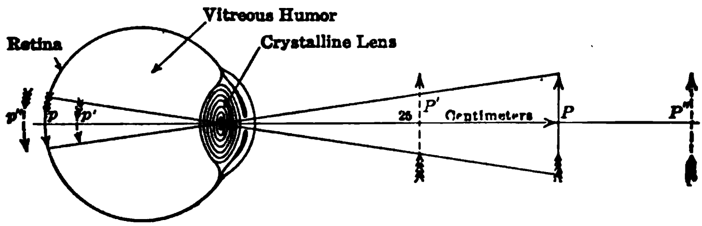


Fig. 157. Defects of Vision.

P'', the eye is far-sighted. In such case the position of a sharp *image* of P is beyond the retina, as at p''.

In case of either of these defects, an auxiliary lens (spectacle lens) may be so combined with the lens of the eye as to make the focal length of the combination such that the image of an object 25 cm. from the eye will be formed, say, 2.3 cm. behind the lens or on the retina. That is, the power $\frac{1}{F'}$ of the eye normally or of the combination should be

$$\frac{1}{F'} = \frac{1}{2.3} + \frac{1}{25} = 0.475.$$

As illustrations, suppose (1) the eye sees print most distinctly at a distance of 10 cm. It is near-sighted. The power of the eye is

$$\frac{1}{f_1} = \frac{1}{2.3} + \frac{1}{10} = 0.535.$$

It must be combined with a concave lens that will reduce the power to 0.475; i.e., a negative lens of power $\frac{1}{f_2} = 0.06$, or one whose focal length is 16.66 cm.

(2) Suppose the eye sees distinctly not nearer than 40 cm.; it is far-sighted. The power of the eye is

$$\frac{1}{f_1} = \frac{1}{2.3} + \frac{1}{40} = 0.46.$$

It must be combined with a positive (convex) lens to bring the power up to 0.475, or have a power 0.015, or a focal length $f_2 = 67$ cm.

The power in these examples must be multiplied by 100 to give the power in diopters; in (1) the power of the auxiliary lens is 6 diopters, in (2) it is 1.5 diopters.

Suppose the distance from the crystalline lens to the retina to have *any* fixed value, say C, in place of 2.5 cms. above, let d be the distance of distinct vision for the eye tested, whether normal or abnormal, and δ the proper distance for distinct vision, like the 25 cms. above, then the power of the eye is

$$\frac{1}{f_1} = \frac{1}{d} + \frac{1}{C}. \quad \dots \dots \dots \dots \quad (a)$$

d may or may not be equal to δ, but it should be so, and a lens of power $\frac{1}{f_2}$ must be combined with the eye so that the combined power shall be

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{\delta} + \frac{1}{C}. \quad \dots \dots \dots \dots \quad (b)$$

The difference between the desired power represented by (*b*) and the actual, represented by (*a*), is the power of the auxiliary lens to be used. Subtracting (*a*) from (*b*) we have $\frac{1}{f_2} = \frac{1}{\delta} - \frac{1}{d}$. Obviously $\frac{1}{f_2}$ is + or − according as d is greater or less than δ. δ is usually taken as 25 cms.

Of course, in practical optometry, many other things have to be taken into account besides the elementary features here considered.

EXAMPLES. —

1. A person holds his book within 9 cm. of his eye to read easily. What spectacle lens does he require?

Ans. A concave lens of 14 cm. focal length.

2. If he sees most distinctly at a distance of 80 cm. what lens should he use?

Ans. A convex lens of 36.5 cm. focal length.

Experiment No. 90, page 350. — Show mirrors, lenses, production of images, foci, reflection, refraction, total reflection, etc., preferably by means of the optical disk if such is available.

238. Photometry. — The term means measurement of light, but is commonly applied to any comparison of the illuminating powers of any different sources of light. This is generally accomplished by comparing the illumination that is afforded by the

lights. By the illumination of a surface is to be understood the ratio of the quantity of light which it receives to the area over which the light is distributed, or the amount of light per unit area. By the illuminating power or the intensity of a light is meant the illumination it can produce at a unit's distance. This depends on the light itself, but the illumination of a surface varies directly with the intensity of the light and inversely with the square of the distance, so that if we call the illumination I, power of the light l, and distance d, we have $I = \frac{l}{d^2}$.

All radiation proceeding from a point in straight lines will cover an area proportional to the square of the distance from the point, and therefore the quantity per unit area will be *inversely* proportional to the square of the distance.

Different lights may be compared in intensity without expressing either one in definite units, but to express the power of either one independently, a standard unit is necessary. For want of a satisfactory standard the measurement of illuminating power is among the least precise of physical determinations. The commonest standard in English and American practice is the British standard candle, a spermaceti candle weighing six to the pound and burning 120 grains per hour. The French standard is the "Carcel," burning 42 grm. of colza oil per hour (equal to about $9\frac{1}{2}$ candles). In Germany the Reichs-Anstalt standard is a lamp burning amylacetate, the flame being adjusted to standard specified conditions.

England, France and the United States of America have now adopted a common "International Candle" which is very nearly equal to the above British standard candle and equals $\frac{10}{9}$ German standard, and 0.104 Carcel units.

If l is the power of the light, d its distance from a surface, and I the illumination produced upon the surface, in general $I = \frac{l}{d^2}$; and for the illumination resulting from two different sources, l_1, l_2, we should have

$$I_1 = \frac{l_1}{d_1^2}, \text{ and } I_2 = \frac{l_2}{d_2^2};$$

and if the two can be so placed with reference to a given surface as to produce equal illumination of it, then $I_1 = I_2$,

and
$$\frac{l_1}{d_1^2} = \frac{l_2}{d_2^2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{d_1^2}{d_2^2};$$

that is, the intensities of the lights are to each other as the squares of their distances from the spot which they illuminate equally.

Experiments Nos. 91 and 92, page 351. — Illustrate with Bunsen and Rumford photometers, and call attention to other forms and to special features in comparison of lights, as difference in color, etc.

The comparison of the intensity (candle power) of two lights is usually effected by deciding when the two lights produce equal illumination of a given surface, i.e., make it appear equally bright, but the relative illumination of two surfaces that are not equally bright is determined by the amount of light per unit area on each surface. The common unit of illumination is the *lux* (plural lux), which means the illumination produced by a light of one candle power at the distance of one meter.

EXAMPLES. —

1. The shadow of a rod is cast upon a screen by a standard candle and by an incandescent lamp. When the shadows are equally lighted, the distance from the candle to the shadow cast by the lamp is 50 cm., and the distance from the lamp to the other shadow is 190 cm. What is the intensity of the lamp? *Ans.* 14.44 c.p.

2. A candle is placed 30 cm. from one side of a cardboard screen, and a 10 c.p. gas flame is on the other side at a distance of one meter. Compare the illumination on the two sides.

Ans. The side toward the gas flame is nine-tenths as bright as that toward the candle.

3. A grease spot in a screen is equally illuminated by a candle on one side of it and a Welsbach incandescent mantle on the other side. The distances of the candle and the Welsbach from the screen are 26 cm. and 200 cm. respectively. Compare the intensity of the two lights.

Ans. Welsbach : candle = 59.17 : 1.

4. Of three equal candles, two are placed together on one side and the third on the other side of a screen containing a translucent grease spot. The distance of the single candle from the other two is 150 cm.

How is the screen situated when the spot is equally illuminated from both sides? *Ans.* 62.13 cm. from the single candle.

5. A lamp of 11 c.p. is placed 3 meters from a standard candle: determine two positions in which a cardboard screen would be equally illuminated by the lamp and the candle.

Ans. On that side of the candle toward the lamp, at a distance of 69.5 cm. from the candle; and on the side away from the lamp at 129.5 cm. from the candle.

239. The Spectrum; Dispersion. — Although all colors falling upon a reflecting surface at a given angle are reflected at this

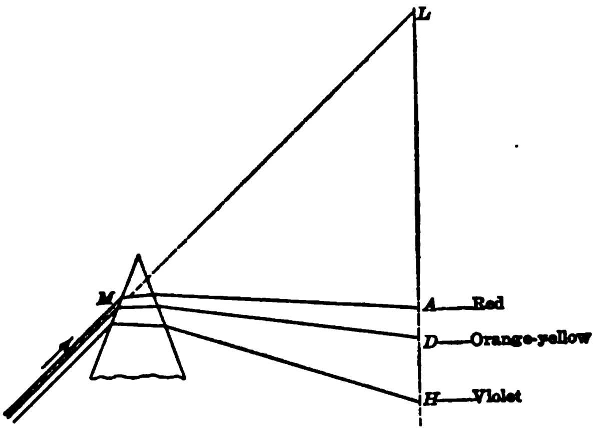


Fig. 158. Spectrum formed by a Prism.

same angle, they are not equally refracted by the same refracting medium. When a narrow beam of white light falls upon the surface of a refracting medium, it is separated into a band of greater width, the most highly refracted part being violet in color and the least refracted red, with intermediate colors indigo, blue, green, yellow, orange, no one color sharply defined in limits but each merging into the next. This band of colors is called a spectrum. It exhibits at once the facts that the light producing it is not simple but complex, and that its constituent colors are unequally refracted, this unequal refraction constituting dispersion.

If the light passes from air through a triangular prism, as in Fig. 158, further refraction occurs at the second face, causing a

further deviation of the light, and the spectrum is easily observed. A very narrowly limited portion, i.e., an elementary portion of any one color, if passed through another prism, will be again refracted, but not further separated. Of prisms of different substances, as glass, water, etc., each has its own refractive power for the several colors of white light. The refractive power is determined by the least angular deviation which a prism of a given angle can produce in a given color. This is obviously greater for violet than for red. One substance, however, may refract the whole spectrum highly, but the violet not much more than the red; there would then be large deviation and small dispersion, while another substance might have lower refractive power for every part of the spectrum, but relatively greater for the violet than for the red; there would then be small deviation with large dispersion. The *dispersive power* of any substance as compared with that of another is the ratio of the difference in the angular deviation of two colors (i.e., the dispersion of those two colors) to the angular deviation of the mean ray between them; for example, the angle between AM and HM divided by the angle LMD. It is connected with the index of refraction by the equation $\frac{\delta_H - \delta_A}{\delta_D} = \frac{\mu_H - \mu_A}{\mu_D - 1}$ (see Watson, Art. 372), in which δ is the angular deviation and μ is the index of refraction. The first member of the equation is the dispersive power of the prism for the extremes of the spectrum.

When the source of light is a narrow slit illuminated by an incandescent solid or liquid, the spectrum is a continuous band of colors, but when the light is that of a gas, the spectrum consists of one or more streaks of color with dark spaces between them. Moreover each gas produces a spectrum peculiar to itself, and distinguished from the spectrum of other gases by the colors of the lines and their situation.

Experiments Nos. 93 and 94, page 351. — Continuous and Discontinuous Spectra.

240. Chromatic Aberration. — Since the faces of a lens at any distance from the axis are inclined to each other, they correspond

to the faces of a prism, and light passing through the lens is decomposed into colors, the violet being bent most toward the thick part of the lens. As a consequence the violet rays are brought to a focus nearer the lens than are the red rays, and the image is a colored one and not sharp. Such "going wrong" on account of color is called "chromatic aberration."

241. Direct Vision Spectroscope. — By arranging two prisms of properly related refractive and dispersive powers, light may be dispersed and yet proceed from the last face in a direction which, for some portion of the spectrum, is parallel to that in which it came to the first face. There will then be dispersion without deviation (see Fig. 159).

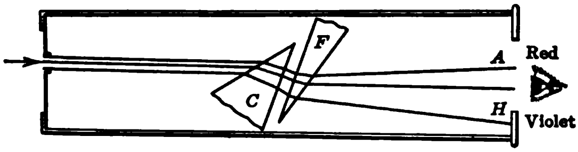


Fig. 159. Deviation corrected while Dispersion is retained.

242. Achromatic Lens. — If the angles and the material of the two prisms of Art. 241 are so chosen that the dispersion of two colors caused by one is neutralized by the other while the refraction is not neutralized, there will be refraction, but the two

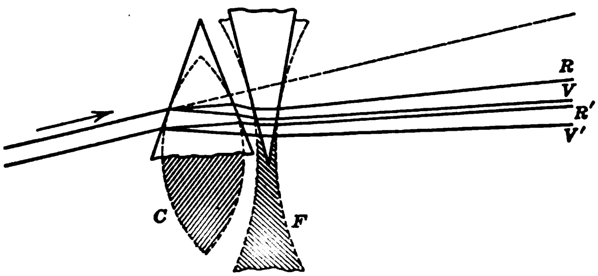


Fig. 160. Dispersion corrected while Deviation is retained.

colored images of the source of light will be brought to the same focus, and chromatic aberration for those two colors will be corrected, as in Fig. 160. The same principles apply to light passing through a corresponding combination of lenses, and the

same results are obtained. A third lens or prism makes it possible to correct for three colors, and if these are blue, yellow and red, the correction is pretty complete for the entire spectrum. Such a lens is called achromatic. It consists usually of a converging (double convex) lens of crown glass, for which the indices of refraction for the extremes of the spectrum are $\mu_A = 1.528$, $\mu_H = 1.55$, and a diverging (double concave) lens of flint glass of which the indices of refraction are $\mu_A = 1.578$, and $\mu_H = 1.614$.

243. Interference of Light. — Regarding light as a wave phenomenon, if two separate beams of light or two parts of one beam, proceeding from a common source and accordingly in the same phase of vibration, are so modified in their subsequent progress as to arrive at a point in opposite phases, they interfere, and a diminution or extinction of light occurs. Any color is "light," and it will be found that light of one color differs from that of another color only in wave length. The retardation of one part of a beam of light as compared with another part is accomplished in various ways, of which that by reflection from the two surfaces of a *thin* medium is one of the simplest.

If the convex surface of a plano-convex lens with large radius of curvature, as C (Fig. 161), be placed upon a plane glass P, the surfaces will be in contact at the point of tangency O, and will separate very gradually, with a thin layer of air between them, whose thickness is calculable for any given distance from O. A ray of monochromatic light IA enters C in the direction AB, at B is in part reflected to E and emerges in the direction ER. The other portion passes from B to C through air, is reflected to D and continues along DFR. If the air layer is very thin FR and ER enter the eye together, and if the path BCD is one half wave length of the light the second part of the light will, on this

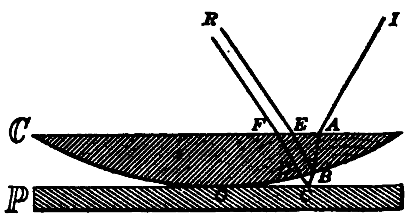


Fig. 161. Formation of Newton's Rings.

account, be a half wave length behind the first part, and *theoretically* the portions ER and FR will interfere, and the eye will perceive a narrow dark circular band of which O is the center and OC the radius. This will occur again at such a radius that the distance BCD is three, or five, or any odd number of half wave lengths; where the distance is any multiple of whole wave lengths there will be a ring of light. This gives the series of so-called Newton's Rings. If the incident light is red, the rings will be only red and dark; if the color is yellow, the rings will be alternately yellow and dark, but the yellow rings will not occur at the same radial distance as the red rings, thus indicating that the different colors have different wave lengths. If the incident light is white there is a succession of circular spectra.

Such, in brief, is the theory of the colors of thin plates.

At O the difference in the length of the paths is zero; here the waves would theoretically have no difference in phase, and the center would be bright; but wherever waves are about to proceed from a denser to a rarer medium reflection occurs with no change of phase, whereas, if reflection takes place at a surface where the wave is about to enter from a rarer into a denser medium, as at C, a reversal of phase occurs, and the reflected wave falls a half period behind in consequence of such reversal, and so the center O is dark (see Art. 118). In the same way, where the distance BCD is a half wave length the total retardation is a whole period, and a bright ring is produced; where the distance BCD is a wave length the retardation is one and a half periods, and the ring is dark, and so on, the reversal of phase modifying the simple theory as at first presented. If the rings are viewed by the light transmitted through P the colors occur in the order first described, but they are not as brilliant as those produced by reflection.

The beautiful colors of soap films, or of a very thin layer of oil on water, are due to interference, but here the thin plate is a liquid, instead of air as above.

NOTE. — The illustration and explanation would be more rigidly correct if the incident light falls upon a plate of uniform thickness, as, e.g., the plate P instead of C, but the presentation here shown is a common one.

Experiments Nos. 95, 96, 97, page 353. — Colors of thin plates.

244. Diffraction; Measurement of Wave Length. — Light passing the edge of an opaque object is deviated both to one side and the other of the edge, from a straight direction, at an angle depending upon the wave length. Such deviation is called diffraction.

In Fig. 162, let AB, BC, CD, etc., represent alternate transparent and opaque spaces constituting the ruling of a grating, of which the clear spaces are equal to one another, and so are the

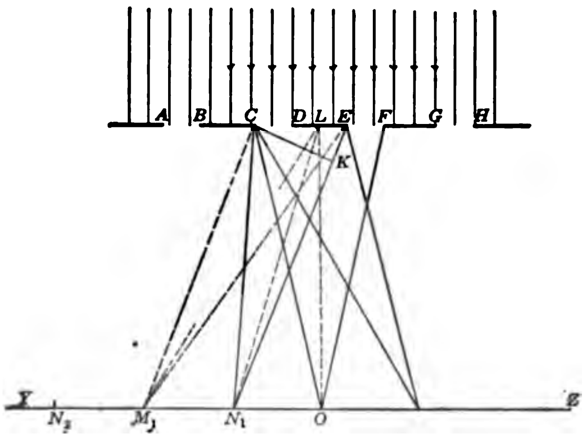


Fig. 162. Interference of Light by Diffraction.

opaque. The total distance occupied by a clear and an opaque space is taken as the distance from line to line of the grating. Suppose monochromatic light to fall normally upon the grating. As the transmitted light falls upon a screen YZ, that from CD will reach a central region as O in the same phase as that from EF; the same is true of AB and GH, and O is light. At a certain distance from O, as at N_1, the light arriving from C by diffraction along CN_1 travels a distance shorter than that from E along EN_1 by the distance EK. This applies to the light from all points between C and D, as compared with that between E and F. If this difference in path, EK, is a half wave length, there will be interference in a limited space about N_1 and therefore darkness. Further on, however, as at M_1, where the difference between

CM_1 and EM_1 is a whole wave length, there will be light, succeeded by interference at N_2 where the difference in paths is three (or any odd number of) half wave lengths. A similar succession of light and dark bands occurs on the other side of O — light or dark according as the waves from C and E are in the same or opposite phases. They will be in the same phase if EN and CN differ by any integral number of wave lengths, i.e., if $EK = n\lambda$ $\left(\text{or } 2n\frac{\lambda}{2}\right)$. They will be in opposite phases and produce interference if $EK = (2n + 1)\frac{\lambda}{2}$.

If the distance CE is called d and the angular deviation N_1LO is θ, $EK = d \sin \theta$. There will be light along YZ at the places where $d \sin \theta = 2n\frac{\lambda}{2}$, and darkness where $d \sin \theta = (2n + 1)\frac{\lambda}{2}$.

Since the regions along YZ, where violet light comes, are nearer the center than where red light comes, we see that the wave length of violet is less than that of red. Moreover, by measuring θ and knowing d, the value of λ is calculable for any color. The construction applies equally to every point between A and B in connection with its corresponding point between C and D.

By measuring the distances ON_1 and LN_1, since $\frac{ON_1}{LN_1} = \sin \theta$, we have, supposing N_1 to be the first band of the color to be measured, from the center,

$$\frac{EK}{EC} = \frac{\lambda}{d} = \frac{ON_1}{LN_1}.$$

Interference will be produced by reflection of light from two surfaces if the reflected waves differ in phase by half a period.

Experiment No. 98, page 354. — Determination of wave length by means of a grating.

245. Polarization. — If transverse waves of various lengths and periods were sent along a cord, and the cord itself were being rotated, the vibrations at one place would be horizontal, at another vertical, and elsewhere in any intermediate angular posi-

tion. Not only so, but at any given point in the cord, the vibrations would not continue in one fixed direction. If such a line of vibrating particles could be viewed end-on, the vibratory motion at any instant within a limited length of the line would present a complex pattern. Fig. 74*b*, Art. 113, is a simple example.

Suppose the vibrating cord at one place passed between two boards; only the motion in the plane of the boards could be maintained, and beyond the boards the vibrations would be reduced to motion in this plane. If, here, the cord had to pass between two other boards in a plane at right angles to that of the motion, the vibrations would be cut off (see Fig. 163*a*, *b*). (See also, Thomson, *Light, Visible and Invisible*, p. 114.)

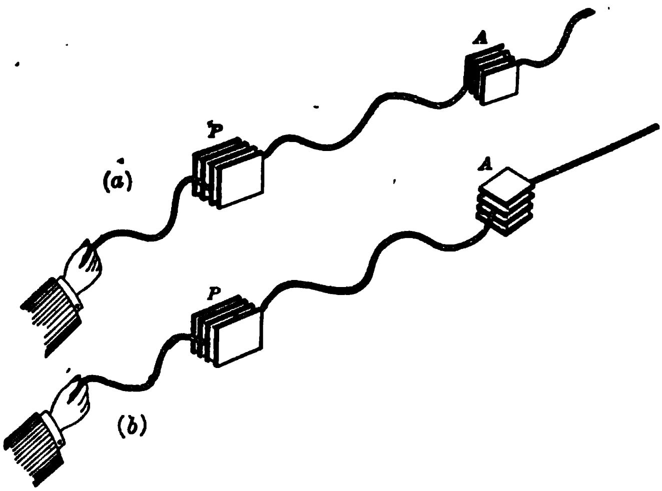


Fig. 163. Transmission and Interception of Waves.

Something analogous to this occurs with light in passing through certain crystalline media, or undergoing reflection under certain conditions.

In general, when action which is not limited as to its direction becomes restricted to some particular direction, this restriction is termed *polarization*, and the body or agent whose action is thus restricted is said to be polarized. Thus, a piece of iron or steel

becomes polarized by magnetization; a relay electromagnet is further polarized because its action is restricted to a current passing in only one direction; light in which the vibrations are restricted in direction is polarized light.

To continue the above illustration, if the vibrations were not those of a single line of particles, but presented to the barrier of boards a wave front of large width, polarization would occur by passing the vibrations through a pile of boards slightly separated, or even through a grating of parallel bars; a second grating with its plane parallel to the first and its bars in the same direction would not further impede the passage of the polarized waves, but if the second grating had its plane parallel to the first and its bars at right angles to those of the former it would intercept the vibrations polarized by the first grating. These two conditions are shown in the above Fig. 163 (*a*) and (*b*).

The second grating permits the polarized waves to pass through it when it is in one position, but intercepts them when it is turned through an angle of ninety degrees. The first one of these two gratings is termed a polarizer and the second an analyzer, marked *P* and *A*, the purpose of the latter being to test whether waves through the first or any other medium are polarized. When the vibrations are reduced to one plane it is called plane polarization.

When ordinary light falls obliquely upon a plate of glass, the light that is reflected and also that which is transmitted is partly polarized, and when examined with an analyzer the reflected light is found to be polarized in a plane at right angles to that in which the transmitted light is polarized. By using a pile of a dozen or more thin plates, a larger portion of the light is reflected and the polarization of both the reflected and the transmitted portions is more marked. The angle of incidence for which the polarization is most nearly complete is the angle whose tangent equals the index of refraction of the substance. That is, for common glass with an index of refraction of 1.5 the incident ray should make an angle with the normal of about 56°, since the tangent of 56° is very nearly 1.5. In Fig. 164 is shown the result of such polarization. *GG* is the glass plate, the incident light is

in the plane ION, perpendicular to the plane of GG, the incident ray making an angle ION of 56°. Suppose the plane of GG to be perpendicular to that of the paper, then the vibrations in the reflected ray OR are perpendicular to the plane of the paper, while those in the transmitted ray OT are parallel to the plane of the paper, but in each case transverse to the direction of propagation of the light. Those in IO are in all directions in planes perpendicular to IO.

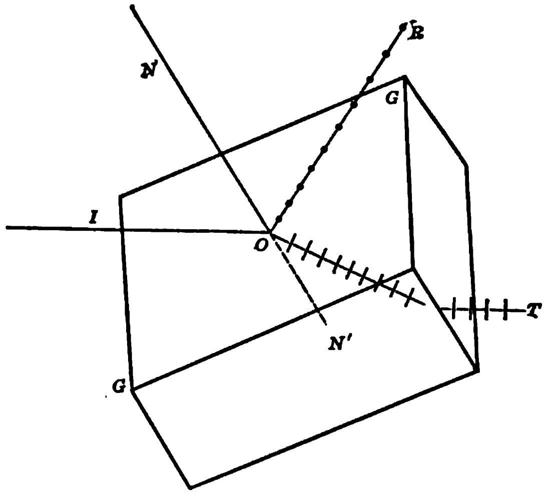


Fig. 164. Polarization of Reflected and Transmitted Ray of Light.

The mineral tourmaline has such a laminar structure that light passing through it is plane polarized, and a second crystal placed in the path of light that has passed through one such crystal transmits the light when in one position but extinguishes it when rotated through 90°.

Polarized light, being simpler in its character than unpolarized, becomes a means of testing some physical or chemical properties of bodies. For example, some substances, as mica, or a solution of sugar, when placed in the path of a beam of polarized light, cause the plane of polarization to rotate through a certain angle. If light is passed through a polarizer and the analyzer is set so as to extinguish the light, and if then a cell containing a solution of sugar is inserted between the polarizer and the analyzer, the latter no longer wholly cuts off the light but will do so when rotated through an angle depending on the nature of the substance inserted and the thickness of it through which the polarized light

passes. Not only so, but some substances cause rotation of the plane of polarization in one direction and others in the opposite direction. Thus cane sugar causes right hand rotation, while sugar from fruits produces left hand rotation. For an account of the many and beautiful phenomena of polarized light the student must refer to more extended works.

The polarization of light was inexplicable until the adoption of the idea, due to Fresnel, that light waves are the result of *transverse vibrations*.

246. Double Refraction. — Besides the means already mentioned for the polarization of light, it is effected by the passage of light through any substance that causes double refraction.

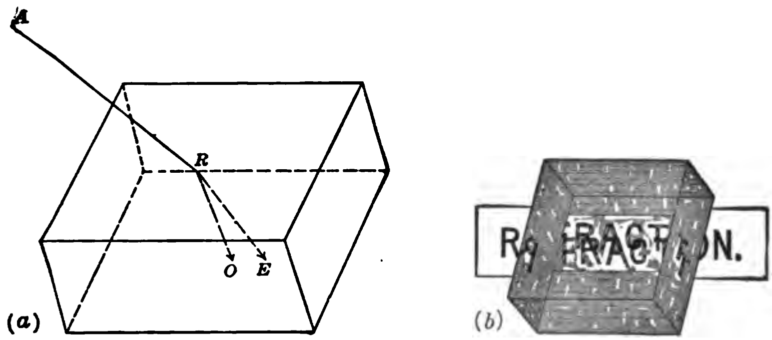


Fig. 165. Double Refraction by Iceland Spar.

If a ray of ordinary light, as AR (Fig. 165*a*), falls upon the face of a crystal of calcite (Iceland spar), it is divided in its passage through the crystal, one portion, as RO, deviating in direction from AR by refraction at an angle whose sine bears a constant ratio to the sine of the angle of incidence. That is, it is refracted in the ordinary way and is called the *ordinary* ray. The other portion RE deviates from the direction AR by a different angle from that made by RO and usually lies in a different plane from ARO. The part RE is called the *extraordinary* ray. Its index of refraction is not constant but may range from zero to a maximum value, depending on the nature of the crystal (which might be something other than calcite) and on the direction of the

incident light relatively to the principal axis of the crystal (to be explained below). This division of the ray is called double refraction, and both portions of the incident light, after emergence from the crystal, will give an image of the source of light, so that an object viewed through the crystal is seen double, and both the ordinary and the extraordinary ray are plane polarized, the vibrations in the one being at right angles to those in the other. Fig. 165*b*.)

There is one direction in which light passes through the crystal without being thus divided. If the crystal were cut with all its edges equal and the natural angles retained, the equilateral rhomb thus formed would have two solid angles at opposite ends of a diagonal, each inclosed within three obtuse plane angles. The diagonal from one of these obtuse solid angles to the other is a principal axis, and any line parallel to this is an optic axis. Along this direction through the crystal, i.e., along any optic axis, the ordinary ray and the extraordinary ray coincide.

A crystal of calcite may be divided and the two portions again cemented together with Canada balsam in such manner that the more refracted, i.e., the ordinary ray, meets the surface of the balsam at an angle so obtuse as to be totally reflected within the prism, while the extraordinary passes through, and thus the crystal transmits a beam of plane polarized light. The crystal so prepared is one of the most common and best forms of polarizer and is known as a Nicol's prism. It may be used either as a polarizer or an analyzer. (Exhibit.)

The explanation of double refraction is an extension of that of single refraction. The latter is understood to be due to the fact that light is propagated at different velocities in the two media, Art. 228. Now the crystalline structure is probably such that light does not travel through it in all directions at the same rate. If light were examined as proceeding from any point within the crystal the progress along the ordinary ray in one plane would give a circular wave front, and since the velocity is constant for the ordinary ray, the complete wave front is spherical, as in any isotropic medium. For the extraordinary ray, in

any one plane, the wave front is elliptical, and for all directions in the crystal the wave front is the surface of a spheroid.

The larger proportion of transparent crystalline bodies, and any transparent substances not isotropic in structure, are double-refracting. This is notably so with glass in a state of strain. Light passing through such bodies is polarized, and when examined with an analyzer displays beautiful color phenomena of interference, accompanying its transmission or extinction as the analyzer is rotated.

Experiment No. 99, page 355. — Double refraction.

247. Fluorescence. — The color of an object is determined by the light which it sends to the eye. If a translucent object is exposed to white light, a portion of the light will be transmitted, a portion will probably be absorbed (and its energy will thereby be transformed) and a portion will be reflected.

If the object is viewed by the transmitted light it will have the color due to the wave lengths in the transmitted light; if it is viewed by reflected light, the wave lengths of the light reflected will determine its color. These may be different from those transmitted.

There are some substances of such structure that light falling upon them produces, at the surface at least, a molecular vibration of a period different from that of the incident light. Then, from the surface of the substance, light of a color different from the incident and the transmitted light proceeds. This emission of light of a period different from that causing it is termed *fluorescence*. Most commonly the fluorescence is of longer waves than the exciting light. It occurs in many instances by exposing the fluorescent substance to the violet or ultra-violet part of the spectrum of white light. Thus, in this nearly or actually invisible part of the spectrum of very short waves, objects may become visible, emitting light of greater wave length and of different colors according to their individual nature. The substance absorbs the energy of the incident light and gives it out again in a lower order of vibration. But a given substance does

not so convert every order of radiation; it will fluoresce only in response to a certain portion of the spectrum. The effect is shown by viewing the substance by transmitted and then by reflected light. For example, petroleum of various degrees of refinement will transmit colors from a pale amber to deep red or dark brown, and reflect various tints of blue, or green, or pink.

Experiment No. 100, page 355. — Exhibition of fluorescence.

248. **Phosphorescence.** — Closely related to the phenomenon of fluorescence is that of phosphorescence. After white light has illuminated certain substances, these glow in the dark, emitting light of a color (i.e., wave length), such as the substance, by selective absorption from the incident light, is in condition to radiate. This property is most characteristic of the sulphides of calcium, barium and strontium, though it is shown to a less degree by many other substances. The phosphorescence continues in some of them for several hours before it quite dies out; in others it persists for only a few seconds.

It is thought that fluorescence is in fact phosphorescence that lasts only a very brief length of time, perhaps only a minute fraction of a second. (See S. P. Thompson, *Light; Visible and Invisible*, pp. 174-176; The MacMillan Co.)

EXPERIMENTS TO ILLUSTRATE CHAPTER VI.

Experiment No. 89, Art. 233. Total Reflection.

Total reflection of light is beautifully shown by the "illuminated jet." A vessel 40 or 50 cm. high, as in *Fig.* 166, has an orifice at *R* about a centimeter in diameter, opposite to which, at *A*, is a lens that concentrates upon *R* the light from the condenser *L* of the lantern. The vessel is filled with water, and when *R* is opened the light from *L* entering the jet illuminates it, but the light strikes the sides of the jet on the inner surface at so large an angle that it cannot emerge into the air but is continuously reflected internally. The whole curved jet is thus luminous though the space about it is dark. A brilliant spot of light appears on the bottom of the vessel where the jet impinges.. The effect is varied by inserting glass of various colors between *L* and *A*.

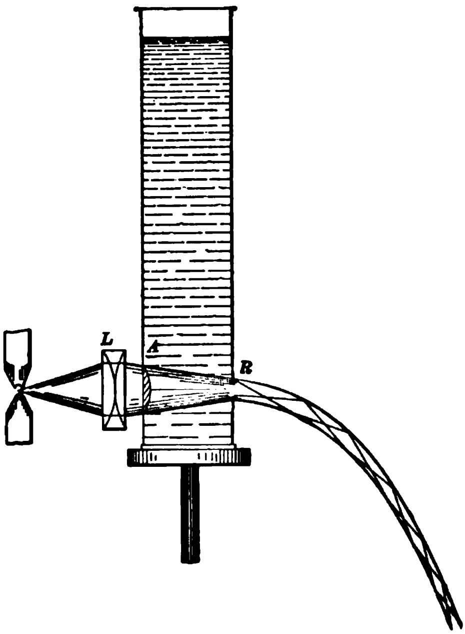


Fig. 166. Luminous Jet.

Experiment No. 90, Art. 237. Reflection and Refraction.

The optical disk is a circular disk against the face of which may be held sectional mirrors or lenses. At the edge of the disk is mounted a thin plate containing one or more slits through which light proceeds to the reflector or refractor, as shown in Fig. 167.

Unless a beam of direct sunlight can be utilized, parallel light may be had by placing the projecting lens (objective) of the lantern in the path of the light from the lantern, at a distance from the focus to which the light is

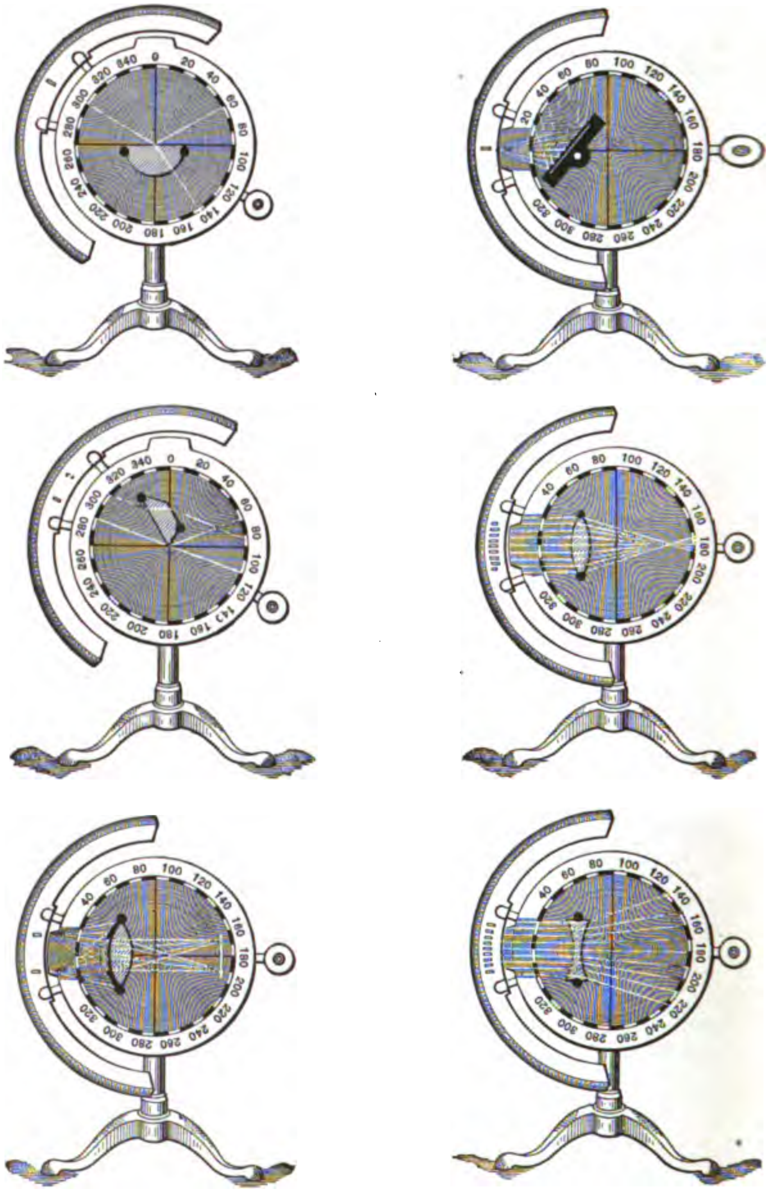


Fig. 167. The Optical Disk, illustrating various Phenomena of Reflection and Refraction.

brought by the condenser, equal to the focal length of the objective. With such a beam of light the optical disk shows all the principal phenomena of reflection and refraction.

Experiment No. 91, Art. 238. Nos. 91 and 92 illustrate Photometry.

The Bunsen photometer has a spot, made by grease or melted paraffin, on a piece of white paper. With a light behind the paper the spot looks bright, with the light in front the spot looks dark; with a light behind and one in front at such relative distances as to illuminate the spot equally from both sides, the spot does not show in contrast with the paper, and then the lights are as the squares of their distances from the spot.

Experiment No. 92, Art. 238.

The Rumford photometer employs an opaque rod in front of a white surface, and the lights to be compared are placed so as to cast shadows of the rod beside each other on the white surface. The shadow from either light would be quite dark but for its illumination by the other light. When the lights are placed at such distances that the shadows are equally intense, these are equally illuminated, and the distance is measured from each light to the shadow it shines upon (not to the shadow it casts). The lights then are to each other as the squares of their respective distances from the surface they illuminate.

Experiment No. 93, Art. 239. The Spectrum formed by a Prism.

Project upon the screen the image of a narrow slit. Hold a prism of transparent substance against the outer face of the projecting lens, the edge of the prism parallel to the slit. The spectrum is formed by deviation of the light through a large angle toward the base of the prism. By turning the prism about an axis parallel to the slit, a position of the spectrum is found such that with further turning of the prism in either direction the spectrum moves so as to increase the deviation of the light. The prism is then in the position of minimum deviation. Prisms of various materials may be used, and of various angles, to show differences in refraction and dispersion. It will be found that with a glass prism of an angle of 90° the light entering one of the faces forming the right angle, perpendicularly, will not emerge through the hypotenuse upon which it strikes but be totally reflected from it as from a mirror, and emerge normally through the other face of the prism.

Experiment No. 94, Art. 239. Continuous and Discontinuous Spectra.

In a projecting lantern with an arc light, separate the carbons so as to make an arc, say, 6 mm. long. Remove the projecting lens and place an

opaque card with a narrow horizontal slit at the focus of the arc as formed by the condenser. If the arc has been placed near the condenser its image will be considerably magnified, and the slit may be placed so as to receive upon it the image of the glowing positive carbon, as at S_1 (Fig. 168), and accordingly be illuminated by an incandescent solid. Project the image of

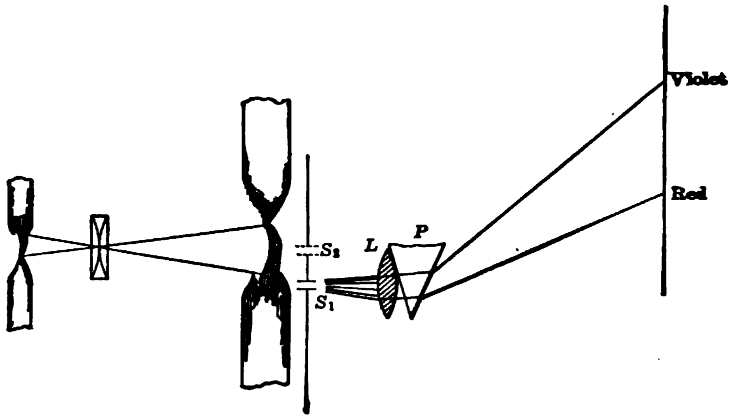


Fig. 168. Projection of Spectrum. (The enlarged carbons and arc represent simply the image of the real carbons and arc, at the left.)

the slit upon the screen by the lens L. By placing the prism P in the light immediately beyond L, the spectrum is shown on the screen in vertical position ranging *continuously* from red to violet.

If the slit be raised so as to receive upon it the image of the arc, as at S_2, it is illuminated by the luminous gas between the carbons, and when L and P are again adjusted so as to form the spectrum from S_2 upon the screen, this is found to consist of bands of various colors intermitted by dark spaces between them. They are separate images of the slit, and the spectrum is *discontinuous*. If a carbon is employed such as is used for the flaming arc lights, the spectrum of the salts with which the carbon is impregnated is shown in brilliant bands from the arc, but the spectrum of the solid is still seen to be continuous.

Experiment No. 95, Art. 226 and Art. 243.

Newton's Rings may be shown in light from any source with the usual combination of plane and curved glasses clamped together. They may be projected by placing them near the focus of the lantern condenser, as in Fig. 169, and then focusing the contact spot S in the reflected light, so as to produce the image of the rings upon the screen as at I. It is best to have a black paper or cloth behind the glass, at B.

Experiment No. 96, Art. 243. Monochromatic Light.

The spectrum of sodium vapor illuminating a slit 2 or 3 mm. in width is a single band of yellow; if the slit is very narrow the spectrum is two yellow strips slightly separated. The light of sodium, then, is only yellow and is one of the best examples of monochromatic light. If a strip of asbestos packing, or even ordinary cloth, is wrapped around the upper part of a Bunsen burner so as to project a little way above the top of the burner, and is soaked in salt water, the flame of the burner will give a strong yellow sodium light. In light of any one color, objects will appear either of that color or none. In the sodium light everything is black or of a pallid hue; objects of various bright colors will all look black except in so far as yellow is a constituent in their color. If not black, therefore, they will range through various tints from a pale to a bright yellow. The complexion of the lecturer and of the spectators has a ghastly hue.

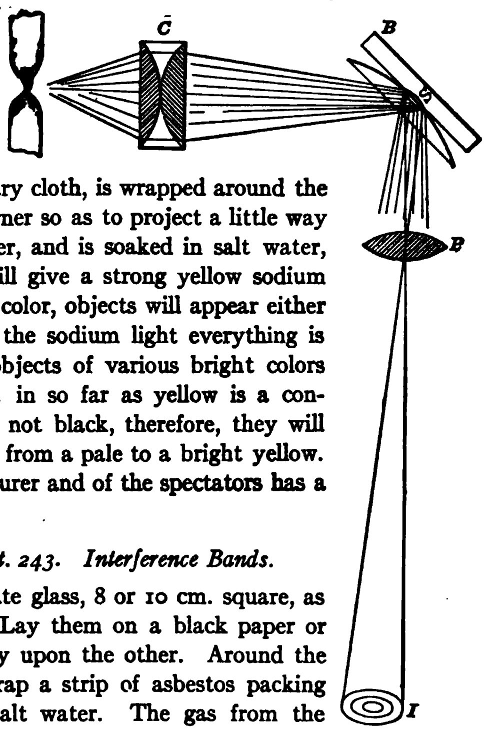


Fig. 169. Projection of Newton's Rings.

Experiment No. 97, Art. 243. Interference Bands.

Obtain two pieces of plate glass, 8 or 10 cm. square, as nearly plane as possible. Lay them on a black paper or cloth, the one fitting snugly upon the other. Around the top of a Bunsen burner wrap a strip of asbestos packing that has been soaked in salt water. The gas from the burner will then give a strong yellow flame that is the light of sodium and is virtually monochromatic.

Its reflection from the glass surfaces on the table shows brilliant bands of yellow and black, straight and uniform if the surfaces are plane, or varying in figure according to the irregularity in thickness of the air film between the glass surfaces. Pressing the glasses together makes the film thinner and the bands correspondingly broader. In this, as in all experiments with light of diminished intensity, the effect is better the darker the room.

To project this, remove the objective and prop the lantern in an inclined position so that the light from the condenser falls upon the plates on the table at a large angle to the normal. Place the objective or any projecting lens in the reflected light and focus the plates on the screen. If the contact between the plates is *sufficiently close* the bands will appear in various colors from ordinary light.

In this, however, as in many other special cases of projecting, the objective of the lantern fails to utilize a good deal of the light that is available, and all is needed that can be had. A convex lens 4 or 5 inches in diameter, such as is sold by opticians as a reading glass, can readily be clamped in any position, and will often serve better than the regular lantern objective.

The effect in this experiment is improved by using monochromatic light. If the lantern has an arc lamp, a fairly good sodium light can be obtained by drilling out the core of one carbon to a depth of about 2 cm. and packing the bore full of fine salt. Make this the lower carbon, and preferably the positive. Then with a moderately long arc the light is chiefly yellow, and the interference bands are yellow and black.

Experiment No. 98, Art. 244. Measurement of Wave Length of Light by Means of a Grating.

If the light falling upon the grating comes from an illuminated slit, as in Fig. 170, which is projected upon the screen YZ, then by placing a

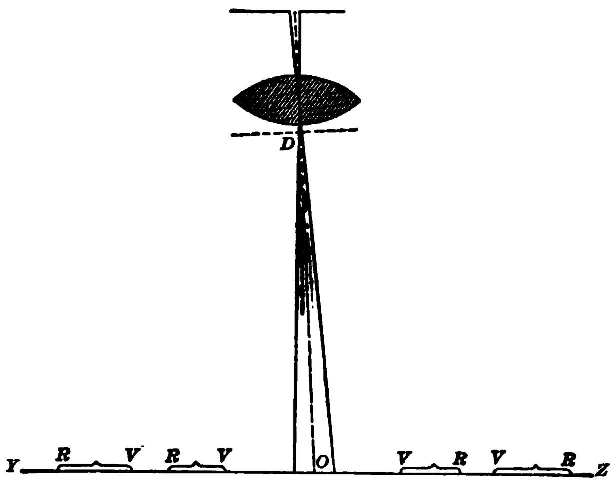


Fig. 170. Spectrum formed by a Grating.

transmission grating ruled with several thousand lines to the inch in front of the projecting lens at D, there will be a white image of the slit at O, containing most of the light passing through the grating, with a series of spectra on each side, that are the colored images resulting from diffraction. If λ_v is the wave length of violet light, and d the distance between the lines of the grating, measure OV and DV; then

$$\frac{\lambda_v}{d} = \frac{OV}{DV}, \text{ or } \lambda_v = d\frac{OV}{DV},$$

and the wave length for any other part of the spectrum may be determined in the same manner.

Experiment No. 99, Art. 246. Double Refraction.

In the slide holder of the lantern place a card or opaque plate with an orifice about 2 mm. in diameter. Focus this on the screen and place a crystal of calcite before the orifice; the image of the orifice on the screen is double. Rotate the crystal and observe the displacement of the images. When the direction of the light through the crystal coincides with the optic axis, only one image is seen.

Experiment No. 100, Art. 247. Fluorescence.

In a well darkened room project as bright a spectrum as possible. In various parts of the spectrum place fluorescent substances as described below and observe their color.

(*a*) A dilute solution of sulphate of quinine in water slightly acidulated with sulphuric acid is colorless by transmitted light, but is a rich blue in the ultra-violet part of the spectrum, and in nearly every color of the spectrum.

(*b*) Green leaves thoroughly macerated in ether or alcohol give a green solution of chlorophyl that is red when placed in violet light, and in the other parts of the spectrum except the extreme red.

(*c*) Eosin (a few drops of red ink will answer), in water, produces a red solution that fluoresces.

(*d*) A solution of thallene shows a rich green in all parts of the spectrum beyond the yellow on the violet side.

(*e*) Petroleum, showing yellow or brown by transmitted light, fluoresces blue or green.

These may all be exhibited by placing them in ordinary test tubes and holding them in the path of the light forming the spectrum. (For preparation and exhibition of these substances, see *Light*, by Lewis Wright, Macmillan Co.)

PHYSICAL DATA

For quantities of variable values a mean is given. Where possible the figures are correct to wi upon temperature and pressure, the figures are for 0° C., and 76 cm. barometer pressure u generally accepted as authoritative.

	Specific Gravity, Density, Grams per cc., Water at 4° C. = 1.	Elasticity; Dynes per Sq.cm. For Solids, Young's Modulus; for Fluids, Bulk Modulus.	Melting Point, ° C.	Boiling Point, ° C.	Specific Heat, Cals. per Gm. per Degree C.	Coeffici of Expa sion p Degree Linear Solids Cubic for Flui
Aluminum	2.65	$7.05\times10''$	657	....	.202	.00002
Beeswax..	.95	....	62	....	....	....
Brass....	8.4	10.0	....	....	.094	.00001
Carbon, diamond	3.51	....	....	....	....	....
Carbon, graphite	2.25	....	....	....	....	....
Catgut...	1.5	.32	....	....	....	....
Copper...	8.94	12.3	1050	....	.095	.00001
Cork.....	.24	....	....	....	....	....
Glass, crown	2.5	7.0	....	....	.2	.00000
Glass, flint	2.9–4.5	....	....	....	....	....
Gold.....	19.32	8.0	1250	....	.0324	.000014
Ice......	.916	....	0	....	.5	.000051
India rubber	.94	.05	....	....	....	....
Iron.....	7.86	21.0	1600	....	.112	.000012
Lead.....	11.36	1.62	325	1470	.032	.000028
Paraffin..	.88	....	52.4	....	.694	....
Platinum.	21.5	16.8	1700	....	.03	.000008
Silver....	10.57	7.9	....	....	....	....
Sulphur..	2.07	....	115	440	.184	.000064
Tin......	7.29	5.43	231	....	.056	.000021
Zinc.....	7.1	9.0	420	930	.094	.000029

IOUS COMMON SUBSTANCES

enth of one per cent; in most instances, to a higher degree of accuracy. For values depending
rwise stated. Compiled from Landolt and Börnstein, Smithsonian, and other tables that are

hermal nductivity. Cals. er Sec. rough a Plate 1 Sq.cm. ross-section, 1 Cm. ng; 1° C. Dif. in Temp. between Ends.	Latent Heat of Fusion; Calories per Gram.	Velocity of Sound; in Solids and Liquids, $\sqrt{\frac{\text{Elasticity}}{\text{Density}}}$; in Gases, $\sqrt{\frac{C_p \text{ Pressure}}{C_v \text{ Density}}}$ Meters per Sec.	Liberated from Solution by Current of 1 Ampere, Grams per Second.	Sp. Resist., Resistance of Conductor 1 Sq.cm. Cross-section, 1 Cm. Long. Ohms.	Refractive Index of Light for Mean D Line; into Solids and Liquids from Air; into Gases from Vacuum.	E.M.F. of Battery Cells, Volts.
.48	76.8	5160	.000094	2.67×10^{-6}		Clark
....	42.3					Standard
.20	....	3460	....	8.5		1.434
....	....	....	....	....	2.42	Daniell (or
						gravity),
....	....	....	....	3000.		1.08
....	....	1460				
.92	42.0	3710	.000329	1.6	0.641	Dry, 1.5+
.00072						
						Leclanché,
.0017	....	5292	....	....	1.5	(sal ammo-
						niac), 1.48
....	....	....	....	....	1.58–1.9	
.75	....	2035	.000681	2.2	0.366	Weston
.004	80.0	....	....	....	1.31	Standard
						1.018
....	....	730				
.161	....	5170	....	10.0		Storage,
.084	5.9	1195	....	20.0		lead plates
....	35.1					2.2
.166	27.2	2796	.001012	11.0	2.06	
....	....	....	.001118	1.47		Storage,
....	9.37					Edison,
.152	14.0	2728	....	11.0		(iron-nickel),
.265	28.13	3560	.000339			1.1

PHYSICAL DATA FOR VARIC

For quantities of variable values a mean is given. Where possible the figures are correct to wit upon temperature and pressure, the figures are for 0° C., and 76 cm. barometer pressure ur generally accepted as authoritative.

	Specific Gravity, Density, Grams per cc., Water at 4° C. = 1.	Elasticity; Dynes per Sq.cm. For Solids, Young's Modulus; for Fluids, Bulk Modulus.	Melting Point, ° C.	Boiling Point, ° C.	Specific Heat, Cals. per Gm. per Degree C.	Coefficient of Expansion per Degree C.; Linear for Solids; Cubical for Fluids.	Therr Condu ity, C per S throug Plate Sq.ci Cross-tion, 1 Long; 1 Dif. Tem betw End
Alcohol, ethyl	.792	.1031×10″	....	78.3	.615	.00108	.000
Alcohol, methyl	.810	....	....	66.3	.613	.00136	
Carbon disulphide	1.292	....	....	46.1	.24	....	.000
Ether....	.736	....	....	35.5	.517	.00163	.....
Glycerine	1.26	....	....	290	.576	.00053	.000
Mercury..	13.596	5.415	−39	356	.0335	.00018	.015
Olive oil..	.92	....	....	....	....	....	.....
Turpentine	.87	....	....	156	.467	.00105	.000
Water....	.9998	.2045	....	100	1 at 15° C.	.00013	.001
Air......	.001293	For gases, elasticity isothermally equals the pressure; adiabatically, $\frac{C_p}{C_v}\times$pressure.	....	−191	C_p, .237 C_v, .168	.00367	.000
Ammonia	.000771		− 75	−38. 5	....	....	.....
Carbon dioxide	.001977		....	−78.2	C_p, .217 C_v, .172	....	.000
Chlorine..	.00317		−102	−33.6	C_p, .121 C_v, .093	....	.....
Hydrogen	.0000899		−257	−253	C_p, 3.409 C_v, 2.411	.00367	.0003
Nitrogen .	.001251		−210	−195	C_p, .244 C_v, .173	.00367	.0000
Oxygen ..	.001429		−227	−182	C_p, .218 C_v, .155	.00367	.0000
Steam.... (100°)	.000596		....	......	C_p, .481 C_v, .37		

)MMON SUBSTANCES—*Continued*

-tenth of one per cent; in most instances, to a higher degree of accuracy. For values depending
erwise stated. Compiled from Landolt and Börnstein, Smithsonian, and other tables that are

atent leat of iporiza-ion, at imp. of Boiling Point, Calories r Gram.	Maximum (Saturation) Pressure of Vapor in Mm. of Mercury Column.	Velocity of Sound; in Solids and Liquids, $\sqrt{\frac{\text{Elasticity}}{\text{Density}}}$; in Gases, $\sqrt{\frac{C_p}{C_v}\frac{\text{Pressure}}{\text{Density}}}$ Meters per Sec.	Liberated from Solution by Current of 1 Ampere, Grams per Second.	Refractive Index of Light for Mean D Line; into Solids and Liquids from Air; into Gases from Vacuum.
02.	0°, 12.7° 20°, 44.5 78°.3, 760	1140	....	1.309
83.8	127.9	....	....	1.643
90.0	0°, 18.4 20°, 432.8 35°.5, 760	....	....	1.357
....	....	....	....	1.47
62.0	.004	2020	....	1.73
....	....	....	....	1.476
69.0	2.1	....	....	1.47
536.0	−5°, 3.13 0°, 4.6 10°, 9.17 20°, 17.39 50°, 91.98 100°, 760	1432	....	1.333
51.	....	331.2	....	1.000292
2.95	....	....	....	1.000377
86.7	....	200.0		
67.0	....	212.0	.000368	1.000768
200.0	....	1268.0	.0000104	1.000138
48.0	....	337.8	....	1.000297
51.0	....	315.0	.0000832	1.000272

WAVE LENGTH OF RADIATION; CMS.

Solar Spectrum. (End of One Color is Beginning of Next.)

Ultra violet, beginning	2948×10^{-8}
Violet, beginning	3970
Indigo, beginning	4240
Blue, beginning	4550
Green, beginning	4920
Yellow, beginning	5750
Orange, beginning	5850
Red, beginning	6470
Infra red, beginning	8100
Infra red, ending	27000
Sodium flame:	
mean D line	5893
X-rays	1
Wireless telegraphy	100–5000m
Fraunhofer lines, cms.	
A, Oxygen, dark red	7594×10^{-8}
B, Oxygen, dark red	6867
C, Hydrogen, red	6563
D_1, Sodium, orange	5896
D_2, Sodium, orange	5890
E, Iron, green	5270
F, Hydrogen blue	4861
G, Iron, indigo	4308
H, Calcium, violet	3968

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14 DAY USE

RETURN TO DESK FROM WHICH BORROWED

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REC'D LD	
MAR 3 [illegible]	